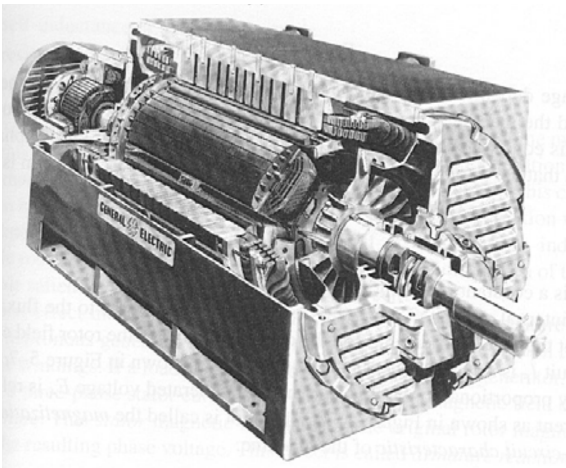
AC Machine Fundamentals

- **Generators**
- **Motors**
- **3-phase**
- **1-phase**
- **Synchronous**
- **Induction**

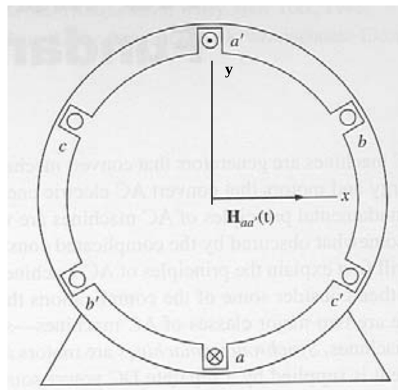


AC Machines Systematic

- **Fundamentals**
  - Principle of generating a rotating magnetic field
  - Magnetomotive force and induced voltage
  - Principle of generating a torque in machines
  - Power flow in AC machines
- **Synchronous Machines**
  - Construction and operating principle
  - Steady state model and phasor diagram
  - Single generator operation
- **Induction ("Asynchronous") Machines**
  - …

Space Vector Notation

- Three windings, aa' ,bb' cc', separated 120° mechanically carry a set of balanced three-phase currents (separated 120° electrically)
- Space vector (in spatial xy plane)
  - Represents the spatial direction of a vector quantity (H, B, possibly time dependent)
  - Not to be confused with phasor diagram (time dependent scalar in Re/Im plane)!



Excitation from winding aa' alone

Rotating Magnetic Field

Symmetric 3~ current system

$$i_{aa'}(t) = I_M \sin(\omega t)$$

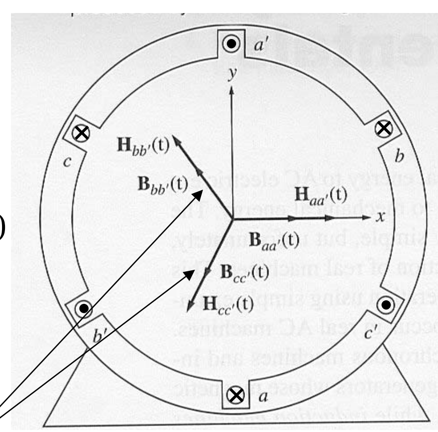
$$i_{bb'}(t) = I_M \sin(\omega t - 120°)$$

$$i_{cc'}(t) = I_M \sin(\omega t - 240°)$$

$$H(t) \propto i(t) \quad B = \mu H \quad \rightarrow \quad B(t) \propto i(t)$$

In linear region of the magnetic material (iron) the magnetic filed is proportional to the winding current

Due to spatial arrangement the three windings produce B-fields in three different directions



Magnitudes shown here are for the positive peak of the corresponding phase b and c current

