EE 102 # Homework #3 Solutions **Fall 2001**

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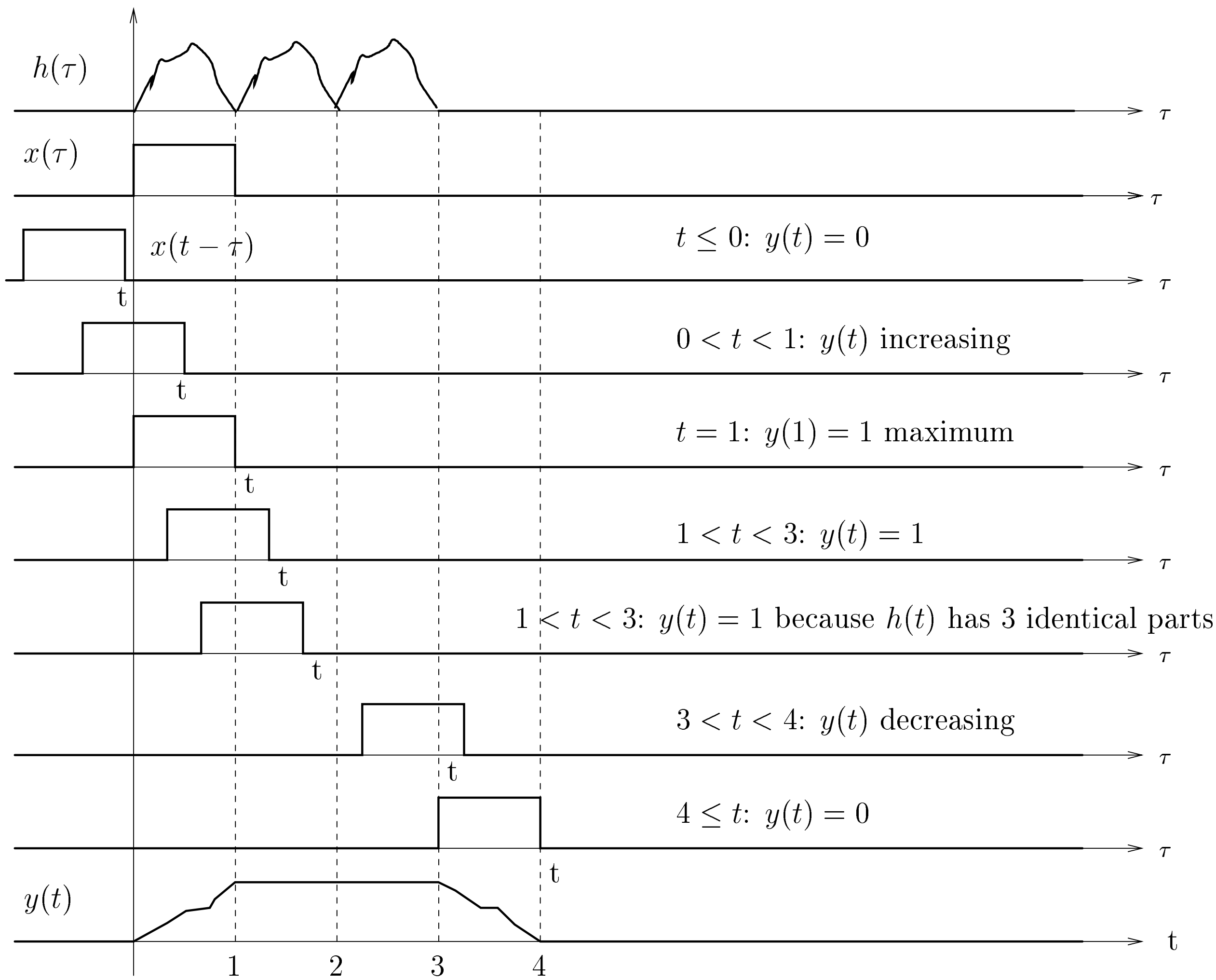
1. The convolution with the input $x(t) = u(t) - u(t-1)$ is depicted in the following figure.

