EECE 360
Lecture 12

Properties of Feedback: Steady-State Error

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Chapter 4.2 - 4.4

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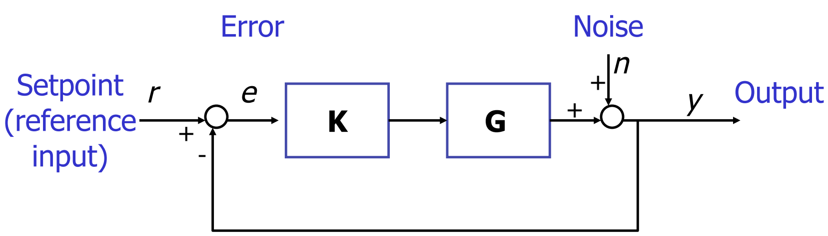
Review: Trade-offs in Control

- Although it seems that all is needed is high gain feedback, there is a cost attached to the use of high-gain feedback
  - Results in very large control actions
  - Increases the risk of instability
  - Increases the sensitivity to measurement noise
- High gain increases performance, but decreases robustness to noise
- **This tradeoff (robustness vs. performance) is the essence of control design**

Review: The Feedback Loop



Sensitivity and Complementary Sensitivity functions

Open-loop:

$$Y(s) = \underbrace{1}_{S_{OL}(s)} \cdot N(s) + \underbrace{GK}_{T_{OL}(s)} \cdot R(s)$$

Closed-loop:

$$Y(s) = N(s) + GKE(s)$$

$$= \underbrace{\frac{1}{1+GK}}_{S_{CL}(s)} N(s) + \underbrace{\frac{GK}{1+GK}}_{T_{CL}(s)} R(s)$$

Review: Sensitivity Function

- S is a function of *s*. If we replace *s* by $j\omega$, we have sensitivity as a frequency response.
- Typically GK is large at low frequencies and small at high frequencies, hence

$$S(0) \approx 0 \text{ while } S(\infty) = 1$$

- This implies

$$T(0) \approx 1 \text{ while } T(\infty) = 0$$

Review

- Inversion as essence of control
- Can be achieved through high gain feedback
- High gain increases performance but decreases robustness
- All control design involves a trade off between performance and robustness

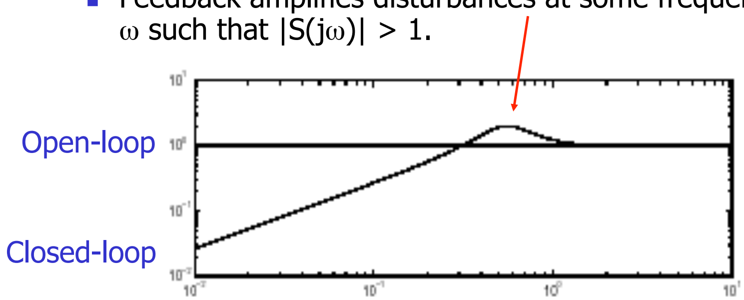
  ($S(s)+T(s) = 1$)

Effect of feedback

- Reduces sensitivity to disturbances at low frequencies
- Close to perfect setpoint tracking at low frequencies
- At high frequencies, when $S(j\omega)\approx 1$ the system has the same sensitivity and disturbance rejection properties as the open-loop plant
- Typically $S(j\omega)$ can be decreased in a frequency range at the cost of an increase in another frequency range

Effect of feedback

- Feedback attenuates disturbances at low frequencies ω such that $|S(j\omega)| << 1$.
- Feedback amplifies disturbances at some frequencies ω such that $|S(j\omega)| > 1$.

