Calculus in MATLAB - Symbolic toolbox

MATLAB provides various ways for solving problems of differential and integral calculus, solving differential equations of any degree and calculation of limits. Best of all, you can easily plot the graphs of complex functions and check maxima, minima and other stationery points on a graph by solving the original function, as well as its derivative.

MATLAB provides the **diff** command for computing symbolic derivatives. In its simplest form, you pass the function you want to differentiate to diff command as an argument.

For example, let us compute the derivative of the function f(t) = 3*t^2 + 2*t^(-2)

Derivative

Create a script file and type the following code into it −

```
syms t
f = 3*t^2 + 2*t^(-2)
```

f =

$$\frac{2}{t^2} + 3\,t^2$$

```
dt=diff(f)
```

dt =

$$6\,t - \frac{4}{t^3}$$

Create a script file and type the following code into it −

```
syms x
y0 = exp(x)
```

y0 = e^x

```
dt0=diff(y0)
```

dt0 = e^x

```
y1 = x^9
```

y1 = x^9

```
dt1=diff(y1)
```

dt1 = $9\,x^8$

```
y2 = sin(x)
```

y2 = $\sin(x)$

```
dt2=diff(y2)
```

dt2 = $\cos(x)$

```
y3 = tan(x)
```

y3 = $\tan(x)$

```
dt3=diff(y3)
```

dt3 = $\tan(x)^2 + 1$

```
y4 = cos(x)
```

y4 = $\cos(x)$

```
dt4=diff(y4)
```

dt4 = $-\sin(x)$

```
y5 = log(x)
```

y5 = $\log(x)$

```
dt5=diff(y5)
```

dt5 =
$$\frac{1}{x}$$

```
y6 = log10(x)
```

y6 =
$$\frac{\log(x)}{\log(10)}$$

```
dt6=diff(y6)
```

dt6 =
$$\frac{1}{x\log(10)}$$

```
y7 = sin(x)^2
```

y7 = $\sin(x)^2$

```
dt7=diff(y7)
```

dt7 = $2\cos(x)\sin(x)$

```
y8 = cos(3*x^2 + 2*x + 1)
```

y8 = $\cos(3\,x^2 + 2\,x + 1)$

```
dt8=diff(y8)
```

dt8 = $-\sin(3\,x^2 + 2\,x + 1)\,(6\,x + 2)$

```
y9 = exp(x)/sin(x)
```

y9 =

$$\frac{e^x}{\sin(x)}$$

```
dt9=diff(y9)
```

dt9 =

$$\frac{e^x}{\sin(x)} - \frac{e^x\cos(x)}{\sin(x)^2}$$

```
f = x*exp(-3*x)
```

f = $x\,e^{-3\,x}$

```
dtf1=diff(f)
```

dtf1 = $e^{-3\,x} - 3\,x\,e^{-3\,x}$

```
dtf2=diff(f, 2)
```

dtf2 = $9\,x\,e^{-3\,x} - 6\,e^{-3\,x}$

A slightly more general example is: dx^n/ dx which can be entered as:

```
syms x n
y10 = x^n
```

y10 = x^n

```
dt10= diff(y10,x)
```

dt10 = $n\,x^{n-1}$

Notice that MATLAB didn't simplify the result, but you can do this using the "simplify" funct

```
simplify(dt10)
```

ans = $n\, x^{n-1}$

MATLAB knows all the rules of differentation, so you can really make it work for you.

For example, use MATLAB to compute:

```
dt11=diff(x^3*log(sin(x)^2),x)
```

dt11 =

$$3\, x^2 \log(\sin(x)^2) + \frac{2\, x^3 \cos(x)}{\sin(x)}$$

Substitution

You can substitute a numerical value for a symbolic variable using the "subs" function.

For example, define the symbolic expression:

```
f = 2*x^2 - 3*x + 1
```

f = $2\, x^2 - 3\, x + 1$

To substitute the value x = 2 in the expression for f, enter the command:

```
subs(f,2)
```

ans = 3

This returns f(2), which has the value 3. When your expression contains more than one variable, you can specify the variable for which you want to make the substitution. For example, define:

```
syms x y
f = x^2*y + 5*x*sqrt(y)
```

f = $x^2\, y + 5\, x\, \sqrt{y}$

```
dtfx=diff(f,x)
```

dtfx = $2\, x\, y + 5\, \sqrt{y}$

```
dtfy=diff(f,y)
```

dtfy =

$$\frac{5\, x}{2\, \sqrt{y}} + x^2$$

To substitute the value x = 3 in the expression for f, enter:

```
subs(f,x,3)
```

ans = $9\, y + 15\, \sqrt{y}$

Integrals:

Create a script file and type the following code in it −

```
syms x n
i1=int(cos(x))
```

i1 = $\sin(x)$

```
i2=int(exp(x))
```

i2 = e^x

```
i3=int(log(x))
```

i3 = $x\ (\log(x) - 1)$

```
i4=int(x^-1)
```

i4 = $\log(x)$

```
i5=int(x^5*cos(5*x))
```

i5 =

$$\frac{24\cos(5\,x)}{3125} + \frac{24\,x\sin(5\,x)}{625} - \frac{12\,x^2\cos(5\,x)}{125} + \frac{x^4\cos(5\,x)}{5} - \frac{4\,x^3\sin(5\,x)}{25} + \frac{x^5\sin(5\,x)}{5}$$

```
i6=int(x^-5)
```

i6 =

$$-\frac{1}{4\,x^4}$$

```
i7=int(sec(x)^2)
```

i7 = $\tan(x)$

```
i8=int((3+5*x-6*x^2-7*x^3)/2*x^2)
```

i8 =

$$-\frac{7\,x^6}{12} - \frac{3\,x^5}{5} + \frac{5\,x^4}{8} + \frac{x^3}{2}$$

Definite Integral

Let us calculate the area enclosed between the x-axis, and the curve y = x3−2x+5 and the ordinates x = 1 and x = 2.

