Notes on Phasor Notation and Diagrams

Mathematically correct

V_m…Magnitude

Complex quantity ("underlined") → $\underline{\mathbf{V}} = V_m e^{j\omega t} = V_m \cos(\omega t) + jV_m \sin(\omega t)$

$$v(t) = \mathrm{Re}(\underline{V}) = V_m \cos(\omega t)$$

or $$v(t) = \mathrm{Im}(\underline{V}) = V_m \sin(\omega t)$$

Practical notation and usage

Complex quantity, but underlining omitted → $I = 10 \text{ A} \angle 45°$

RMS value instead of magnitude

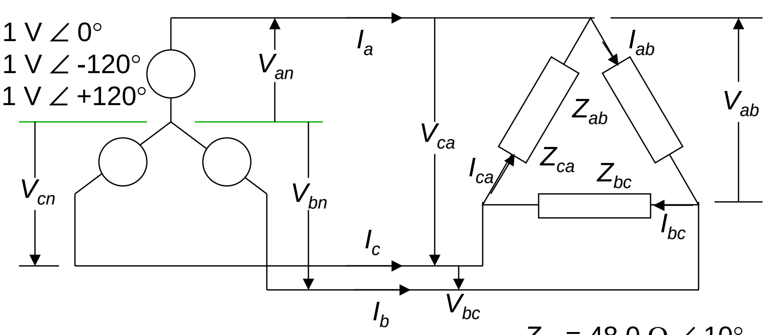
For practical use phasor diagrams can are drawn with RMS values

RMS values are given if not noted otherwise. If in doubt, ask!

Example: 3~ Balanced Circuit

For the following circuit, find all the voltage and current values and phasors and use them to find the power flowing in each part of the circuit

V_{an} = 277.1 V ∠ 0°
V_{bn} = 277.1 V ∠ -120°
V_{cn} = 277.1 V ∠ +120°



Z_{ab} = 48.0 Ω ∠ 10°
Z_{bc} = 48.0 Ω ∠ 10°
Z_{ca} = 48.0 Ω ∠ 10°

Example: 3~ Balanced Circuit

$$V_{ab} = V_{an} - V_{bn} = 277.1\angle 0° - 277.1\angle -120° = 480\angle 30°$$

$$|V_{L-L}| = 277.1 \cdot \sqrt{3} = 480V$$

$$I_{ab} = \frac{V_{ab}}{Z_{ab}} = \frac{480\angle 30°}{48\angle 10°} = 10\,A\angle 20°$$

$$I_{bc} = I_{ab} \cdot 1\angle -120° = 10\,A\angle -100°$$

$$I_{ca} = I_{ab} \cdot 1\angle 120° = 10\,A\angle 140°$$

$$I_a = I_{ab} - I_{ca} = 10\sqrt{3}\angle -30° + 20°$$

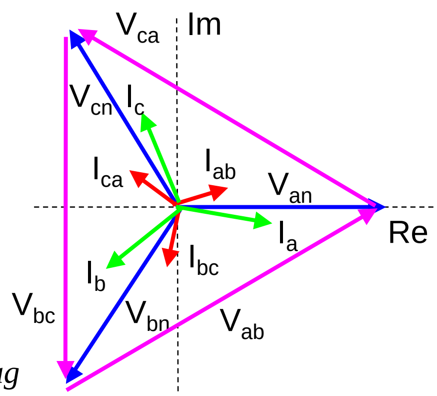
$$= 17.3\,A\angle -10°$$

$$S = 3V_p I_p^* = 277.1 \cdot 17.3\,A\angle 0 + 10°$$

$$= 4.8\,kVA\angle 10°, PF = \cos(\phi) = 0.985, lag$$

$$P = S \cdot PF = 4.8 \cdot 0.985 = 4.72\,kW$$

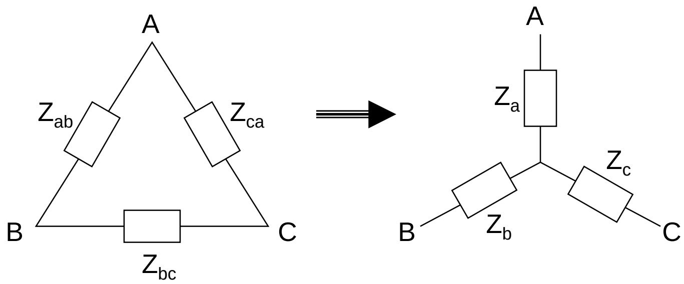
$Q = S \cdot \sin(\phi) = 0.83\,kVAr$ delivered from source to load



