EECE 360
Lecture 23

Controller Design with Freq. Methods

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Chapter 10.1-10.3, 10.4,10.6

Today's Lecture

- Review: Using Bode diagrams
  - Phase and gain margin
- Today
  - Lead and lag control
  - PID control
  - Examples

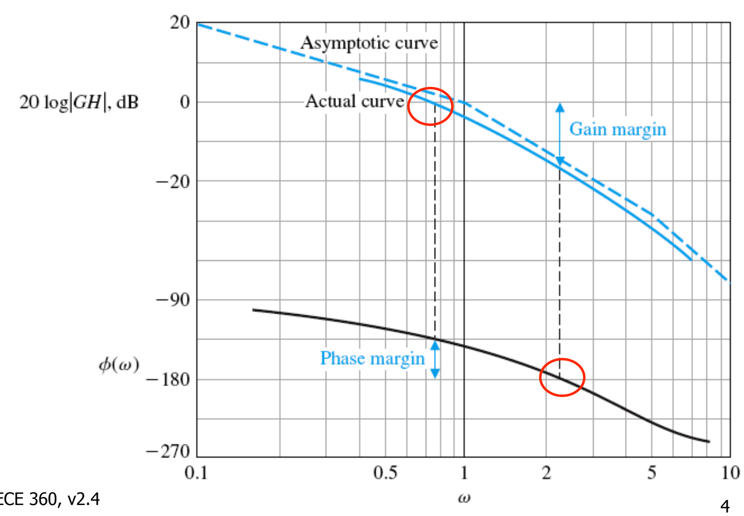
Review: Gain / Phase Margins

Informal definitions:

- The **gain margin** is the factor by which the gain can be increased before instability results.
- The **phase margin** is the amount of phase by which $G(j\omega)$ exceeds -180 degrees when $|KG(j\omega)|=1$
- These are easily measured on Bode diagrams.

Derivations and formal definitions will be provided when we investigate the Nyquist criterion (next chapter).

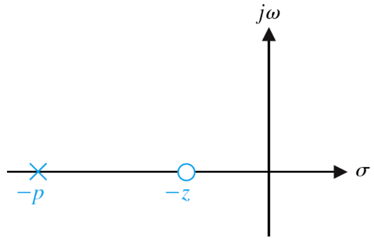
Review: Gain / Phase Margins



Phase-Lead Control

$$G_c(s) = \frac{K(s+z)}{(s+p)}$$

When $|p| > |z|$



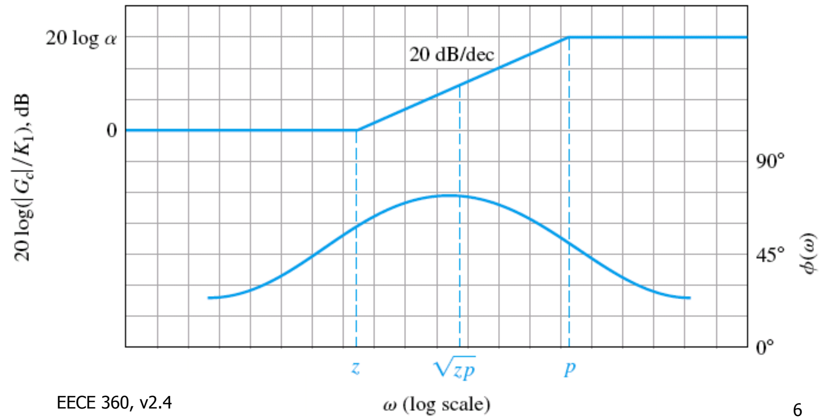
$$G_c(j\omega) = \frac{K(j\omega+z)}{(j\omega+p)} = \frac{K_1(1+j\omega\alpha\tau)}{(1+j\omega\tau)}$$