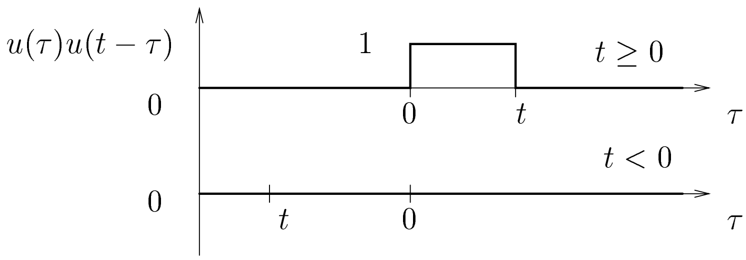


The convolution $y(t) = \int_{-\infty}^{\infty} h(t-\tau)x(\tau)d\tau$ is rewritten as $y(t) = \int_{-\infty}^{\infty} x(t-\tau)h(\tau)d\tau$ in order to get a simpler graphical convolution, where $x(\tau)$ is shifted by t.

2. For the following convolutions the evaluation of the product

$$u(\tau)u(t-\tau) = \begin{cases} 1 & \text{if } t \geq 0, \text{ for } 0 \leq \tau \leq t \\ 0 & \text{if } t < 0, \text{ for all } \tau \end{cases}$$

is helpful. $u(\tau)u(t-\tau)$ is drawn as a function of τ in the next figure.



(a)

$$u * f = \int_{-\infty}^{\infty} e^{-\tau} u(\tau)u(t-\tau)d\tau = \begin{cases} \int_0^t e^{-\tau} d\tau & \text{if } t \geq 0 \\ 0 & \text{else} \end{cases} = \underline{u(t)\left(1-e^{-t}\right)}$$

(b)

$$f * f = \int_{-\infty}^{\infty} e^{-\tau} e^{-(t-\tau)} u(\tau)u(t-\tau)d\tau$$

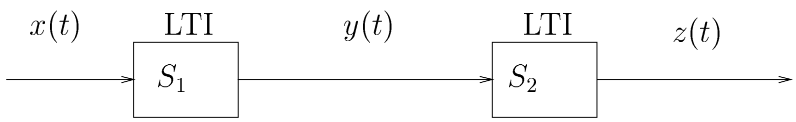
$$= e^{-t} \int_{-\infty}^{\infty} u(\tau)u(t-\tau)d\tau = \begin{cases} e^{-t} \int_0^t d\tau & \text{if } t \geq 0 \\ 0 & \text{else} \end{cases} = \underline{u(t)\ t\ e^{-t}}$$

(c)

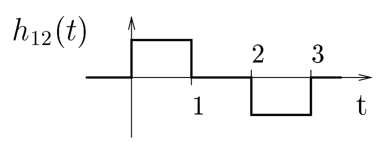
$$u * u = \int_{-\infty}^{\infty} u(\tau)u(t-\tau)d\tau = \begin{cases} \int_0^t d\tau = t & \text{if } t \geq 0 \\ 0 & \text{else} \end{cases} = \underline{u(t)t}$$

3. Cascade of Linear time-invariant (LTI) systems

As seen in class, an LTI system commutes with differentiation: that is, if we apply the derivative of a certain input, we get the derivative of the output. We use this property to derive the necessary information about the systems, as summarized in the figure below. In it we use the notation $r(t) := tu(t)$ (the "ramp" function).



line 1	$\delta(t)$	$\longrightarrow$	$u(t) - u(t-2)$		
line 2			$tu(t) = r(t)$	$\longrightarrow$	$r(t) - r(t-1)$
line 3			$\frac{dr(t)}{dt} = u(t)$	$\longrightarrow$	$\frac{dr(t)}{dt} - \frac{r(t-1)}{dt} = u(t) - u(t-1)$
line 4			$u(t-2)$	$\longrightarrow$	$u(t-2) - u(t-3)$
line 5	$\delta(t)$	$\longrightarrow$	$u(t) - u(t-2)$	$\longrightarrow$	$u(t) - u(t-1) - u(t-2) + u(t-3)$



In this picture the input of the whole system, $x(t)$, is represented in the leftmost column. In the middle column $y(t)$, the input to the second subsystem S_2 can be seen and the column on the rightmost side represents the output, $z(t)$, of the system.

The first and second lines of formulas are given in the problem, where the given graph of $z(t)$ could be found as $r(t) - r(t-1)$.

Formula line 3 makes use of the fact that the derivative of an input gives the derivative of the output. Line 4 follows by shifting, using time invariance. Line 5, which is under the horizontal bar gives the result of the linear combination of lines 3 and 4. The impulse response of the whole system is $h_{12} = u(u) - u(t-1) - u(t-2) + u(t-3)$ and is plotted.