EECE 360
Lecture 20

Controllers in Root Locus

Dr. Oishi

Electrical and Computer Engineering

University of British Columbia

http://courses.ece.ubc.ca/360

eece360.ubc@gmail.com

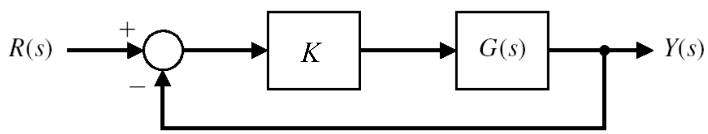
Chapter 7.2 – 7.4, 7.12

Review

- Complete root locus sketches
  - Number of poles, zeros, and asymptotes
  - Segments on the real axis
  - Center/angle of asymptotes
  - Departure/arrival angles
  - Imaginary axis crossings
- Today: Standard Controllers
  - PID
  - Lead
  - Lag

Root Locus Method

- Unity feedback



- Characteristic equation $1 + KG(s) = 0$
- The root locus originates at the poles of G(s) and terminates on the zeros of G(s).

Gain and Phase Criterion

- A location s* is on the locus if
  $$1 + K\,G(s^*) = 0$$
  which is equivalent to
  $$G(s^*) = -1/K$$
- Recall that *s* is a complex number (therefore it has a *magnitude* and a *phase*) and **assume K > 0**

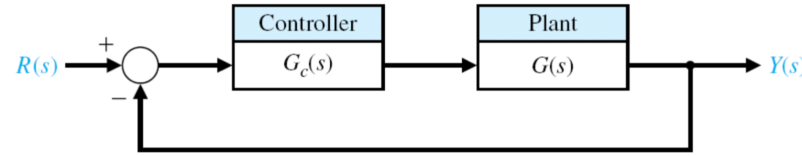
- Phase condition
  $$\angle[G(s^*)] = 180^\circ \pm 360^\circ n$$
  determines which points are on the locus
- The magnitude condition
  $$|G(s^*)| = 1/K$$
  determines the value of K at s*

Root Locus for Design

- Location of roots indicates performance of the closed-loop system.
- Move roots along the root locus by varying K.
- Pick K to accommodate transient or steady-state constraints (e.g. desired damping ratio, natural frequency, overshoot, etc.)
- Poles closest to the imaginary axis are **dominant** in because they are slowest (e.g. they take the longest to die out).

Root Locus for Design

- Unity feedback with controller $G_c(s)$



- The controller $G_c(s)$ can take many forms
- These controllers alter the root locus in different ways.

Common controllers

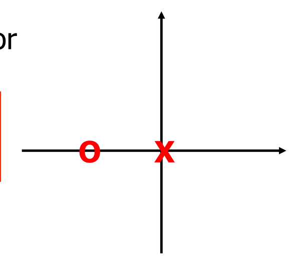
- Proportional-Integral (PI)
- Proportional-Derivative (PD)
- Proportional-Integral-Derivative (PID)
- Phase lead
- Phase lag

PI Control

- PI controller
  - Improves steady-state behavior

$$\text{PI controller: } G_c(s) = K_P + \frac{K_I}{s}$$

$$u(t) = K_P e(t) + K_I \int e(\tau) d\tau$$



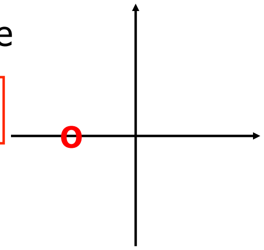
  - One zero, one pole at the origin

PD Control

- PD controller
  - Improves transient response

PD controller: $G_c(s) = K_P + K_D s$

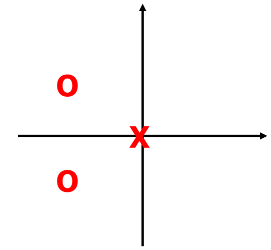
$$u(t) = K_P e(t) + K_I \int e(\tau) d\tau$$



  - One zero

PID Control

- Good performance in a wide range of operating conditions
- Dependent on 3 parameters
- Two zeros and one pole
- Standard PID controller:

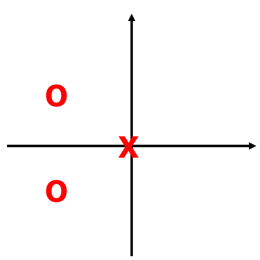


$$G_c(s) = K_P + \frac{K_I}{s} + K_D s$$

which corresponds to

$$u(t) = K_P e(t) + K_I \int e(\tau) d\tau + K_D \frac{de(t)}{dt}$$

PID Control



Consider the PID controller

$$G_c(s) = K_1 + \frac{K_2}{s} + K_3 s = \frac{K_3 s^2 + K_1 s + K_2}{s}$$

$$= \frac{K_3(s^2 + as + b)}{s} = \frac{K_3(s+z_1)(s+z_2)}{s}$$

The PID controller introduces a pole at the origin and two zeros

PID Controller

- Consider the system

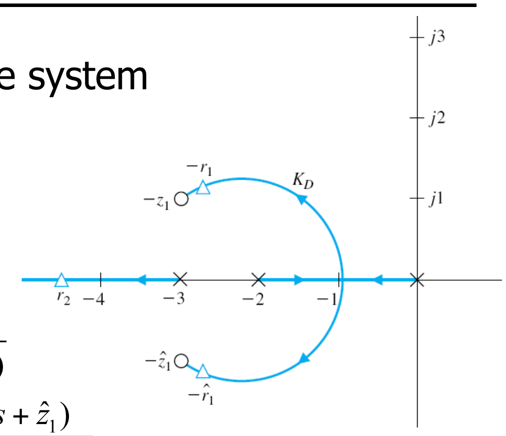
$$G(s) = \frac{1}{(s+2)(s+3)}$$

Using a PID s.t.

$$z_{1,2} = -3 \pm j$$

$$T(s) = \frac{G(s)G_c(s)}{1 + G(s)G_c(s)}$$

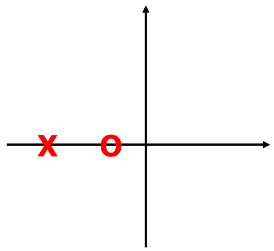
$$= \frac{K_3(s+z_1)(s+\hat{z}_1)}{(s+r_2)(s+r_1)(s+\hat{r}_1)}$$



Phase lead control

- Lead controller
  - Improves transient response

$$G_c(s) = K\frac{s+z}{s+p}, \quad z < p$$

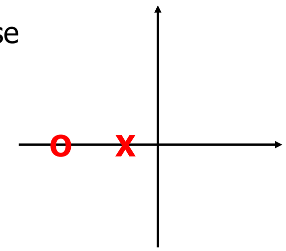


  - One zero, one pole
  - Zero closer to the origin than the pole

Phase lag control

- Lag controller
  - Improves steady-state response

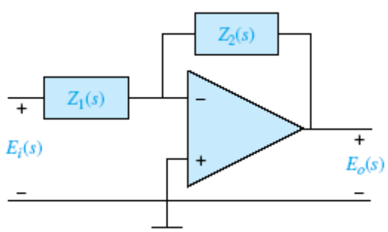
$$G_c(s) = K\frac{s+z}{s+p}, \quad p < z$$



  - One zero, one pole
  - Pole closer to the origin than the zero

Controller Design

- Recall that RLC Op-amp circuits are common ways to build controllers
- The constants K_P, K_I, K_D, K are determined by appropriate choices of R,C



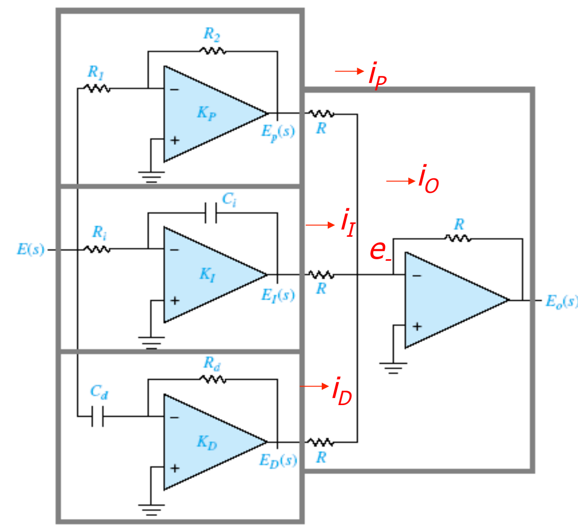
$$G(s) = \frac{E_o(s)}{E_i(s)} = -\frac{Z_2(s)}{Z_1(s)}$$

