Module 8 : Numerical Relaying I : Fundamentals

Lecture 31 : Fourier Algorithms

Objectives

In this lecture, we will learn

- Phasor estimation using least square method.
- Half cycle and full cycle fourier algorithms.
- Frequency response of algorithms.
- Role of mimic impedance in distance relaying.

31.1 Full Cycle Fourier Algorithm

So far we have used number of sample points required in estimation method to define the length of data window. Alternatively, length of data window can be characterized by it's time span. For example, for a 3-sample data window, time span of data window is $2\Delta t$, thus, higher the sampling frequency, smaller the time span. We now consider the case when length of the data window is one cycle, though we have a freedom to choose number of samples in a window subject to the constraint N > 2.

Let the sampling frequency be such that (K>2) K samples be acquired in a cycle. With one cycle data window the eq (5) can be simplified dramatically. For example, for the first cycle (samples 0, 1, 2......K-1), LS estimation model with K samples per cycle in the data window is given by follows eqn.

$$\begin{bmatrix} \sin\theta_{K-1} & \cos\theta_{K-1} \\ \sin\theta_{K-2} & \cos\theta_{K-2} \\ | & | \\ | & | \\ \sin\theta_0 & \cos\theta_0 \end{bmatrix} \begin{bmatrix} V_m\cos\phi_v \\ V_m\sin\phi_v \end{bmatrix} + \begin{bmatrix} e_{K-1} \\ e_{K-2} \\ | \\ | \\ e_0 \end{bmatrix} = \begin{bmatrix} v_{K-1} \\ v_{K-2} \\ | \\ | \\ v_0 \end{bmatrix}$$

The solution to the LS estimate problem is given by,

$$\begin{bmatrix} \sum_{j=0}^{K-1}\sin^2\theta_j & \sum_{j=0}^{K-1}\sin\theta_j\cos\theta_j \\ \sum_{j=0}^{K-1}\sin\theta_j\cos\theta_j & \sum_{j=0}^{K-1}\cos^2\theta_j \end{bmatrix} \begin{bmatrix} V_m\cos\phi_v \\ V_m\sin\phi_v \end{bmatrix} = \begin{bmatrix} \sum_{j=0}^{K-1}\sin\theta_j v_j \\ \sum_{j=0}^{K-1}\cos\theta_j v_j \end{bmatrix}$$

Now,

$$\sum_{j=0}^{K-1}\sin\theta_j\cos\theta_j = \sum_{j=0}^{K-1}\frac{\sin 2\theta_j}{2} = \sum_{j=0}^{K-1}\frac{\sin(2\omega_0 j\Delta t)}{2}$$

Since $K\omega_0\Delta t = 2\pi \;\Rightarrow\; \sum_{j=0}^{K-1}\sin 2\omega_0 j\Delta t = \sum_{j=0}^{K-1}\sin\frac{4\pi}{K}j$ (1)

Equation (1) represent the numerical integral of $\sin 2\omega_0 t$ over a time interval of $2\pi/\omega_0$ (see exercise 1).

Since $\int_0^{2\pi/\omega_0} \sin 2\omega_0 t dt = 0$, it is not surprising to find out that above numerical integration is also zero.

Similarly, it can be shown that $\sum_{j=0}^{K-1} \sin^2 \theta_j$ and $\sum_{j=0}^{K-1} \cos^2 \theta_j$ are numerical equivalent of $\int_0^{2\pi/\omega_0} \sin^2 \omega_0 t dt$ and $\int_0^{2\pi/\omega_0} \cos^2 \omega_0 t dt$.

Consequently, they are non-zero and it can be shown that,

$$\sum_{j=0}^{K-1} \sin^2 \theta_j = \sum_{j=0}^{K-1} \cos^2 \theta_j = K/2$$

Thus, with one cycle data window, coefficient matrix in (15) becomes diagonal. Hence the equation simplifies to

$$\begin{bmatrix} K/2 & 0 \\ 0 & K/2 \end{bmatrix} \begin{bmatrix} V_m \cos \phi_v \\ V_m \sin \phi_v \end{bmatrix} = \begin{bmatrix} \sum_{j=0}^{k-1} v_j \sin \theta_j \\ \sum_{j=0}^{k-1} v_j \cos \theta_j \end{bmatrix}$$

$$V_m \cos \phi_v = \frac{2}{K} \sum_{j=0}^{K-1} v_j \sin \theta_j \quad (2)$$

where $\theta = 2\pi/K$

$$V_m \sin \phi_v = \frac{2}{K} \sum_{j=0}^{K-1} v_j \cos \theta_j \quad (3)$$

31.1 Full Cycle Fourier Algorithm (contd..)

The voltage signal $v(t) = V_m \sin(\omega t + \phi_v)$ is also represented in literature as
$V_m \cos \phi_v \sin \omega t + V_m \sin \phi_v \cos \omega t = V_s \sin \omega t + V_c \cos \omega t$

With these notation $V_s = V_m \cos \phi_v$ and $V_c = V_m \sin \phi_v$

The estimating equations (1) and (2) can be generalized for L^{th} data window as follows

$$V_s^L = \frac{2}{K} \sum_{j=L-K+1}^{L} v_j \sin \theta_j \quad (4)$$

$$V_c^L = \frac{2}{K} \sum_{j=L-K+1}^{L} v_j \cos \theta_j \quad (5)$$

Infact, these equations are identical to rectangular form of DFT to be discussed in later lectures.

