Module 8 : Numerical Relaying I : Fundamentals

Lecture 29 : Least Square Method for Estimation of Phasors - I

Objectives

In this lecture, we will formulate the phasor estimation problem

- In particular we will learn 2-sample approach to estimation.
- The role of noise in estimation will be brought out.

29.1 Phasor Estimation

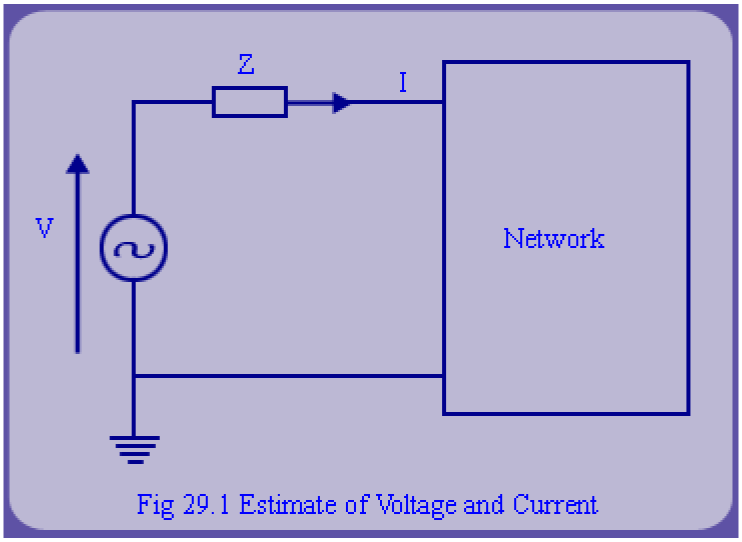

Fig 29.1 Estimate of Voltage and Current

Consider the problem of estimating the impedance seen by a relay. In a numerical relaying setup, voltage and current signals would be sampled at appropriate frequency and acquired by a micro processor or a DSP. For relaying decision making we need to estimate the voltage and current phasor. For simplicity, imagine a single phase circuit as shown below. Also assume that the frequency of the supply (e.g. 50/60Hz) is known.

The voltage waveform can be represented by

$$v(t) = V_m \sin(\omega t + \phi_v) \qquad (1)$$

and current waveform by

$$i(t) = I_m \sin(\omega t + \phi_i) \qquad (2)$$

Let us concentrate on the problem of estimating voltage phasor ($V_m \angle \phi_v$). Since there are two unknowns, minimum number of samples required in a cycle to estimate unknowns is two.

Let us assume that a sample is acquired after every Δt second. Let the first sample be obtained at t_1 and the next one at t_2. Then, voltage samples at t_1 and t_2 are given by,

$$v_1(t_1) = v_1 = V_m \sin(\omega t_1 + \phi_v)$$

$$v_1 = V_m \sin\theta_1 \cos\phi_v + V_m \cos\theta_1 \sin\phi_v$$ where, $\theta_1 = \omega t_1$ and $\theta_2 = \omega t_2$

$$v_2 = V_m \sin\theta_2 \cos\phi_v + V_m \cos\theta_2 \sin\phi_v$$

29.1 Phasor Estimation (contd..)

Henceforth, we use the convention that first sample is obtained at $t = 0$ and angle corresponding to jth sample is given by $\theta_j = j\omega_0 \Delta t$

Now, treating and as unknowns, we get

$V_m \cos\phi_v \qquad V_m \sin\phi_v$

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \sin\theta_1 & \cos\theta_1 \\ \sin\theta_2 & \cos\theta_2 \end{bmatrix} \begin{bmatrix} V_m \cos\phi_v \\ V_m \sin\phi_v \end{bmatrix} \qquad (3)$$

$$\begin{bmatrix} V_m \cos\phi_v \\ V_m \sin\phi_v \end{bmatrix} = \begin{bmatrix} \sin\theta_1 & \cos\theta_1 \\ \sin\theta_2 & \cos\theta_2 \end{bmatrix}^{-1} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$= \frac{1}{\sin(\theta_1 - \theta_2)} \begin{bmatrix} \cos\theta_2 & -\cos\theta_1 \\ -\sin\theta_2 & \sin\theta_1 \end{bmatrix} \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}$$

$$V_m \cos\phi_v = \frac{v_1 \cos\theta_2 - v_2 \cos\theta_1}{\sin(\theta_1 - \theta_2)} \qquad (4)$$

and $$V_m \sin\phi_v = \frac{v_2 \sin\theta_1 - v_1 \sin\theta_2}{\sin(\theta_1 - \theta_2)} \qquad (5)$$

It is clear from eqns. (4) and (5) that if $\theta_2 - \theta_1 < \pi$, m an integer, then $\sin(\theta_1 - \theta_2) = 0$, which implies the singularity of coefficient matrix. In such an event, it is not possible to proceed with estimation. For example, with $m = 1$, we get $\omega t_2 = \omega t_1 + \pi$. Corresponding samples are shown in fig 29.2.