PID

Proportional

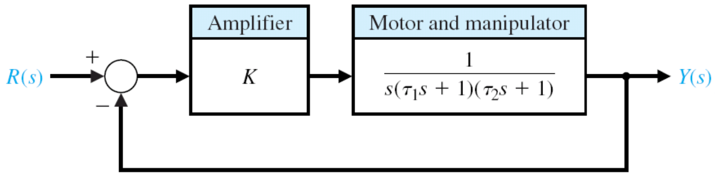
Integral

Derivative



Laser Manipulator Control

The use of lasers for surgery requires high accuracy for position and velocity response.



Use a DC motor and manipulator for the laser.

$$T(s)=\frac{KG(s)}{1+KG(s)}=\frac{K}{s(0.1s+1)(0.2s+1)+K}$$

$$=\frac{K}{0.02s^3+0.3s^2+s+K}=\frac{50K}{s^3+15s^2+50s+50K}$$

Goal: Find K s.t. $e_{ss}\le 0.1$ mm for a ramp input $r(t)=At$ where $A=1$ mm/s

Laser Manipulator Control

For a ramp input $R(s)=1/s^2$, $e_{ss}=\frac{1}{K_v}$ where $K_v=\lim_{s\to 0} s\cdot K\cdot G(s)=K$

Hence $e_{ss}=\frac{1}{K}\le 0.1 \Rightarrow$ $K\ge 10.$

$s^3+15s^2+50s+50K=0$

s^3	1	50
s^2	15	$50K$
s^1	$\frac{750-50K}{15}$	0
s^0	$50K$	

$15s^2-750=0=15(s^2-50)=15(s+7.07j)(s-7.07j)$

$\frac{750-50K}{15}\ge 0 \Rightarrow$ $K\le 15$

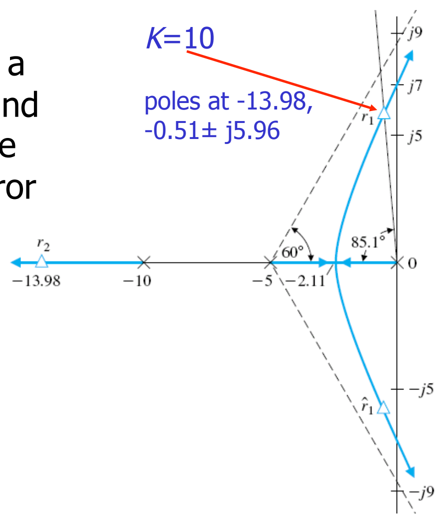
To ensure a stable system: 0<K<15.

Laser Manipulator Control

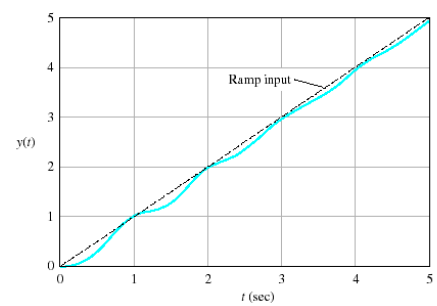
- K=10 results in a stable system and also satisfies the steady-state error specification.
- What about transient performance characteristics?

K=10

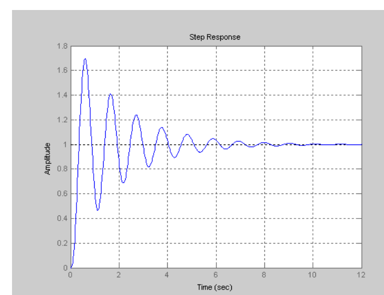
poles at -13.98, -0.51± j5.96



Laser Manipulator Control



The response to a ramp input for a laser control system



The system response to a step input (highly oscillatory).

-> cannot be tolerated for laser surgery

Summary

- Design with root locus
  - Choosing K to meet transient/steady-state response criteria

- Common controllers
  - PID
  - Lead
  - Lag