Table 1 : Performance of Full Cycle Fourier Algorithm (K = 10)

Randn multiplier(E)	Mean	Standard deviation
0.1	10.0058	0.0441
0.2	10.0118	0.0882
0.3	10.0180	0.1323
0.4	10.0243	0.1764
0.5	10.0308	0.2205
0.6	10.0374	0.2646
0.7	10.0442	0.3087
0.8	10.0512	0.3528
0.9	10.0583	0.3969
1.0	10.0656	0.4409
1.1	10.0731	0.4850
1.2	10.0807	0.5290
1.3	10.0885	0.5730
1.4	10.0964	0.6170
1.5	10.1045	0.6610
1.6	10.1128	0.7050
1.7	10.1212	0.7489
1.8	10.1298	0.7928
1.9	10.1386	0.8367
2.0	10.1475	0.8806
2.1	10.1566	0.9244
2.2	10.1658	0.9682
2.3	10.1752	1.0120
2.4	10.1848	1.0558
2.5	10.1945	1.0995
2.6	10.2044	1.1432
2.7	10.2144	1.1869
2.8	10.2246	1.2305
2.9	10.2350	1.2740
3.0	10.2455	1.3176

Table 1 illustrates the results of the estimation when full cycle data window is used. It can be seen that standard deviation associated with measurement reduces even further to 1.3176 for E = 3. This should be contrasted with 3-sample data window where corresponding σ was 2.7638. This brings out an important aspect of relaying discussed earlier that accuracy of estimation is improved by increasing the length of data window. (see Exercise - 2)

Example : 1

The algorithm that we have discussed is known as **Full Cycle Fourier Algorithm**. In this example, we evaluate the capability of full cycle Fourier algorithm to filter out harmonics. Input signal corresponds to a 50 Hz square wave shown below. The harmonic spectrum of such wave form is given by

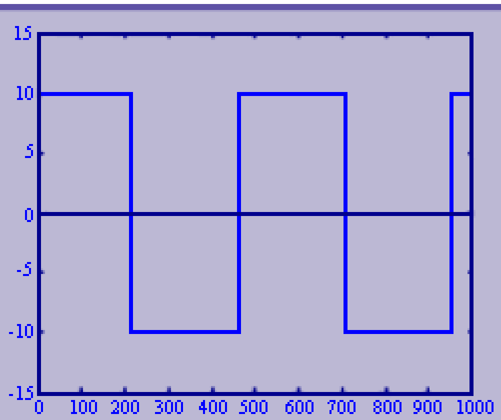


Fig 31.1 Square Periodic Wave

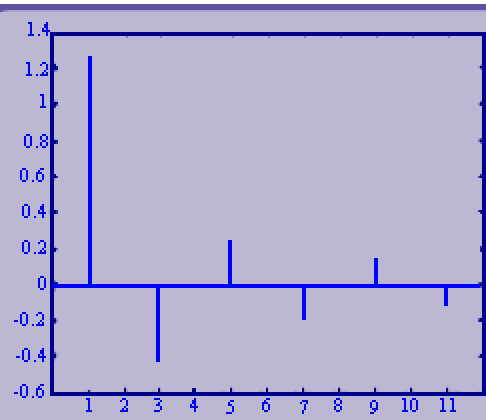


Fig 31.2 Harmonic Spectrum

This signal is sampled at a rate of 10 samples per cycle and full cycle Fourier method is applied to estimate the fundamental. In addition noise is introduced using random number generator. The true value of fundamental component is $\frac{4}{\pi} \times 10$ = 12.7324.

Example : 1 (contd..)

Table 2 : Harmonic + noise filtering capability of full cycle algorithm

Randn multiplier(E)	Mean	Standard deviation
0.1	12.9512	0.0444
0.2	12.9583	0.0888
0.3	12.9655	0.1332
0.4	12.9728	0.1776
0.5	12.9802	0.2220
0.6	12.9878	0.2664
0.7	12.9955	0.3108
0.8	13.0033	0.3552
0.9	13.0112	0.3996
1.0	13.0193	0.4440
1.1	13.0275	0.4884
1.2	130358	0.5328
1.3	13.0442	0.5772
1.4	130527	0.6216
1.5	13.0614	0.6660
1.6	13.0702	0.7104
1.7	13.0791	0.7548
1.8	13.0881	0.7992
1.9	13.0972	0.8435
2.0	13.1065	0.8879
2.1	13.1159	0.9323
2.2	13.1254	0.9766
2.3	13.1350	1.0209
2.4	13.1448	1.0653
2.5	13.1546	1.1096
2.6	13.1646	1.1539
2.7	13.1747	1.1982
2.8	13.1849	1.2425
2.9	13.1953	1.2867

3.0	13.2057	1.3310

Table 2 summarizes the response of full cycle algorithm in the presence of harmonics. It is seen that the full cycle algorithm also filters harmonics effectively. Note that mean and average are calculated over 100 consecutive estimation.

Example : 2

To improve speed, we can even restrict the data window to half a cycle. When this is done, we get half cycle Fourier algorithm. With K number of samples per half cycle, the relevant equations are given by (see exercise - 4)

$$V_c^L = \frac{2}{K} \sum_{j=L-K+1}^{L} v_j \cos\theta_j \qquad (6)$$

$$V_s^L = \frac{2}{K} \sum_{j=L-K+1}^{L} v_j \sin\theta_j \qquad (7)$$

Notice that our convention is that the latest sample corresponds to the window number. Therefore, first K - window are incomplete because K - samples are not available with them. To complete the incomplete windows, adequate number of zeros are padded in the beginning. Correct estimates are obtained only after $L \geq K$.