Observations at ωt = 0°

$$B_{aa'} = B_M \sin(0) = 0$$

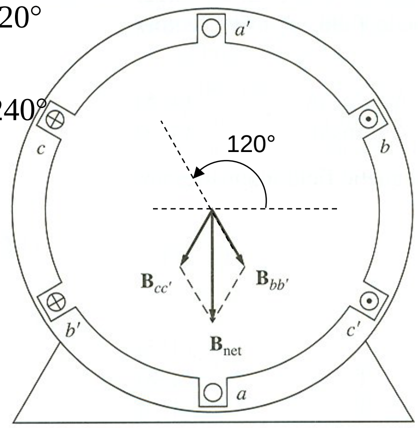
$$B_{bb'} = B_M \sin(-120°)\angle 120° = -\frac{\sqrt{3}}{2} B_M \angle 120°$$

$$B_{cc'} = B_M \sin(-240°)\angle 240° = +\frac{\sqrt{3}}{2} B_M \angle 240°$$

electrical, time domain

mechanical, spatial domain

$$B_{net} = B_{aa'} + B_{bb'} + B_{cc'} = \frac{3}{2} B_M \angle -90°$$



$\omega t = 0°$

Observations at ωt = 90°

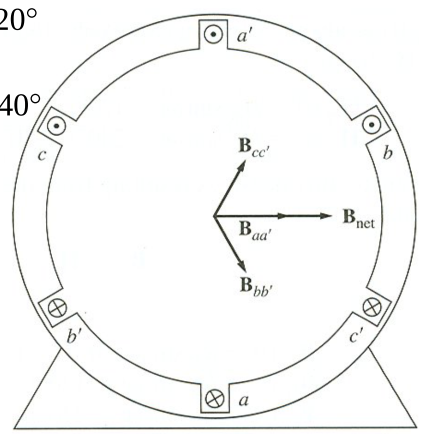
$$B_{aa'} = B_M \sin(90°) = B_M \angle 0°$$

$$B_{bb'} = B_M \sin(-30°)\angle 120° = -\frac{1}{2} B_M \angle 120°$$

$$B_{cc'} = B_M \sin(210°)\angle 240° = -\frac{1}{2} B_M \angle 240°$$

$$B_{net} = B_{aa'} + B_{bb'} + B_{cc'} = \frac{3}{2} B_M \angle 0°$$

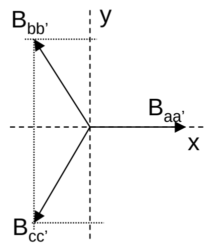
It appears that with evolving time a vector of constant magnitude $B_{net} = 1.5\ B_M$ rotates clockwise in the machine



$\omega t = 90°$

Rotating Magnetic Field - Proof

$$B_{aa'} = B_M \sin(\omega t)$$
$$B_{bb'} = B_M \sin(\omega t - 120°)\angle 120°$$
$$B_{cc'} = B_M \sin(\omega t - 240°)\angle 240°$$
$$B_{net} = B_{aa'} + B_{bb'} + B_{cc'}$$



Decomposition of all three space vectors B_{nn} into their spatial

x components and **y** components

$$B_{aa'} = B_M \sin(\omega t)$$
$$B_{bb'} = -\tfrac{1}{2} B_M \sin(\omega t - 120°)$$
$$B_{cc'} = -\tfrac{1}{2} B_M \sin(\omega t - 240°)$$

$$B_{aa'} = 0$$
$$B_{bb'} = +\tfrac{\sqrt{3}}{2} B_M \sin(\omega t - 120°)$$
$$B_{cc'} = -\tfrac{\sqrt{3}}{2} B_M \sin(\omega t - 240°)$$

Rotating Magnetic Field - Proof

$$\sin(\omega t - 120°) = -\tfrac{1}{2}\sin(\omega t) - \tfrac{\sqrt{3}}{2}\cos(\omega t) \qquad \sin(\omega t - 240°) = -\tfrac{1}{2}\sin(\omega t) + \tfrac{\sqrt{3}}{2}\cos(\omega t)$$

x components

$$B_{aa'} = B_M \sin(\omega t)$$
$$B_{bb'} = -\tfrac{1}{2} B_M \left[-\tfrac{1}{2}\sin(\omega t) - \cancel{\tfrac{\sqrt{3}}{2}\cos(\omega t)} \right]$$
$$B_{cc'} = -\tfrac{1}{2} B_M \left[-\tfrac{1}{2}\sin(\omega t) + \cancel{\tfrac{\sqrt{3}}{2}\cos(\omega t)} \right]$$