where $\tau = 1/p$, $p = \alpha z$ and $K_1 = K/\alpha$ $\qquad \alpha > 1$

$$\phi(\omega) = \tan^{-1}\alpha\omega\tau - \tan^{-1}\omega\tau$$

Phase-Lead Control

- Lead controllers **add** phase angle



Phase-Lead Control

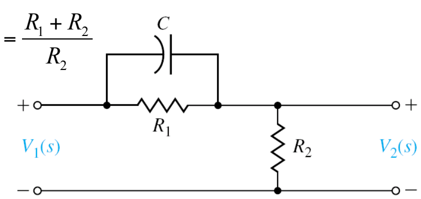
- Constructing a phase-lead controller

$$G_c(s) = \frac{V_2(s)}{V_1(s)} = \frac{R_2}{R_2 + \{R_1(1/Cs)/[R_1+(1/Cs)]\}}$$

$$= \left(\frac{R_2}{R_1+R_2}\right)\frac{R_1Cs+1}{\frac{R_1R_2}{R_1+R_2}Cs+1}$$

$$\tau = \frac{R_1R_2}{R_1+R_2}C \text{ and } \alpha = \frac{R_1+R_2}{R_2}$$

$$G_c(s) = \frac{1+\alpha\tau s}{\alpha(1+\tau s)}$$



Phase-Lead Control

- The amount of phase added is at most

$$\phi = \tan^{-1}\frac{\alpha\omega\tau - \omega\tau}{1+\omega\tau^2}$$

$$\phi \text{ is maximum at } \omega_m = \sqrt{zp} = \frac{1}{\tau\sqrt{\alpha}}$$

$$\tan\phi_m = \frac{(\alpha/\sqrt{\alpha})-(1/\sqrt{\alpha})}{1+1} = \frac{\alpha-1}{2\sqrt{\alpha}} \Rightarrow \boxed{\sin\phi_m = \frac{\alpha-1}{\alpha+1}}$$

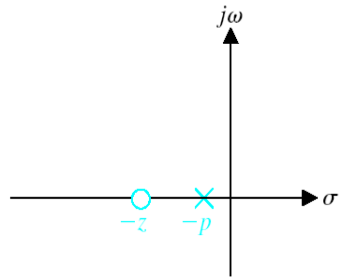
which is about 70 degrees for large α

- ** What if more phase is required?

Phase-Lag Control

$$G_c(s) = \frac{K(s+z)}{(s+p)}$$

When $|\,p\; < |\,z\,|$



$$G_c(j\omega) = K\frac{1+j\omega\tau}{1+j\omega\alpha\tau}$$

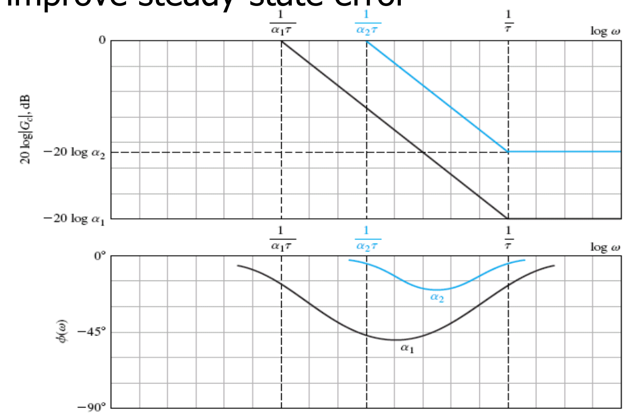
where $z = 1/\tau$ and $p = 1/(\alpha\tau)$