Table 3 : Performance of Half Cycle Fourier Algorithm

Randn multiplier(E)	Mean	Standard deviation
0.1	10.0058	0.0614
0.2	10.0119	0.1228
0.3	10.0183	0.1842
0.4	10.0251	0.2456
0.5	10.0322	0.3070
0.6	10.0396	0.3684
0.7	10.0474	0.4298
0.8	10.0555	0.4911
0.9	10.0639	0.5525
1.0	10.0727	0.6138
1.1	10.0817	0.6751
1.2	10.0912	0.7364
1.3	10.1009	0.7977
1.4	10.1110	0.8589
1.5	10.1214	0.9201
1.6	10.1321	0.9813
1.7	10.1431	1.0424
1.8	10.1545	1.1034
1.9	10.1662	1.1645
2.0	10.1783	1.2254
2.1	10.1906	1.2863
2.2	10.2033	1.3472
2.3	10.2163	1.4080
2.4	10.2297	1.4687
2.5	10.2434	1.5294
2.6	10.2574	1.5899
2.7	10.2717	1.6504
2.8	10.2864	1.7108
2.9	10.3014	1.7711
3.0	10.3168	1.8314

Table 3 summarizes the performance of half cycle algorithm for the standard sinusoidal signal used in all our examples. In presence of harmonics, it can be shown that the accuracy of the algorithm is not as good as full cycle algorithm. (see example - 5)

Example : 2

To improve speed, we can even restrict the data window to half a cycle. When this is done, we get half cycle Fourier algorithm. With K even number of samples per half cycle, the relevant equations are given by (see exercise - 4)

$$V_c^L = \frac{2}{K}\sum_{j=L-K+1}^{L} v_j \cos\theta_j \qquad (6)$$

$$V_s^L = \frac{2}{K}\sum_{j=L-K+1}^{L} v_j \sin\theta_j \qquad (7)$$

Notice that our convention is that the latest sample corresponds to the window number. Therefore, first K - window are incomplete because K - samples are not available with them. To complete the incomplete windows, adequate number of zeros are padded in the beginning. Correct estimates are obtained only after $L \geq K$.

Table 3 : Performance of Half Cycle Fourier Algorithm

Randn multiplier(E)	Mean	Standard deviation
0.1	10.0058	0.0614
0.2	10.0119	0.1228
0.3	10.0183	0.1842
0.4	10.0251	0.2456
0.5	10.0322	0.3070
0.6	10.0396	0.3684
0.7	10.0474	0.4298
0.8	10.0555	0.4911
0.9	10.0639	0.5525
1.0	10.0727	0.6138
1.1	10.0817	0.6751
1.2	10.0912	0.7364
1.3	10.1009	0.7977
1.4	10.1110	0.8589
1.5	10.1214	0.9201
1.6	10.1321	0.9813
1.7	10.1431	1.0424
1.8	10.1545	1.1034
1.9	10.1662	1.1645
2.0	10.1783	1.2254
2.1	10.1906	1.2863
2.2	10.2033	1.3472
2.3	10.2163	1.4080
2.4	10.2297	1.4687
2.5	10.2434	1.5294
2.6	10.2574	1.5899
2.7	10.2717	1.6504
2.8	10.2864	1.7108
2.9	10.3014	1.7711
3.0	10.3168	1.8314

Table 3 summarizes the performance of half cycle algorithm for the standard sinusoidal signal used in all

our examples. In presence of harmonics, it can be shown that the accuracy of the algorithm is not as good as full cycle algorithm. (see example - 5)

31.1 Full Cycle Fourier Algorithm

So far we have used number of sample points required in estimation method to define the length of data window. Alternatively, length of data window can be characterized by it's time span. For example, for a 3-sample data window, time span of data window is $2\Delta t$, thus, higher the sampling frequency, smaller the time span. We now consider the case when length of the data window is one cycle, though we have a freedom to choose number of samples in a window subject to the constraint N > 2.

Let the sampling frequency be such that (K>2) K samples be acquired in a cycle. With one cycle data window the eq (5) can be simplified dramatically. For example, for the first cycle (samples 0, 1, 2......K-1), LS estimation model with K samples per cycle in the data window is given by follows eqn.

$$\begin{bmatrix} \sin\theta_{K-1} & \cos\theta_{K-1} \\ \sin\theta_{K-2} & \cos\theta_{K-2} \\ | & | \\ | & | \\ \sin\theta_0 & \cos\theta_0 \end{bmatrix} \begin{bmatrix} V_m\cos\phi_v \\ V_m\sin\phi_v \end{bmatrix} + \begin{bmatrix} e_{K-1} \\ e_{K-2} \\ | \\ | \\ e_0 \end{bmatrix} = \begin{bmatrix} v_{K-1} \\ v_{K-2} \\ | \\ | \\ v_0 \end{bmatrix}$$

The solution to the LS estimate problem is given by,

$$\begin{bmatrix} \sum_{j=0}^{K-1}\sin^2\theta_j & \sum_{j=0}^{K-1}\sin\theta_j\cos\theta_j \\ \sum_{j=0}^{K-1}\sin\theta_j\cos\theta_j & \sum_{j=0}^{K-1}\cos^2\theta_j \end{bmatrix} \begin{bmatrix} V_m\cos\phi_v \\ V_m\sin\phi_v \end{bmatrix} = \begin{bmatrix} \sum_{j=0}^{K-1}\sin\theta_j v_j \\ \sum_{j=0}^{K-1}\cos\theta_j v_j \end{bmatrix}$$

Now,

$$\sum_{j=0}^{K-1}\sin\theta_j\cos\theta_j = \sum_{j=0}^{K-1}\frac{\sin 2\theta_j}{2} = \sum_{j=0}^{K-1}\frac{\sin(2\omega_0 j\Delta t)}{2}$$

Since $K\omega_0\Delta t = 2\pi \Rightarrow \sum_{j=0}^{K-1}\sin 2\omega_0 j\Delta t = \sum_{j=0}^{K-1}\sin\frac{4\pi}{K}j$ (1)

Equation (1) represent the numerical integral of $\sin 2\omega_0 t$ over a time interval of $2\pi/\omega_0$ (see exercise 1).