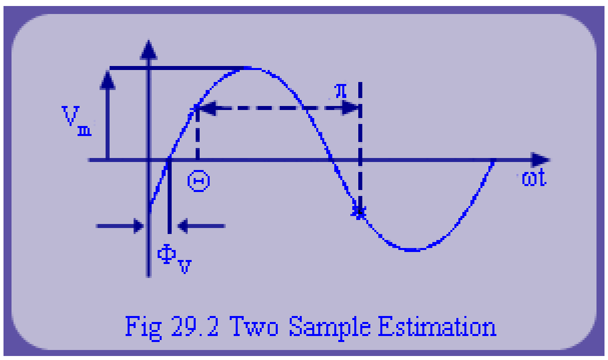

Fig 29.2 Two Sample Estimation

$\phi_v = -10^0$

This corresponds to a sampling rate of two samples per cycle. If power system frequency is f_0, this implies that sampling rate should be higher than $2f_0$, so that $\theta_2 - \theta_1 < \pi$. In fact, this is in agreement with the well known sampling theorem (seen in previous lecture).

29.1 Phasor Estimation (contd..)

Henceforth, we use the convention that first sample is obtained at $t = 0$ and angle corresponding to jth sample is given by $\theta_j = j\omega_0 \Delta t$

Now, treating $V_m \cos\phi_v$ and $V_m \sin\phi_v$ as unknowns, we get

$$\begin{bmatrix} v_1 \\ v_2 \end{bmatrix} = \begin{bmatrix} \sin\theta_1 & \cos\theta_1 \\ \sin\theta_2 & \cos\theta_2 \end{bmatrix} \begin{bmatrix} V_m \cos\phi_v \\ V_m \sin\phi_v \end{bmatrix} \qquad (3)$$

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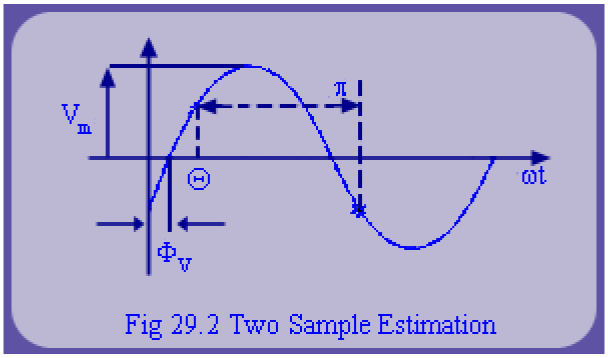

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29.1 Phasor Estimation (contd..)

Consequently V_m and ϕ_v can be computed by following equations.

$$V_m = \sqrt{V_m^2 \cos^2\phi_v + V_m^2 \sin^2\phi_v}$$

and $$\phi_v = \tan^{-1}\left(\frac{V_m \sin\phi_v}{V_m \cos\phi_v}\right)$$

A generic form of the estimation is given by the following equation.

$$V_m^k \cos\phi_v^k = \frac{v_{k-1} \cos\theta_k - v_k \cos\theta_{k-1}}{\sin(\Delta\theta)} \quad (6)$$

(7)

$$V_m^k \sin \phi_v^k = \frac{v_k \sin \theta_{k-1} - v_{k-1} \sin \theta_k}{\sin(\Delta \theta)}$$

Infact, we have replaced the sample-2 by the most recent sample k^{th} sample. Similarly sample-1 is replaced by sample $k-1$. We also say that sampling window has 2 samples per window (see fig 29.4). In any relaying application, computations have to be completed before the arrival of next sample. Note that, when uniform sampling is used with time space of Δt sec. $\theta_{k-1} - \theta_k = \Delta\theta$, a constant.

29.1 Phasor Estimation (contd..)

Example 1

Consider a signal $V(t) = 10\sin(2\pi \times 50 \times t + 30^0)$, and a sampling time of 5 msec i.e. sampling rate 4 samples per cycle. Let the sampling be initiated at $t_0 = 0$ sec.

(a) Write down the sequence of first 10 voltage samples.

Ans: The sequence is summarized below.

t msec	0	5	10	15	20	25	30	35	40	45
V	5.00	8.66	-5.00	-8.66	5.00	8.66	-5.00	-8.66	5.00	8.66

(b) Compute the voltage phasor for 100 samples

Fig 29.3 shows the estimate of voltage magnitude V_m as a function of samples. It is obvious that in the absence of noise, estimate gives the right value of V_m.
Fig 29.4 also introduces the concept of data window for estimation. This window contains the 'active' set of samples which are currently being processed for phasor estimation. In the present case, we say that we are using a 2-sample window. Each consecutive window, differs from the previous window by adding a new sample and by removing the oldest active sample.