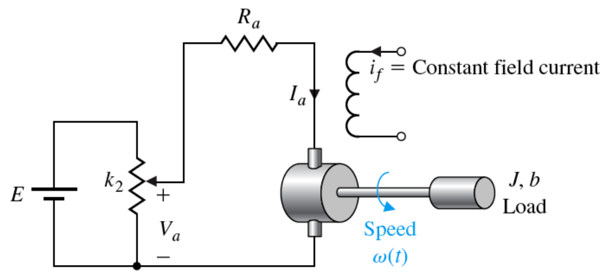


Transient Response

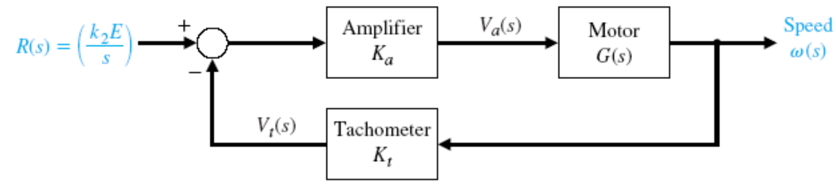
- Transient response is the response of a system as a function of time
- Generally refers to phenomena in the short-term (as opposed to t --> ∞)
- Quantified in overshoot, settling time, time-to-peak, rise time, and other measures.
- Transient response can be drastically improved through feedback

Example: Speed Control

- **Goal:** Motor should track a step-change in desired speed in a short amount of time.
- Open-loop configuration



Example: Speed Control



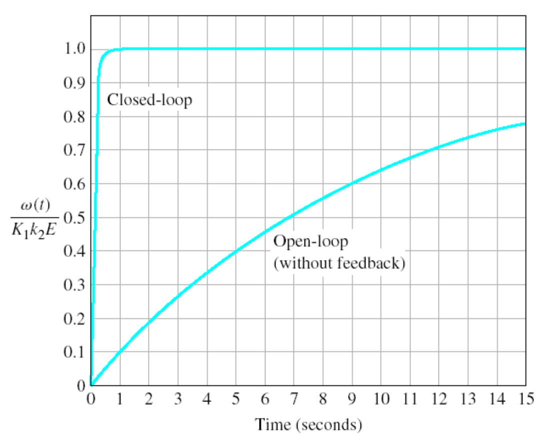
$$\frac{\omega(s)}{R(s)} = \frac{K_a G(s)}{1 + K_t K_a G(s)}$$

$$= \frac{K_a K_1}{\tau_1 s + 1 + K_t K_a K_1}$$

$$G(s) = \frac{K_1}{\tau_1 s + 1}$$

$$= \frac{\frac{K_a K_1}{\tau_1}}{s + \frac{1 + K_t K_a K_1}{\tau_1}}$$

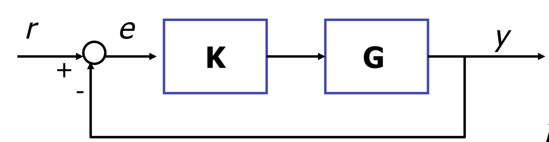
Example: Speed Control



Output response to a step input

Steady-State Error

- Steady-state error is the difference between the actual and desired output as t --> ∞ in response to a **step input**
- Steady-state error is improved through feedback
- For a standard feedback system (no noise)



$$E(s) = \frac{1}{1+KG} R(s)$$

$$= \frac{1}{1+KG} \cdot \frac{1}{s}$$

Steady-State Error

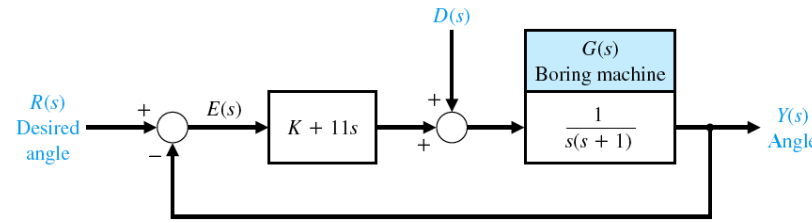
- Using the Final Value Theorem, we know that

$$\lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s)$$
$$= \lim_{s\to 0} s \cdot \frac{1}{1+K(s)G(s)} \cdot \frac{1}{s}$$
$$= \frac{1}{1+K(0)G(0)}$$

- If *G(0)* is finite, *K(s)* must contain one integrator *1/s*
- Either *G(s)* or *K(s)* must contain an integrator *1/s* for the closed-loop system to have **zero** steady-state error.