Since $\int_0^{2\pi/\omega_0}\sin 2\omega_0 t\,dt = 0$, it is not surprising to find out that above numerical integration is also zero.

Similarly, it can be shown that $\sum_{j=0}^{K-1}\sin^2\theta_j$ and $\sum_{j=0}^{K-1}\cos^2\theta_j$ are numerical equivalent of $\int_0^{2\pi/\omega_0}\sin^2\omega_0 t\,dt$ and $\int_0^{2\pi/\omega_0}\cos^2\omega_0 t\,dt$.

Consequently, they are non-zero and it can be shown that,

$$\sum_{j=0}^{K-1}\sin^2\theta_j = \sum_{j=0}^{K-1}\cos^2\theta_j = K/2$$

Thus, with one cycle data window, coefficient matrix in (15) becomes diagonal. Hence the equation simplifies to

$$\begin{bmatrix} K/2 & 0 \\ 0 & K/2 \end{bmatrix} \begin{bmatrix} V_m \cos\phi_v \\ V_m \sin\phi_v \end{bmatrix} = \begin{bmatrix} \sum_{j=0}^{k-1} v_j \sin\theta_j \\ \sum_{j=0}^{k-1} v_j \cos\theta_j \end{bmatrix}$$

$$V_m \cos\phi_v = \frac{2}{K}\sum_{j=0}^{K-1} v_j \sin\theta_j \qquad (2)$$

where $\theta = 2\pi/K$

$$V_m \sin\phi_v = \frac{2}{K}\sum_{j=0}^{K-1} v_j \cos\theta_j \qquad (3)$$

TOP

We conclude this lecture by summarizing the effect of length of data window on delay in post fault estimation of voltage and current signals.

31.2 Issues Related to Fault Current Estimation

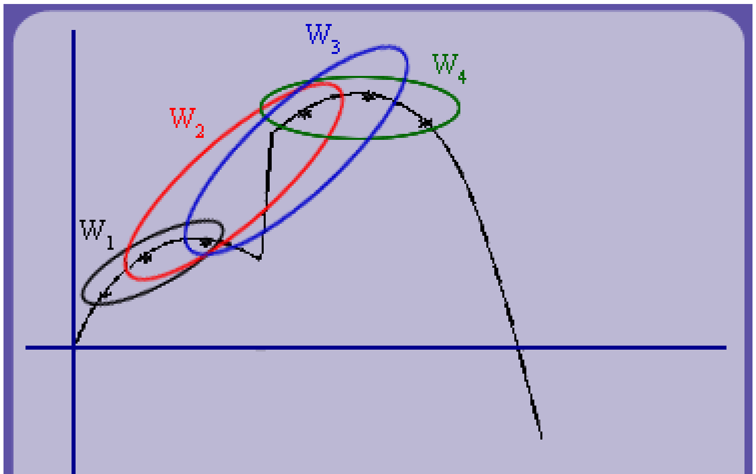


Fig 31.3 Data Window during Fault

Fig 31.3 shows pre-fault to post-fault current waveform. A 3-sample full cycle data window is considered. The window W_1 contains only pre-fault data. Thus it can be used to correctly estimate the pre-fault current. The first post-fault sample is seen in data window W_2. Window W_2 contains one post-fault current sample and two pre-fault current samples. Hence it does not correspond to either pre fault or post fault phasor. Hence, it's estimation is completely erroneous. When, we reach window W_4, we find that it is populated completely with post fault data. Consequently, it's phasor estimated corresponds to the post fault phasor.

Thus, the delay introduced in measuring post-fault signal is equal to the length of data window. Thus, 1 cycle data window introduces a delay of 1 cycle in estimation. It is likely that CT may be driven into saturation by DC offset current. While $1/2$ cycle window will reduce accuracy of estimation, with it's use one can strike a compromise between the problem of CT saturation and improving accuracy of estimation.

The next example considers the effect of delaying DC offset current of the fundamental on estimation.

Example : 3

Consider a current signal which does not have noise but it has DC offset. This represents fault current on an unloaded system.