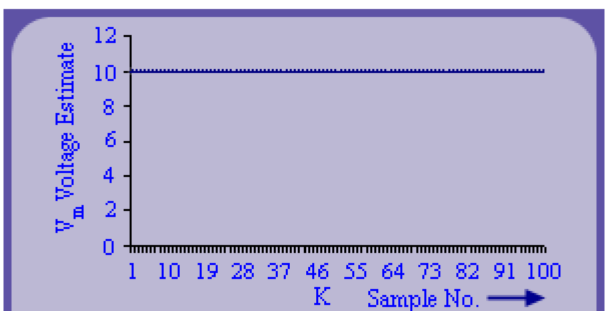

Fig 29.3 Result Estimation under Ideal Conditions

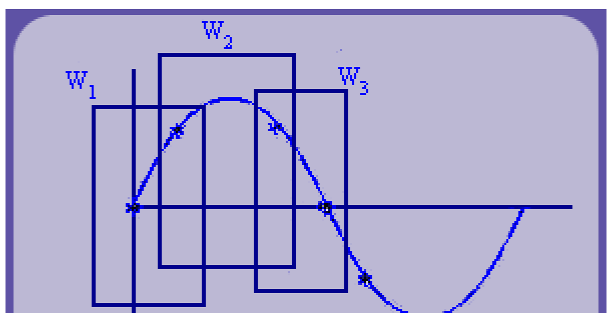

Fig 29.4 2-sample window for estimation

29.1 Phasor Estimation (contd..)

Example 2

In real life, the voltage signal will not be a perfect sinusoid. Further, transducers like voltage transformer, A/D converter etc. introduce inaccuracies which we can model as noise. The noise has a zero mean, and it's standard deviation measures the accuracy of the meter. Typically, noise is

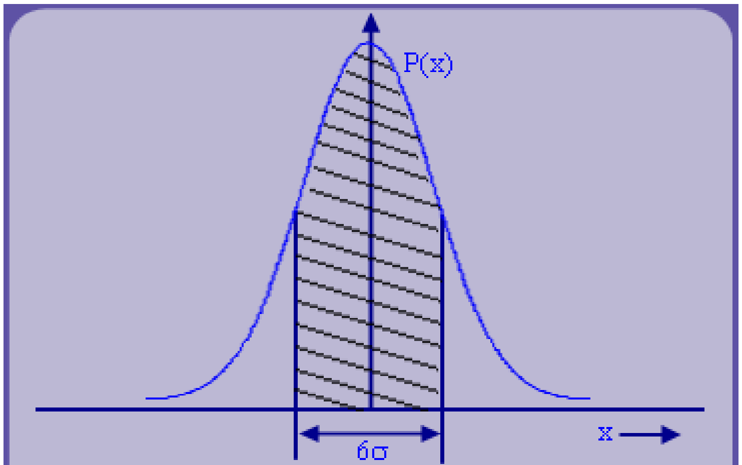

Fig 29.5 Gaussian Distribution: Sample will lie in shaded region 99% of Time

modeled by zero mean Gaussian distribution. Therefore, measurement value will be within 36 around the true value, 99% of the time. Consequently, if a 0-100V voltmeter has a standard deviation of 1%, then it implies that a measurement of a signal having magnitude of 100V will be measured any where between 97-103V, 99% of time.

For our simulations, let us model the voltage samples by

$$v(k\Delta t) = 10\sin(2\pi \times 50 \times t + 30^0) + E \bullet randn \qquad (8)$$

Function randn is obtained from a random number generator for normal distribution in MATLAB. E models the standard deviation of noise. A smaller value of E implies lower level of noise and vice versa. Now we conduct an experiment to estimate voltage phasor in presence of noise for 100 samples using 2-sample method. The mean and standard deviation of V_m for different levels of noise is shown in table 1. They are calculated as per following equations. Let X_1, X_2 ..... X_N be N samples under consideration. Then, mean

$$M = \frac{\sum_{i=1}^{N} X_i}{N}$$

and standard deviation

$$\sigma = \sqrt{\frac{\sum_{i=1}^{N} (X_i - M)^2}{N}}$$

the square of standard deviation (σ^2) is called variance.