α > 1

$$\phi(j\omega) = \tan^{-1}(\omega\tau) - \tan^{-1}(\omega\alpha\tau)$$

Phase-Lag Control

- Lag controllers provide attenuation and improve steady-state error



Phase-Lag Control

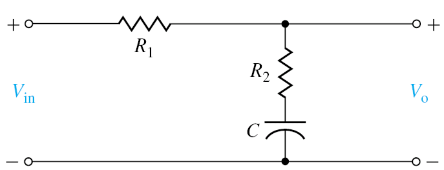
- Constructing phase-lag controllers

$$G_c(s) = \frac{V_o(s)}{V_{in}(s)} = \frac{R_2 + (1/Cs)}{R_1 + R_2 + (1/Cs)} = \frac{R_2Cs+1}{(R_1+R_2)Cs+1}$$

where $\tau = R_2C$ and $\alpha = (R_1+R_2)/R_2$

$$G_c(s) = \frac{1+\tau s}{1+\alpha\tau s} = \frac{1}{\alpha}\frac{(s+z)}{(s+p)}$$

where $z = 1/\tau$ and $p = 1/\alpha\tau$



Phase-Lag Control

- Similar to phase-lead, the minimum phase occurs at

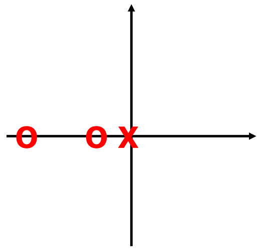
$$\omega_m = \sqrt{zp}$$

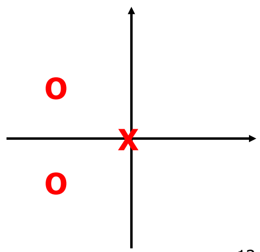
PID Control

- General PID control

$$G_c(s) = K_P + \frac{K_I}{s} + K_D s$$
$$= \frac{K_I(\tau s + 1)(\frac{\tau}{\alpha}s + a)}{s}$$

- PID improves steady-state response as well as transient response
- With zeros on real axis, PID is a notch (or bandstop) compensator

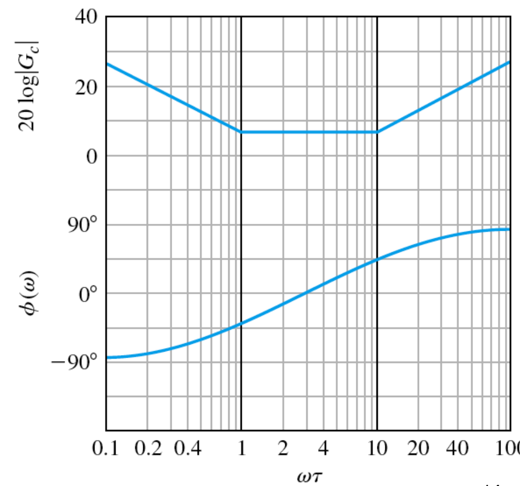




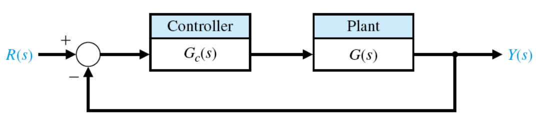
PID Control

$$G_c(s) = \frac{K_I(\tau s + 1)(\frac{\tau}{\alpha}s + a)}{s}$$

- Frequency response of notch (PID) controller
- Phase gain of 180^0

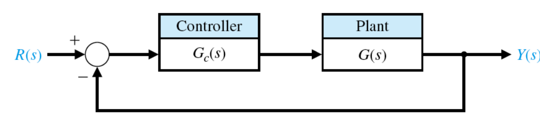


Lead Controller Design



- 1. Pick gain of uncompensated system G(s) so that error constants are satisfied
- 2. Evaluate the phase margin of the uncompensated system G(s)
- 3. Determine additional phase lead ϕ_M required, including a margin of safety
- 4. Evaluate α, the ratio between the lead pole and zero

Lead Controller Design



- 5. Determine the frequency where the uncompensated magnitude curve 20 log|G(jω)| is equal to -10 log α dB. This frequency is ω_m, and is the new crossover frequency.
- 6. Calculate the pole $p = \omega_m\sqrt{\alpha}$, and zero $z = p/\alpha$.
- 7. Draw the compensated frequency response, check the resulting phase margin. Raise the gain of the amplified to account for the attenuation $1/\alpha$.