$$i(t) = 10\sin(2\pi\times 50\times t - 30^0) + 5e^{-t\times 2\pi\times 50/10}$$

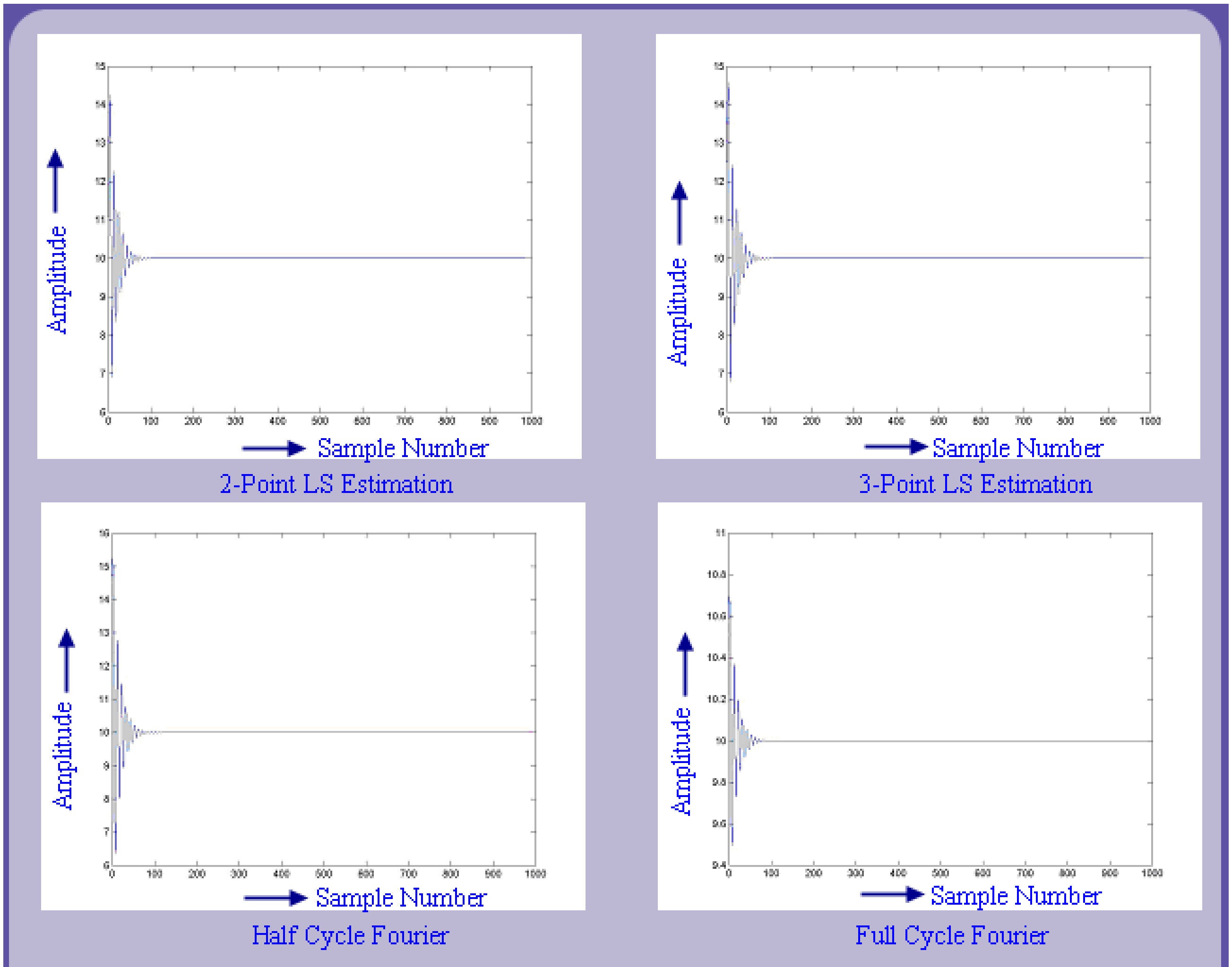


Fig 31.4 Effect on Dc-offset Current on Estimate of Fundamental

Fig 31.4 show the estimated magnitude of I_m, measured for 5-fundamental cycles using 2-point, 3-point $\frac{1}{2}$ cycle and full cycle Fourier algorithms. It can be seen that, significant errors are seen in all estimation methods. Also, accuracy of full cycle fourier algorithm is seen to be the most accurate algorithm. The reason is quite obvious. Even if we view DC offset current as noise, it is apparent that it does not have a zero mean. Thus, least square based estimation algorithms are expected to fail under such situations.

One way out of this imbroglio is that we should use some other filtering method for dc offset current. This is usually achieved in hardware by what is known as mimic impedance. (refer Q. 7)

31.3 Mimic impedance :

Mimic impedance is an impedance whose X/R ratio is identical to the X/R ratio of transmission lines. In that sense, it mimics a transmission line.

Fig 31.5 shows a current source having sinusoidal component and dc offset current connected to the $R+jX$ impedance. The sinusoidal voltage developed across the impedance is given by

$$v(t) = R_1 i + L_1 \frac{di}{dt}$$

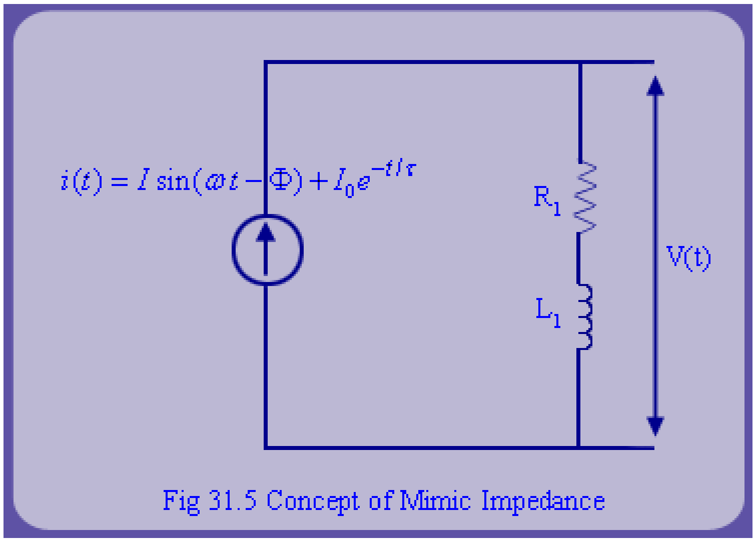


Fig 31.5 Concept of Mimic Impedance

$$= R_1 I \sin(\omega t - \phi) + \omega L_1 I \cos(\omega t - \phi) + R_1 I_0 e^{-t/\tau} - \frac{L}{\tau} I_0 e^{-t/\tau}$$

$$= |Z_1|\, I \sin(\omega t - \phi + \theta) + I_0 e^{-t/\tau}\left[R_1 - \frac{L_1}{\tau}\right]$$

$$= |Z_1|\, I \sin(\omega t - \phi + \theta) + L_1 I_0 e^{-t/\tau}\left[\frac{R_1}{L_1} - \frac{1}{\tau}\right]$$

where $Z_1 = R_1 + j\omega L_1 = |Z_1| \angle\theta$

One way out of this imbroglio is that we should use some other filtering method for dc offset current. This is usually achieved in hardware by what is known as mimic impedance. (refer Q. 7)