29.1 Phasor Estimation (contd..)

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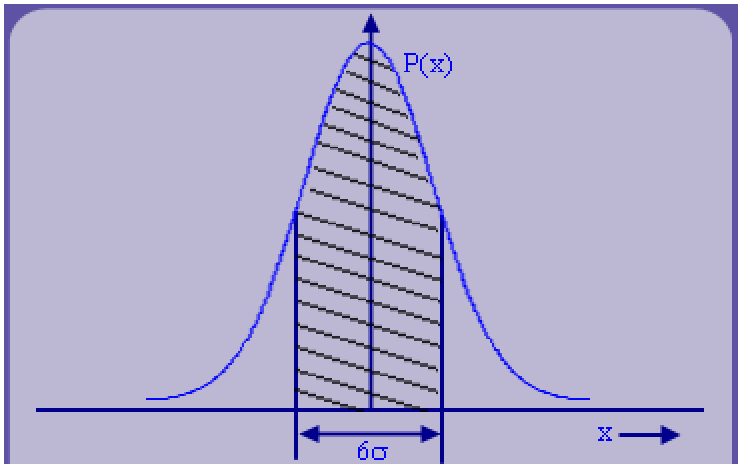

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29.1 Phasor Estimation (contd..)

Example 2
(contd..)

Table 1 : Effect of Gaussian Noise on Estimation of Phasors using 2-Sample Window

Randn multiplier(E)	Mean	Standard deviation
0.1	10.0069	0.1596
0.2	10.0162	0.3194
0.3	10.0282	0.4793
0.4	10.0426	0.6392
0.5	10.0596	0.7991
0.6	10.0791	0.9587
0.7	10.1011	1.1182
0.8	10.1256	1.2772
0.9	10.1527	1.4358
1.0	10.1824	1.5938
1.1	10.2146	1.7510
1.2	10.2495	1.9073

1.3	10.2871	2.0624
1.4	10.3275	2.2162
1.5	10.3707	2.3683
1.6	10.4169	2.5186
1.7	10.4662	2.6666
1.8	10.5188	2.8116
1.9	10.5750	2.9531
2.0	10.6346	3.0919
2.1	10.6975	3.2284
2.2	10.7638	3.3625
2.3	10.8336	3.4940
2.4	10.9065	3.6240
2.5	10.9825	3.7529
2.6	11.0613	3.8809
2.7	11.1430	4.0080
2.8	11.2277	4.1241
2.9	11.3152	4.2590
3.0	11.4055	4.3830

29.1 Phasor Estimation (contd..)

Example 2 (contd..)

Fig 29.6 shows the plot of estimated value of V_m as a function of sample number for E = 0.5.

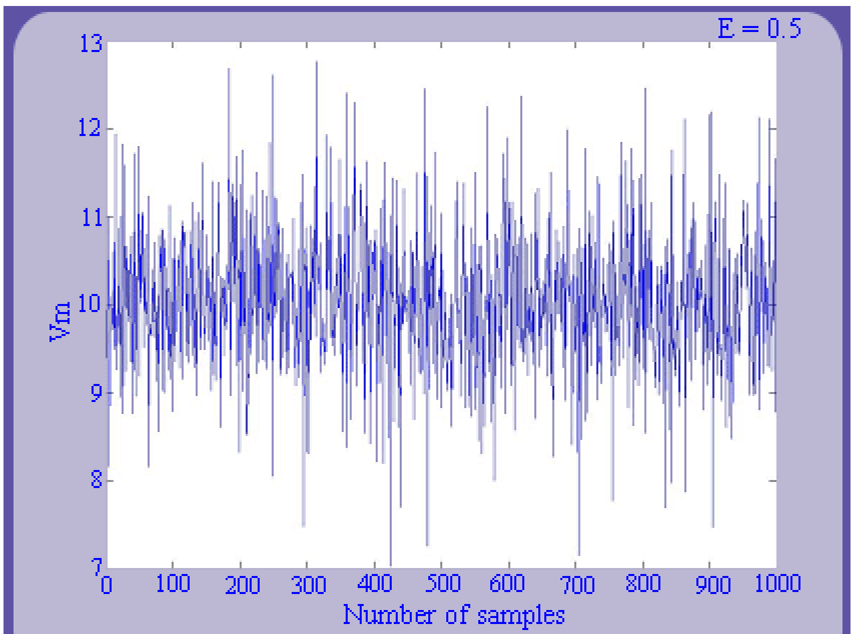

Fig 29.6 Result of Estimation in Presence of Noise

From fig 29.6 and table 1, it is evident that

1. As magnitude of zero mean noise increases, the standard deviation associated with magnitude increase.
2. Mean of V_m is nearly 10.
3. Actual estimates seldom match with 10V.

This brings out an important fact that with bare minimum number of measurements, the noise affects the accuracy. Therefore, in real-life, we have to device a procedure to filter noise and then estimate V_m and ϕ_v. To filter out noise, we need to consider redundant measurement. Redundancy in measurement is defined as ratio of actual number of measurement used for estimation to minimum number of measurement required for estimation. This issue is elaborated in the subsequent lecture.

Review Questions

1. Using MATLAB, repeat Example 1 and Example 2.

2. Derive two-sample estimation technique. Discuss it's limitation.

Recap

In this lecture we have learnt the following:

- How to approach the phasor estimation problem in numerical relaying.

- The significance of noise in estimation was brought out through 2-sample estimation method.