Create a script file and type the following code −

```
f = x^3 - 2*x +5;
```
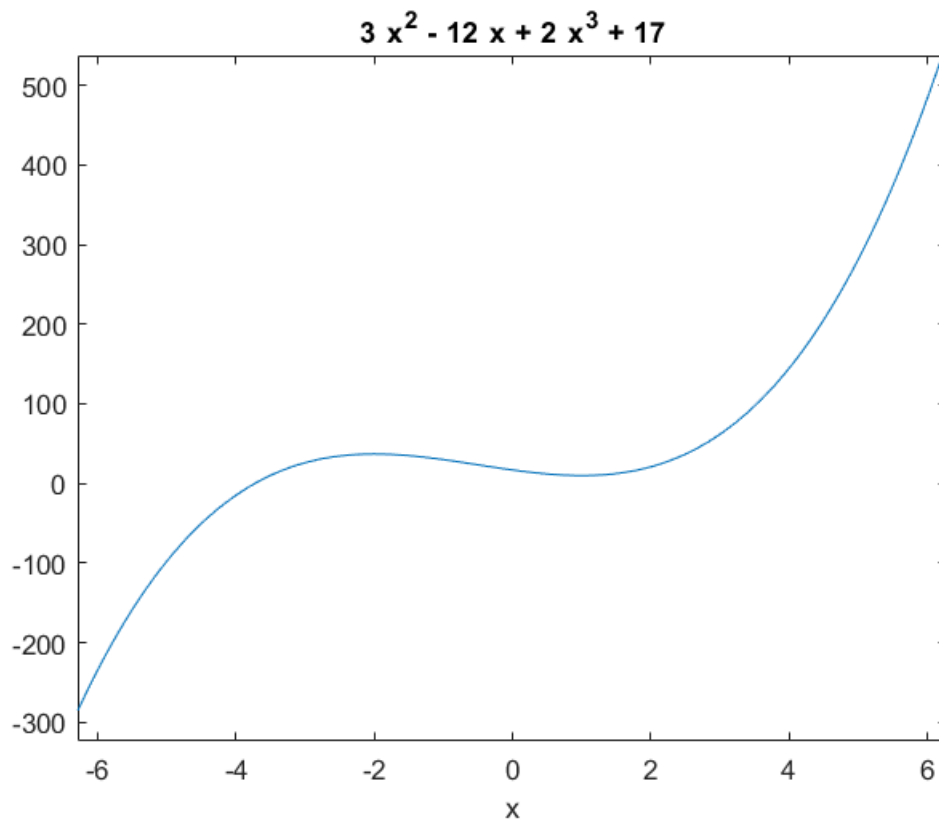
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```
a = int(f, 1, 2)
```

a =

$$\frac{23}{4}$$

```
display('Area: '), disp(double(a));
```

Area:
    5.7500

Function and plot.

```
syms x
y = 2*x^3+3*x^2-12*x+17;    % defining the function
ezplot(y)
```


MATLAB executes the code and returns the following plot

Calculating Limits

For example, let us calculate the limit of a function f(x) = (x3 + 5)/(x4 + 7), as x tends to zero.

```
syms x
limit((x^3 + 5)/(x^4 + 7))
```

ans =

$$\frac{5}{7}$$

For example, let us calculate limit of a function f(x) = (x-3)/(x-1), as x tends to 1.

```
limit((x - 3)/(x-1),1)
```

ans = NaN

Let's take another example,

```
limit(x^2 + 5, 3)
```

ans = 14

Finding solutions to equations

Lets defined the quadratic function f = 3x2 + 4x − 7. We can use MATLAB to find a solution to this function.

The MATLAB function "solve(expression,variable)" will try to find the values of "variable" which satisfy the equation "expression=0".

For example, type "solve(f,x)":

```
f = 3*x^2 + 4*x - 7
```

f = $3 x^2 + 4 x - 7$

```
solve(f,x)
```

ans =

$$\begin{pmatrix} -\dfrac{7}{3} \\ 1 \end{pmatrix}$$

MATLAB finds the two values of x which satisfy f = 0, in this case 1 and -7/3.

Of course, sometimes no solutions to a given equation exist. To see what MATLAB does in this case, try solving the equation 1/x = 0 for x. You should get the message "Explicit solution could not be found". (Try it!)

Note that the solve function will try to find a numeric solution if possible. If no numeric solution is possible, solve will try to find a symbolic solution.

You can also try factoring expressions using the "factor" function. Try factoring the function f we defined above:

```
factor(f)
```

ans = $(3 x + 7 \quad x - 1)$

Symbolic built-in functions

expand multiplies out factor factors the expression

```
syms a b
expand((a-b)^2)
```

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ans = $a^2 - 2\,a\,b + b^2$

```
factor(ans)
```

ans = $(a - b \quad a - b)$

pretty makes it look nicer collect collect terms

simplify simplifies expressions

subs replaces variables with number or expressions

solve replaces variables with number or expressions

```
g = 3*a +4*b-1/3*a^2-a+3/2*b;
collect(g)
```

ans =

$$\frac{11\,b}{2} + 2\,a - \frac{a^2}{3}$$

```
subs(g,[a,b],[0,1])
```

ans =

$$\frac{11}{2}$$