31.3 Mimic impedance :

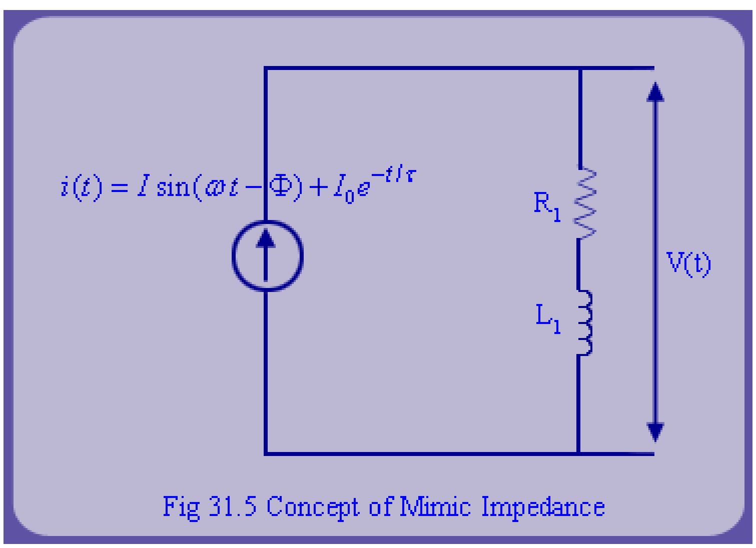


Fig 31.5 Concept of Mimic Impedance

Mimic impedance is an impedance whose X/R ratio is identical to the X/R ratio of transmission lines. In that sense, it mimics a transmission line.
Fig 31.5 shows a current source having sinusoidal component and dc offset current connected to the $R + jX$ impedance. The sinusoidal voltage developed across the impedance is given by

$$v(t) = R_1 i + L_1 \frac{di}{dt}$$

$$= R_1 I \sin(\omega t - \phi) + \omega L_1 I \cos(\omega t - \phi) + R_1 I_0 e^{-t/\tau} - \frac{L}{\tau} I_0 e^{-t/\tau}$$

$$= Z_1 I \sin(\omega t - \phi + \theta) + I_0 e^{-t/\tau}[R_1 - \frac{L_1}{\tau}]$$

$$= Z_1 I \sin(\omega t - \phi + \theta) + L_1 I_0 e^{-t/\tau}[\frac{R_1}{L_1} - \frac{1}{\tau}]$$

31.3 Mimic impedance : (contd..)

Time constant τ is the L/R ratio of the line. Now, if we choose $\frac{L_1}{R_1} = \tau$, then it is obvious that the voltage waveform is devoid of the dc offset component and is given by

$$v(t) = Z_1 I \sin(\omega t - \phi + \theta)$$

Infact this is the sinusoidal steady response for the mimic impedance circuit.
The current is scaled by magnitude $|Z_1|$ of mimic impedance and in phase by θ. Thus by an inverse operation, we get back the sinusoidal current waveform devoid of dc offset component. Filtering algorithm discussed earlier will then give satisfactory results. Mimic impedances are routinely used in distance relays used for transmission line relaying where the problem of decaying dc offset is most serious. Mimic impedance can also be implemented in software.

31.4 Frequency response of Estimation algorithms

By now we have deduced that:

1. Full cycle fourier algorithm gives the best performance in filtering harmonics and noise.
2. Half cycle does improve speed of response at the cost of accuracy.
3. Three sample algorithm is quite fast but the accuracy of estimation is poor.

Any of the above estimation algorithms can be viewed as a digital filter whose job is to extract fundamental in presence of harmonics and noise. The presentation so far was biased towards elimination of noise. Filtering of harmonic can be discussed more neatly by evaluating the frequency response of the estimation algorithms.