y components

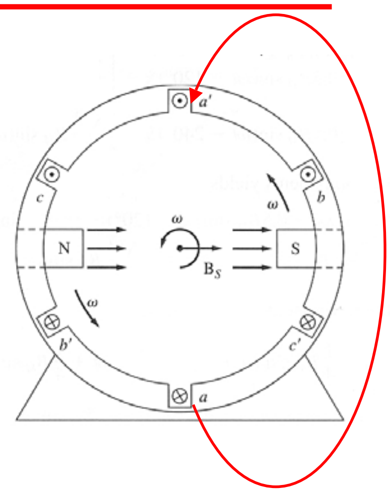
$$B_{aa'} = 0$$
$$B_{bb'} = +\tfrac{\sqrt{3}}{2} B_M \left[\cancel{-\tfrac{1}{2}\sin(\omega t)} - \tfrac{\sqrt{3}}{2}\cos(\omega t) \right]$$
$$B_{cc'} = -\tfrac{\sqrt{3}}{2} B_M \left[\cancel{-\tfrac{1}{2}\sin(\omega t)} + \tfrac{\sqrt{3}}{2}\cos(\omega t) \right]$$

$$B_{net}(x, y, t) \;=\; 1.5 B_M \sin(\omega t)\vec{\mathbf{x}} \quad - 1.5 B_M \cos(\omega t)\vec{\mathbf{y}}$$

This is a vector of constant magnitude B_{net} = 1.5 B_M rotating clockwise with the angular frequency ω. In the chosen reference frame B_{net} points in the –y direction for t = 0.

Rotating Magnetic Field - Equivalent

- **The magnetic field B_S**
  - generated by a set of symmetrical three-phase AC currents at angular power frequency ω_e
  - flowing in a set of circularly arranged (symmetrical) three-phase windings
  - is equivalent to a permanent magnet rotating with the angular speed $\omega_m = \omega$ in a 2-pole machine

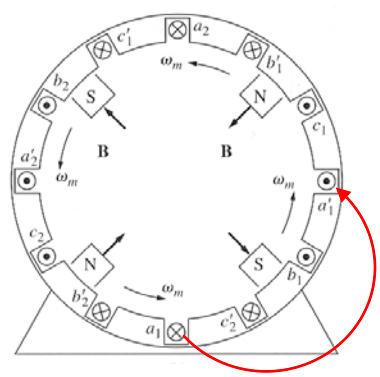


Winding aa' closes within 180° mechanically

Number of Poles >2

- **The rotating magnetic field B_S**
  - in an P - pole machine rotates with an angular frequency of $\omega_m = \omega_e/(P/2)$
  - Therefore, the mechanical speed of the rotating magnetic field can only be reduced by increasing the number of pole-pairs
  - Often used in machine theory: Number of pole-pairs p = P/2 so the number of poles become P = 2p
  - The mechanical speed is often given in rounds-per-minute (rpm)

$$n = \frac{2}{P} 60 f_e = \frac{120 f_e}{P}$$



Winding aa' closes within 90° mechanically

$$\omega_e = \frac{P}{2}\omega_m, \quad f_e = \frac{P}{2} f_m$$

$$\Theta_e = \frac{P}{2}\Theta_m$$ Mechanical angle $\theta_m = \omega_m t$

Speed Reversal by Sequence Change

Altering two phases (e.g. b and c) changes the sequence from abc to acb

x components

$$B_{aa'} = B_M \sin(\omega t)$$

$$B_{bb'} = -\tfrac{1}{2} B_M \left[-\tfrac{1}{2}\sin(\omega t) + \cancel{\tfrac{\sqrt{3}}{2}\cos(\omega t)} \right]$$

$$B_{cc'} = -\tfrac{1}{2} B_M \left[-\tfrac{1}{2}\sin(\omega t) - \cancel{\tfrac{\sqrt{3}}{2}\cos(\omega t)} \right]$$

y components

$$B_{aa'} = 0$$

$$B_{bb'} = +\tfrac{\sqrt{3}}{2} B_M \left[\cancel{-\tfrac{1}{2}\sin(\omega t)} + \tfrac{\sqrt{3}}{2}\cos(\omega t) \right]$$

$$B_{cc'} = -\tfrac{\sqrt{3}}{2} B_M \left[\cancel{-\tfrac{1}{2}\sin(\omega t)} - \tfrac{\sqrt{3}}{2}\cos(\omega t) \right]$$

$$B_{net}(x,y,t) = 1.5 B_M \sin(\omega t)\vec{\mathbf{x}} \quad + 1.5 B_M \cos(\omega t)\vec{\mathbf{y}}$$