Simplified Analysis, Balanced System

- **Single-phase analysis can be applied to balanced three-phase systems**
- **Convert a network into a wye-connected system**
  - Convert delta connected load and sources to wye connected equivalent
  - Connect a neutral conductor between all the neutral points of the wye connected loads and sources
- **Split the network into three equal single-phase equivalents**
- **Solve one of the single-phase equivalents**
- **Combine solutions of the three single-phase equivalents. Remember to adjust the phase angles of phase b and c.**
- **Compute the line voltages and currents**

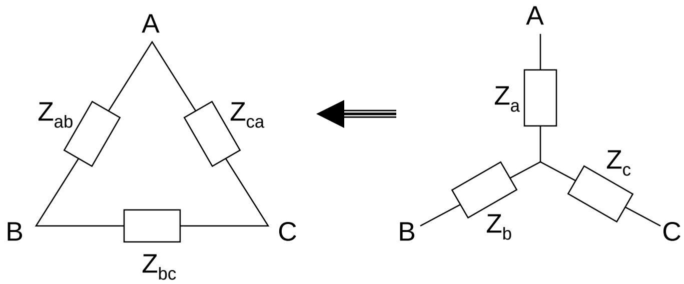
Δ-Y Transformation



$$Z_a = \frac{Z_{ab} \cdot Z_{ca}}{Z_{ab} + Z_{bc} + Z_{ca}} \qquad Z_b = \frac{Z_{ab} \cdot Z_{bc}}{Z_{ab} + Z_{bc} + Z_{ca}} \qquad Z_c = \frac{Z_{bc} \cdot Z_{ca}}{Z_{ab} + Z_{bc} + Z_{ca}}$$

For $Z_{ab} = Z_{bc} = Z_{ca}$ $$Z_Y = \frac{1}{3} Z_\Delta$$

Y-Δ Transformation



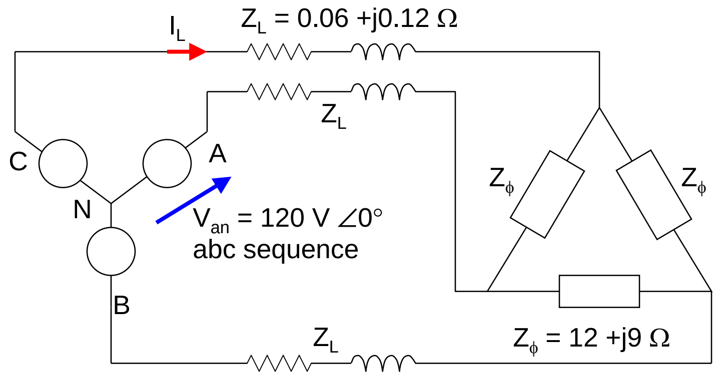
$$Z_{bc} = Z_b + Z_c + \frac{Z_b \cdot Z_c}{Z_a}$$