Example: Eurotunnel

- Consider the following control system



- How can we select *K* to minimize steady-state error?

Example: Eurotunnel

- Output $Y(s) = T(s)R(s) + T_d(s)D(s)$

$$= \frac{K+11s}{s^2+12s+K}R(s) + \frac{1}{s^2+12s+K}D(s)$$

- Error $E(s) = R(s) - Y(s)$

$$= (1-T(s))R(s) - T_d(s)D(s)$$
$$= \left(1 - \frac{K+11s}{s^2+12s+K}\right)R(s) - \frac{1}{s^2+12s+K}D(s)$$
$$= \frac{s(s+1)}{s^2+12s+K}R(s) - \frac{1}{s^2+12s+K}D(s)$$

Example: Eurotunnel

- Steady-state value with step input

$$\lim_{t\to\infty} y_R(t) = \lim_{s\to 0} s \cdot \frac{11s+K}{s^2+12s+K} \cdot \frac{1}{s} = 1$$
$$\lim_{t\to\infty} y_D(t) = \lim_{s\to 0} s \cdot \frac{1}{s^2+12s+K} \cdot \frac{1}{s} = \frac{1}{K}$$

- Steady-state error

$$\lim_{t\to\infty} e_R(t) = \lim_{s\to 0} s \cdot \frac{s(s+1)}{s^2+12s+K} \cdot \frac{1}{s} = 0$$
$$\lim_{t\to\infty} e_D(t) = \lim_{s\to 0} s \cdot \frac{-1}{s^2+12s+K} \cdot \frac{1}{s} = -\frac{1}{K}$$

- Increasing *K* will reduce error due to disturbance.

Example: Eurotunnel

- Now consider the transient response
- Characteristic equation

$$0 = 1 + G(s)K(s) \quad \Rightarrow \quad 0 = s^2 + 12s + K$$

- Response for various *K*

K	$\lambda_{1,2}$	Response	Steady - state dist.
20	$-2,-10$	overdamped	0.050
36	$-6,-6$	critical damping	0.028
72	$-6 \pm 6j$	underdamped	0.014
180	$-6 \pm 12j$		0.006

Example: Eurotunnel

english1.m

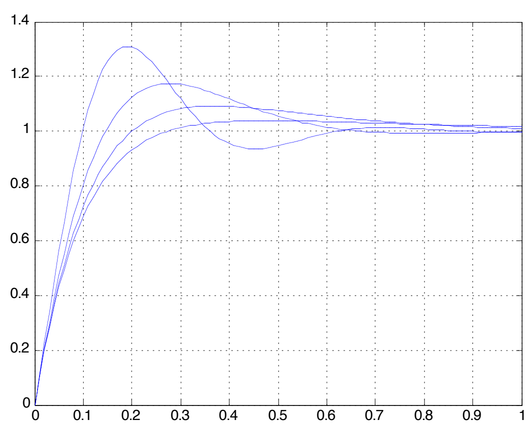
```
% Response to a Unit Step Input R(s)=1/s for K=20 and K=100
%
numg=[1]; deng=[1 1 0]; sysg=tf(numg,deng);
K1=100; K2=20;
num1=[11 K1]; num2=[11 K2]; den=[0 1];
sys1=tf(num1,den);
sys2=tf(num2,den);
%
sysa=series(sys1,sysg); sysb=series(sys2,sysd);
sysc=feedback(sysa,[1]); sysd=feedback(sysb,[1]);
%
t=[0:0.01:2.0];
[y1,t]=step(sysc,t); [y2,t]=step(sysd,t);
subplot(211),plot(t,y1), title('Step Response for K=100')
xlabel('Time (seconds)'),ylabel('y(t)'), grid
subplot(212),plot(t,y2), title('Step Response for K=20')
xlabel('Time (seconds)'),ylabel('y(t)'), grid
```

Closed-loop transfer functions.

Choose time interval.

Create subplots with *x* and *y* axis labels.