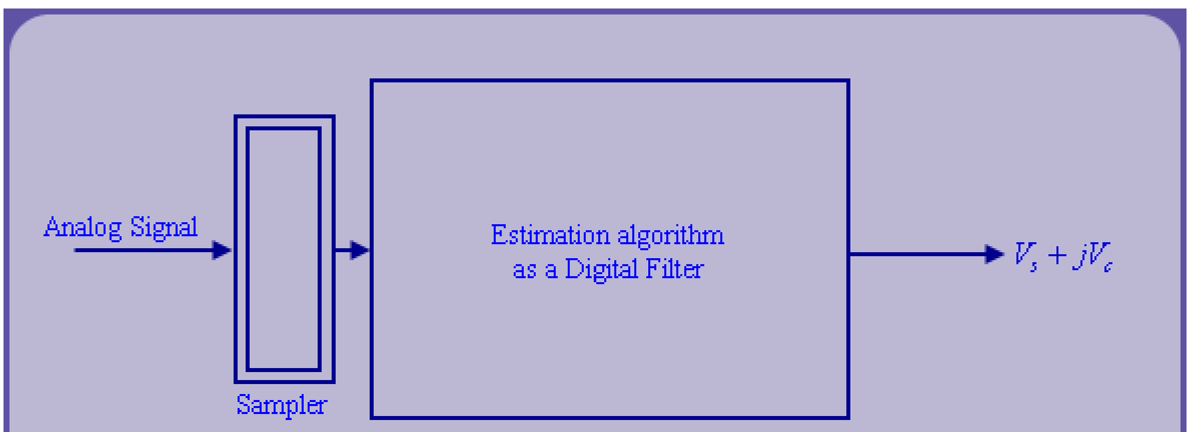


Fig 31.6 Fourier Algorithms as Filters

31.4 Frequency response of Estimation algorithms (contd..)

Input to the filter is stream of samples at frequency mf_0, m_{int} $(0, \pm 1, \pm 2, ...)$. Since, in relaying we are primarily interested in extracting the fundamental component. The output of the estimation algorithm is viewed by the relay logic as the fundamental component of the signal. Thus, if $m = \pm 1$, the output should follow input. On the other hand, if $m \neq 1$, ideally, the output should be zero.

The frequency response can be evaluated by analytical tools. However, to simplify presentation, we restrict the treatment to experimental (by simulation) evaluation of the frequency response. The frequency response for 3-sample, half cycle and full cycle algorithms are shown in fig 31.7.

Salient observations arising out of fig 31.7 are as follows:

1. Full cycle algorithm rejects dc component as well as harmonics (both even and odd) very efficiently. This can be explained by the fact that

$$\int_0^{T_0} \sin(m\omega_0 t + \phi)\sin \omega_0 t \, dt = 0 \;\&$$

$$\int_0^{T_0} \sin(m\omega_0 t + \phi)\cos \omega_0 t \, dt = 0$$

For m - an integer, greater than unity.

2. Half cycle algorithm rejects odd harmonics efficiently but not the even harmonics. This can be explained by
the fact that

$$\int_0^{T_0/2} \sin(m\omega_0 t + \phi)\sin \omega_0 t = 0 \;\&$$

$$\int_0^{T_0/2} \sin(m\omega_0 t + \phi)\cos \omega_0 t \, dt = 0$$

For m - an odd number, greater than unity.

3. 3-sample does not have good harmonic rejection properties.
4. Acharacteristic frequencies are wrongly interpreted by all algorithms as fundamental. Infact, the full cycle Fourier algorithm is identical to DFT. Therefore, it is not surprising to find out that this behavior can be explained by what is known as 'DFT leakage'. We will consider this issue in more detail in later lectures.

31.4 Frequency response of Estimation algorithms (contd..)

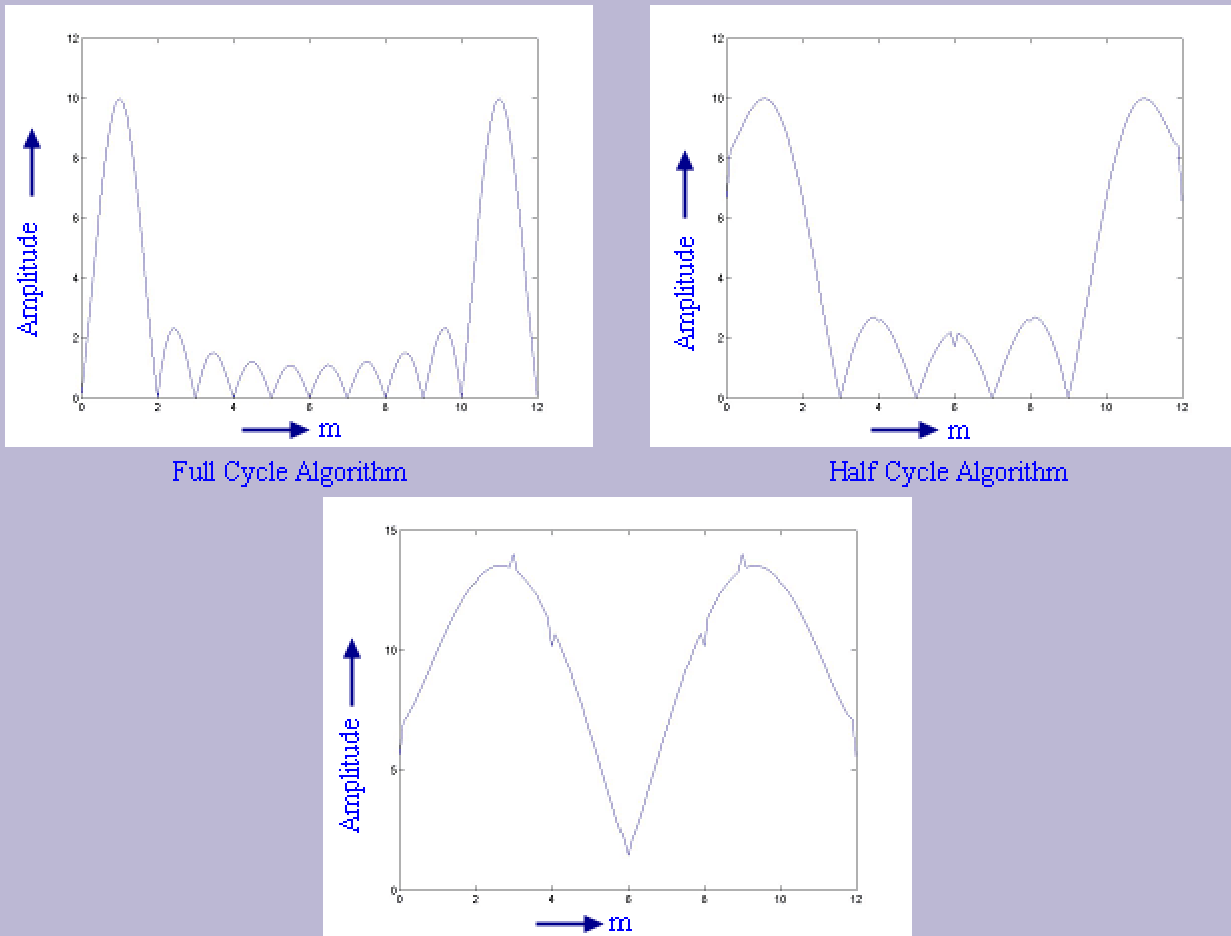
Input to the filter is stream of samples at frequency mf_0, m_{int} $(0,\pm1,\pm2,......)$. Since, in relaying we are primarily interested in extracting the fundamental component. The output of the estimation algorithm is viewed by the relay logic as the fundamental component of the signal. Thus, if $m = \pm 1$, the output should follow input. On the other hand, if $m \neq 1$, ideally, the output should be zero.