$$Z_{ab} = Z_a + Z_b + \frac{Z_a \cdot Z_b}{Z_c}$$

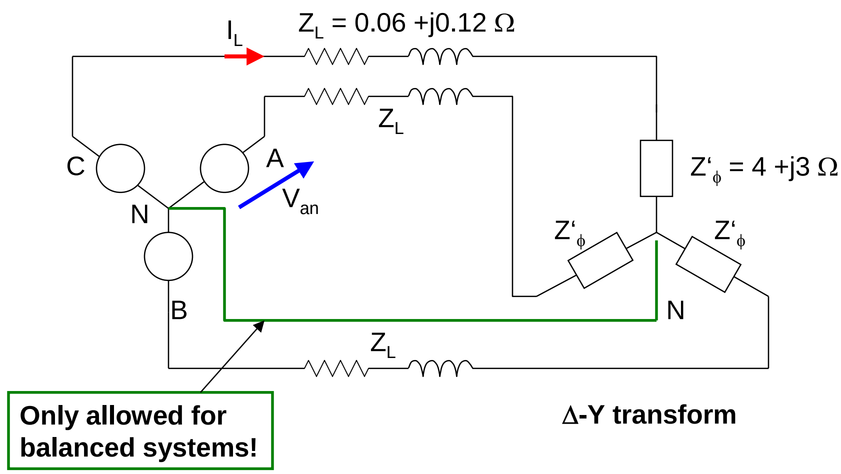
$$Z_{ca} = Z_c + Z_a + \frac{Z_c \cdot Z_a}{Z_b}$$

For $Z_a = Z_b = Z_c$ $$Z_\Delta = 3 \cdot Z_Y$$

Example: 3~ power system (book p.72, 2-2)

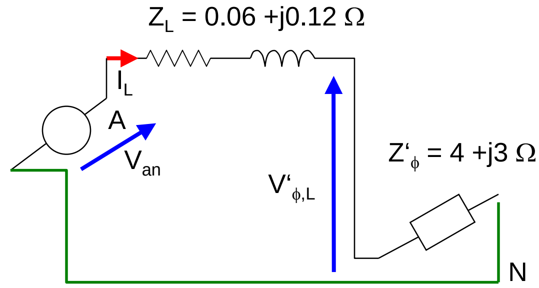


Example: 3~ power system (book p.72, 2-2)



Example: 3~ power system (book p.72, 2-2)

Single phase equivalent



$$I_L = \frac{V_{an}}{Z_L + Z_\phi'} = \frac{120V}{4.06 + j3.12\Omega} = 23.4A\angle -37.5°$$

$$V_{\phi,L}' = I_L Z_\phi' = 117V\angle -0.6°$$

Homework 2

See web site
http://www.eng.fsu.edu/~steurer/eel3216.html

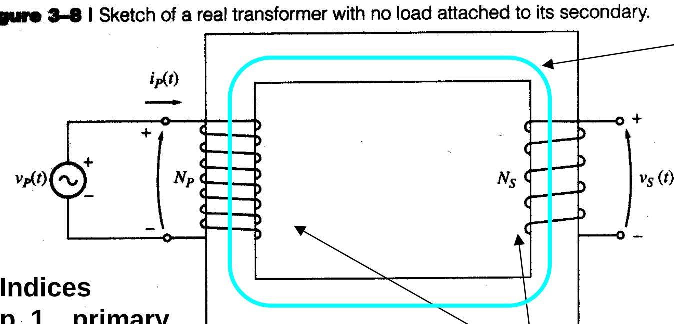
IMPORTANT:
Change of homework policy

Homework solutions will NOT be accepted after the due date (see also updated policies on web site).

Transformers

Transformer Basics

Figure 3–8 | Sketch of a real transformer with no load attached to its secondary.



Flux path give by iron core with low magnetic resistance

Indices
p, 1…primary
s, 2…secondary

Turns ratio $a = \frac{N_1}{N_2} = \frac{v_1}{v_2}$

Real transformer typically has concentric windings for better coupling

Why Power Transformers ?

- **By stepping up the voltage the current is reduced proportionally for the same power**

$$P_1 = V_1 I_1 \cos(\phi) \qquad V_2 = aV_1 \quad I_2 = \frac{I_1}{a} \qquad P_2 = V_2 I_2 \cos(\phi) = P_1$$

(ideal transformer)

- **Smaller currents require less conductor material**
  - e.g. for same losses in transmission line conductors on primary and secondary side of transformer

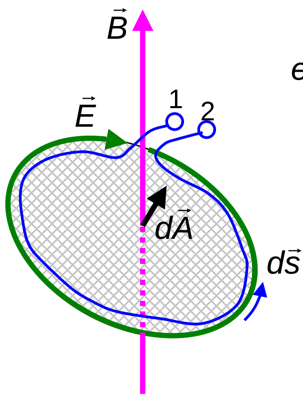
$$P_{Loss,1} = I_1^2 R_1$$

$$P_{Loss,2} = I_2^2 R_2 = \left(\frac{I_1}{a}\right)^2 R_2 = I_1^2 R_1 \rightarrow R_2 = a^2 R_1$$

Why Transformers ?