Example: Phase-Lead Design

- Consider the plant transfer function

$$G(s) = \frac{K_1}{s^2}$$

- Whose uncompensated closed-loop response

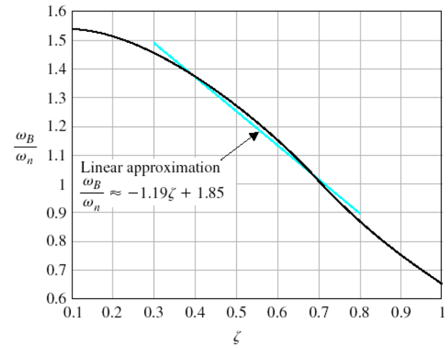
$$T(s) = \frac{Y(s)}{R(s)} = \frac{K_1}{s^2 + K_1}$$

does not meet the transient response specifications

$$T_s \le 4 \text{ seconds}, \quad \zeta \ge 0.45$$

$$T_s = \frac{4}{\zeta\omega_n} \Rightarrow \omega_n = \frac{1}{\zeta} = \frac{1}{0.45} = 2.22$$

Phase-Lead Design Example



$$\omega_n = \sqrt{K_1}$$

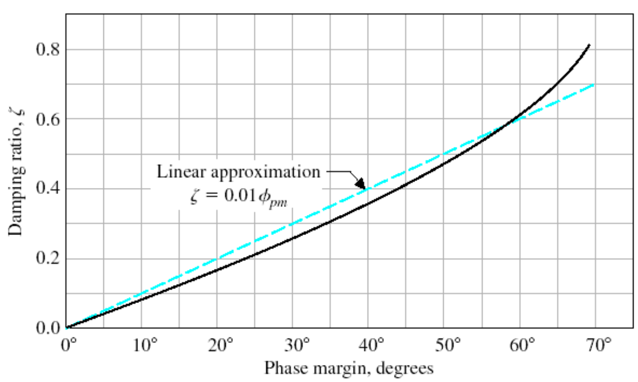
$$\sqrt{K_1} = 2.22$$

$$K_1 \ge 5$$

We choose $K_1 = 10$

$$\omega_B = 1.33\omega_n = 1.33 \times 2.22 = 3.00$$

Phase-Lead Design Example



$$\phi_m = \frac{\zeta}{0.01} = \frac{0.45}{0.01} = 45^\circ$$

Phase-Lead Design Example

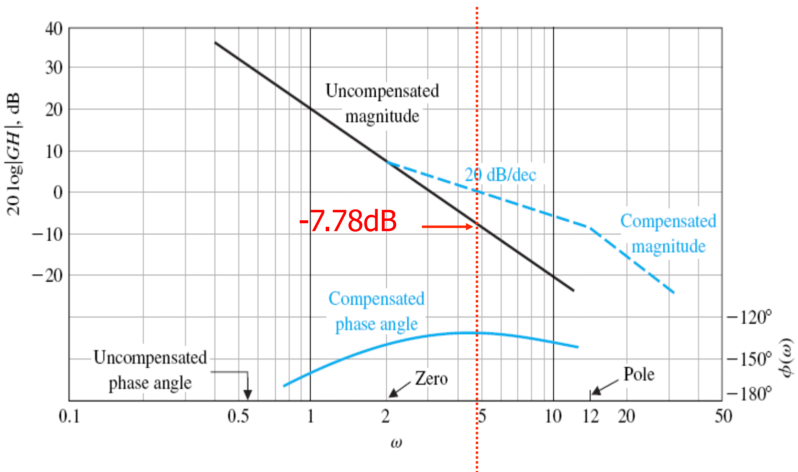
- We need phase lead of 45 degrees at the crossover frequency

$$\frac{\alpha - 1}{\alpha + 1} = \sin\phi_m = \sin 45^\circ \Rightarrow \alpha = 5.8$$