This is a vector of constant magnitude $B_{net} = 1.5\ B_M$ rotating **counterclockwise** with the angular frequency ω. In the chosen reference frame B_{net} points in the +y direction for t = 0.

Generating Torque – Forces on Conductors

- **The rotating field B_S of the stator interacts with currents flowing in the rotor**
- **B_S is distributed sinusoidal around the circumference of the stator/rotor**

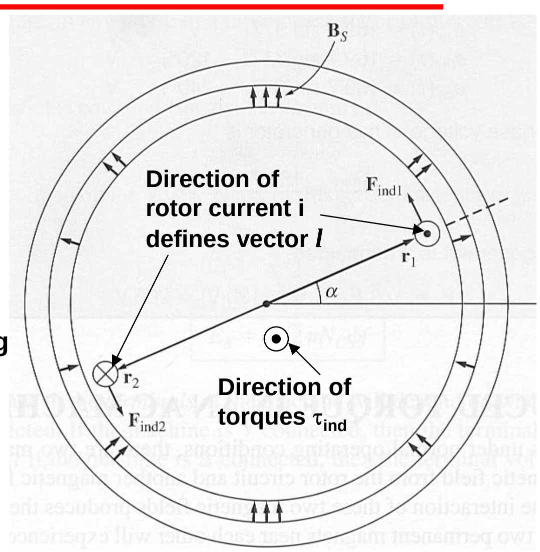
$$B_S(\alpha) = B_S \sin(\alpha)$$

- **Force and torque on a rotor conductor of length *l* carrying a current i**

$$\vec{F} = i\left(\vec{l} \times \vec{B}\right) \qquad \vec{\tau} = \vec{r} \times \vec{F}$$

$$F_{ind} = ilB_S \sin(\alpha)$$

$$\tau_{ind} = 2rilB_S \sin(\alpha), CCW$$



Generating Torque – Interacting B-fields

- Currents in the rotor generate their own magnetic field (C… const)

$$B_R = \mu H_R = \mu C i$$

- Therefore, the torque

$$\tau_{ind} = 2 r i l B_S \sin(\alpha)$$

can be written as

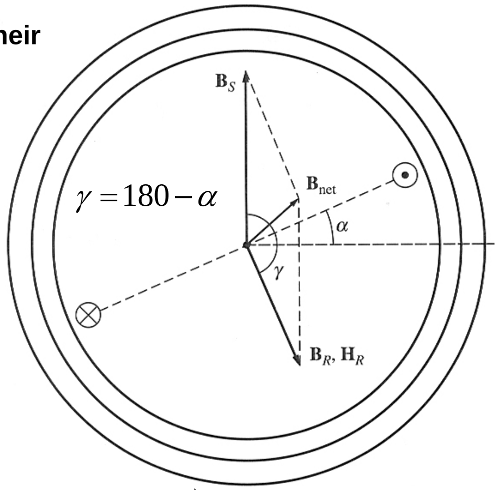
$$\tau_{ind} = k B_R B_S \sin(\alpha)$$

with the machine constant

$$k = {2rl}/{\mu C}$$

- This is equivalent to

$$\vec{\tau}_{ind} = k\vec{B}_R \times \vec{B}_S = k\vec{B}_R \times \vec{B}_{net} \quad \left(\vec{B}_S = \vec{B}_{net} - \vec{B}_R\right)$$



Generating Torque – “Permanent Magnet Rotor”

- Using only the rotor field B_R, the total field B_{net}, and the torque angle δ the torque becomes

$$\tau_{ind} = k B_R B_{net} \sin(\delta)$$

This observation is important for synchronous machines (permanent magnet and electromagnet)