The frequency response can be evaluated by analytical tools. However, to simplify presentation, we restrict the treatment to experimental (by simulation) evaluation of the frequency response. The frequency response for 3-sample, half cycle and full cycle algorithms are shown in fig 31.7.

From the fig 31.7, it can be seen that

1. Full cycle algorithm rejects dc component as well as harmonics (both even and odd) very efficiently
2. Half cycle algorithm rejects odd harmonics efficiently but not the even harmonics
3. 3-sample does not have good harmonic rejection properties.
4. Acharacteristic frequencies are wrongly interpreted by all algorithms as fundamental. Infact, the full cycle fourier algorithm is identical to DFT. Therefore, it is not surprising to find out that this behavior can be explained by what is known as 'DFT leakage'. We will consider this issue in more detail in later lectures.

31.4 Frequency response of Estimation algorithms (contd..)



Full Cycle Algorithm

Half Cycle Algorithm

3-Sample Algorithm

Fig 31.7 Frequency Response of Various Algorithms

Review Questions

Exercise 1:

Consider evaluation $\int_0^{2\pi/\omega_0} \sin 2\omega_0 t dt$ by trapezoidal rule of integration. This is average of the second harmonic signal over 2-cycles which is known to be zero. Consider sampling this signal at rate of K - samples per cycle corresponding to fundamental frequency. The sampels are at start t = 0,.... $\frac{2\pi}{K}$ (2K-1). Now append K+1 sample at the end clearly, $\sin(2\overline{K}\theta) = 0$ and $\theta = \frac{2\pi}{K}$. Addition of this additional sample, allows us to cover one full cycle length of fundamental on x - axis. Now show that $\sum_{j=0}^{K-1} \sin \frac{4\pi}{K} j$ is numerical evaluation of this integral. Hence, deduce that $\sum_{j=0}^{K-1} \sin \frac{4\pi}{K} j = 0$. Illustrate your result geometrically.

Exercise 2:

Assuming a sampling rate of 32 samples per cycle, generate samples for a 50 Hz sinusoidal signal with $V_m = 10$ at different levels of noise. Now choose noise parameter choose E = 0.5. Now consider standard deviation of estimation obtained after 100 estimations. Plot the (curves of 6 vs K; the no. of cycles in data window) where K is varied from 1 - 4. Hence, show that increasing the length of data window reduces error. Interpret this result in terms of speed vs accuracy conflict in relaying.

Exercise 3:

Repeat exercise 2 for E = 0.1, 1, 2, 3 and 4.

Review Questions (contd..)

Exercise 4: (contd..)

Consider LS estimate of phasor using half cycle data window i.e. K-samples per half cycle at nominal frequency. Show that the estimate equations are given as below:

$$\begin{bmatrix} \sum_{j=0}^{K-1} \sin^2 \theta j & \sum_{j=0}^{K-1} \sin \theta j \cos \theta j \\ \sum_{j=0}^{K-1} \sin \theta j \cos \theta j & \sum_{j=0}^{K-1} \cos^2 \theta j \end{bmatrix} \begin{bmatrix} V_m \cos \phi_v \\ V_m \sin \phi_v \end{bmatrix} = \begin{bmatrix} \sum_{j=0}^{K-1} V_j \sin \theta j \\ \sum_{j=0}^{K-1} V_j \cos \theta j \end{bmatrix}$$

Further, show that for cycle with,

$\sum_{j=0}^{K-1} \sin \theta j \cos \theta j = 0$ and $\sum_{j=0}^{K-1} \sin^2 \theta j = \sum_{j=0}^{K-1} \cos^2 \theta j = K/2$. Hence, derive a simple expression for calculating V_s and V_c. Compare and contrast with the full cycle window results.

Exercise 5:

Evaluate fundamental component of the square wave in Example - 1 using half cycle fourier algorithm. What conclusions do you draw.

Exercise 6:

Suppose that square in Example - 1 also had a superposed dc component of 5v. Repeat Q. 5. Hence, refine your conclusions.

Exercise 7:

One way to account for decaying dc offset current during estimation of fundamental is to account for it in the signal model. Hence, consider the signal model to be $V(t) = V_m \sin(\omega_0 t + \phi) + V_0 e^{-t/\tau} + e(t)$. Assuming that time constant 'τ' is known, develop a LS method to estimate V_m, ϕ and V_0. Compare the accuracy of this method with full cycle and half cycle algorithm.

Exercise 8:

Extend full cycle algorithm to measure 3rd and 5th harmonic in a signal. Assume suitable sampling frequency.

Recap

In this lecture we have learnt the following:

- Phasor estimation using least square method.
- Half cycle and full cycle fourier algorithm.
- Frequency response of various algorithms.
- Role of mimic impedance in distance relaying.