- We choose $\alpha = 6 \Rightarrow 10\log\alpha = 7.78 \text{ dB}$

- Now examine Bode diagram to find new crossover frequency ω_m

Phase-Lead Design Example



Phase-Lead Design Example

$$p = \omega_m \sqrt{\alpha} = 4.95\sqrt{6} = 12.0 \quad z = p/\alpha$$

$$G_c(j\omega)G(j\omega) = \frac{10[j\omega/2+1]}{(j\omega)^2[j\omega/12+1]}$$

$$G_c(s) = \frac{1+\alpha\tau s}{\alpha(1+\tau s)} = \frac{1}{6}\frac{1+s/2}{1+s/12}$$

We need to adjust K_1 to account for the $1/\alpha$ term in $G_c(s)$.

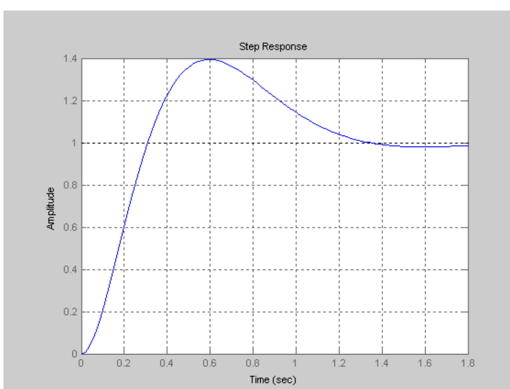
Thus we need to increase K_1 from 10 to 60

The closed-loop transfer function is then

$$T(s) = \frac{GG_c}{1+GG_c} = \frac{\dfrac{60(s+2)}{s^2(s+12)}}{1+\dfrac{60(s+2)}{s^2(s+12)}} = \frac{60(s+2)}{(s^2+6s+20)(s+6)}$$

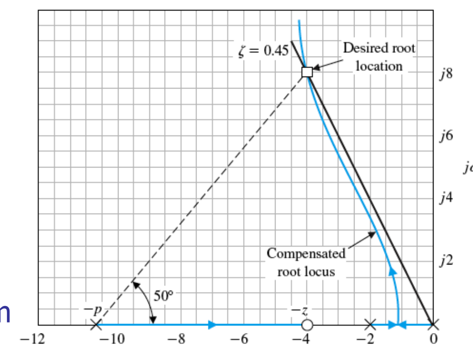
Phase-Lead Design Example

Compensated system closed-loop response



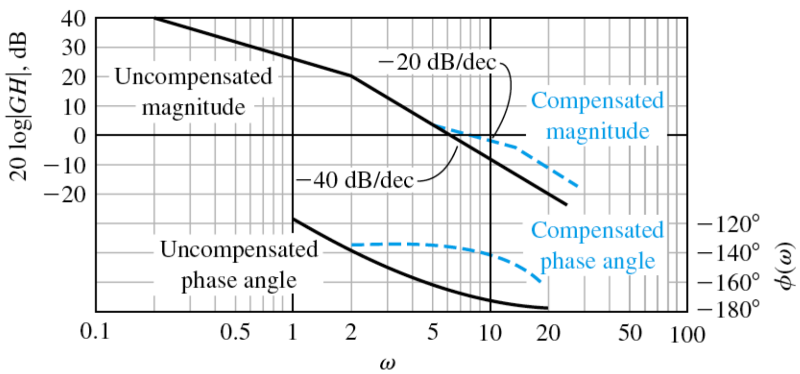
Phase-Lead Design Example

- Root locus plot of compensated system
- Bode diagrams allow analysis of open-loop system to synthesize controller for closed-loop system



Example 2: Phase Lead

- Try example 10.2, Dorf p. 592



Summary

- Today
  - Lead and lag controllers
  - PID controllers
  - Phase lead design through Bode diagrams
- Next
  - Phase lag design through Bode
  - PID design through Bode
  - Examples