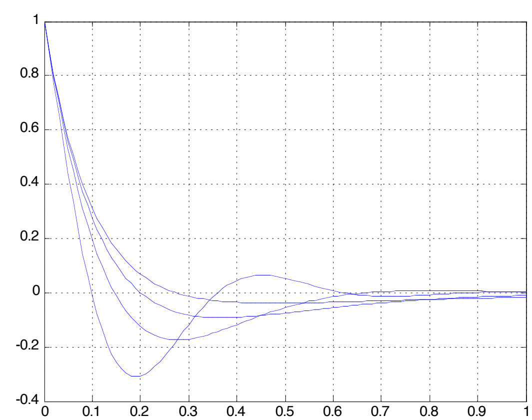
Step Responses



*r(t)=**1**(t)*

d(t)=0

Error Responses

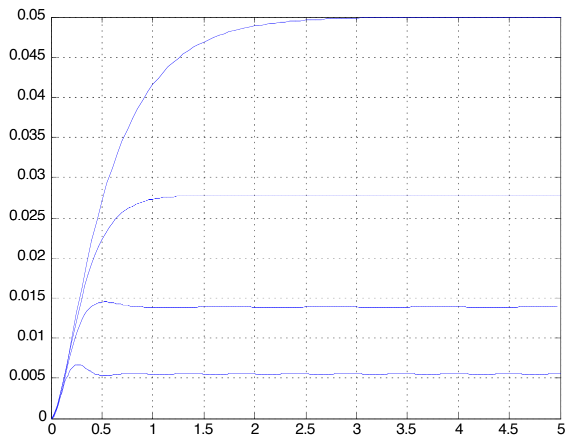


e(t)=r(t)-y(t)

d(t)=0

*r(t)=**1**(t)*

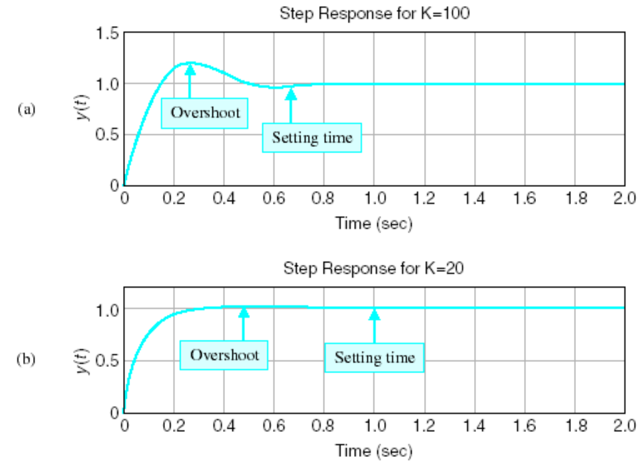
Disturbance Responses



$d(t)=\mathbf{1}(t)$

$r(t)=0$

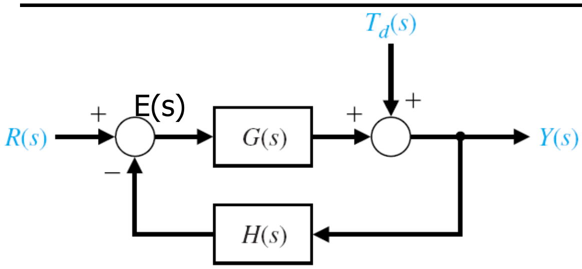
Common Step Response Parameters



$r(t)=\mathbf{1}(t)$

$d(t)=0$

Example: E4.9



$$G(s)=\frac{K}{s+10}$$

$$H(s)=\frac{14}{s^2+5s+6}$$

- Find $Y(s)/R(s)$, $Y(s)/T_d(s)$
- Find steady-state error e_{ss} due to a unit step reference input $r(t)$
- Find the steady-state response $y(t)$ due to unit step disturbance input $T_d(s)=1/s$

Summary

- All control design involves a trade off between **performance** and **robustness**
- Sensitivity function (*S(s)*) measures effect of noise on the output
- Complementary sensitivity function (*T(s)*) measures effect of reference input on output
- Feedback improves transient response
- Feedback improves steady-state error
- Control gain affects transient response and steady-state error