- **Galvanic isolation between sub-systems**
  - Reduce ground fault currents
- **Transition between 3 and 4 wire systems**
  - Delta-wye
- **Voltage regulation**
- **Instrumentation transformers**
  - Potential transformers (PT) to measure high voltages
  - Current transformers (CT) to measure high currents

Faraday's Law



scalar — vectors — (!) — vectors — scalar

$$e_{ind,1-2} = \oint_S \vec{E} \cdot d\vec{s} = -\frac{d}{dt}\int_A \vec{B} \cdot d\vec{A} = -\frac{d\phi}{dt}$$

$d\vec{s}$ defines direction of $d\vec{A}$

Conductor loop defines path of integration

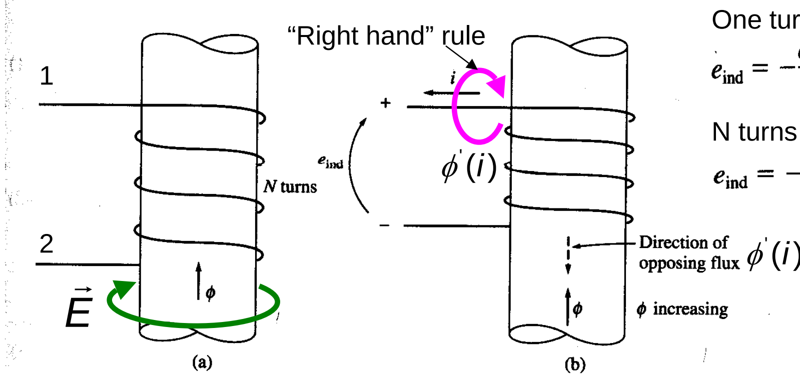
$$\frac{d}{dt}\int_A \vec{B} \cdot d\vec{A} = \int_A \frac{\partial \vec{B}}{\partial t} \cdot d\vec{A}$$

Here: for $\frac{\partial \vec{B}}{\partial t} > 0$, polarity of "1" is positive

The voltage induced in a conductor loop is proportional to the negative rate of change of the magnetic flux through that loop.

Lentz's Law

Figure 1–14 | The meaning of Lenz's law: (a) A coil enclosing an increasing magnetic flux; (b) determining the resulting voltage polarity.



One turn

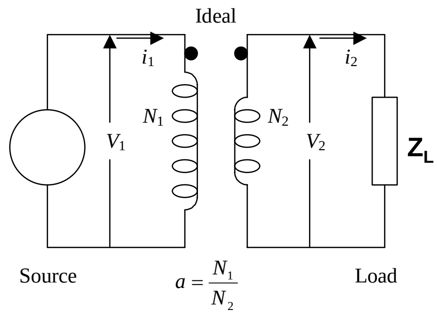
$$e_{ind} = -\frac{d\phi}{dt}$$

N turns

$$e_{ind} = -N\frac{d\phi}{dt}$$

Polarity of e_{ind} is such that if the terminals where shorted a current i would flow which creates a flux ϕ'(i) opposing the original flux ϕ.
=> minus sign often omitted for practical purpose

Ideal 1~ Transformer Model



Flux linkages (winding flux)

$$\lambda_1 = N_1\phi, \quad \lambda_2 = N_2\phi$$

$$v_1 = e_1, \quad v_2 = e_2$$

$$a = \frac{N_1}{N_2} = \frac{v_1}{v_2} = \frac{e_1}{e_2} = \frac{i_2}{i_1}$$

- **Dot convention**
  - Current into doted end produces positive MMF (N·I) or “ampere-turns”
  - Therefore, orientation of i_1 and i_2 must be as shown to cancel MMF which is necessary to maintain a finite flux in an “ideal (iron) core”