EECE 360
Lecture 24

Controller Design with Bode (cont'd)

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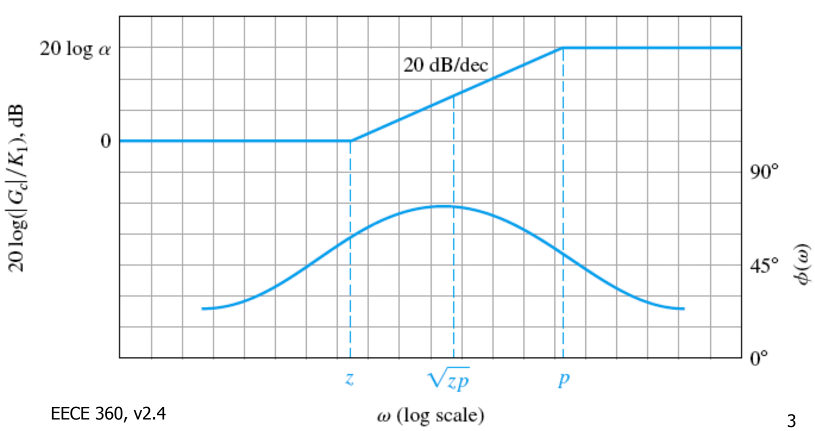
Chapter 10.8

Today's lecture

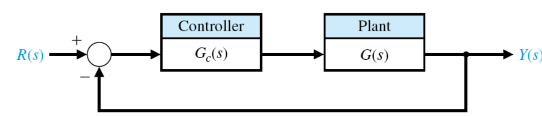
- Review: Using Bode diagrams
  - Lead control design
- Today
  - Lag control
  - Examples

Review: Phase-Lead Controller

- Lead controllers **add** phase angle

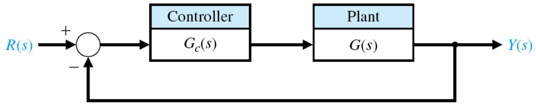


Review: Lead Control Design



- 1. Pick gain of uncompensated system G(s) so that error constants are satisfied
- 2. Evaluate the phase margin of the uncompensated system G(s)
- 3. Determine additional phase lead ϕ_M required, including a margin of safety
- 4. Evaluate α, the ratio between the lead pole and zero

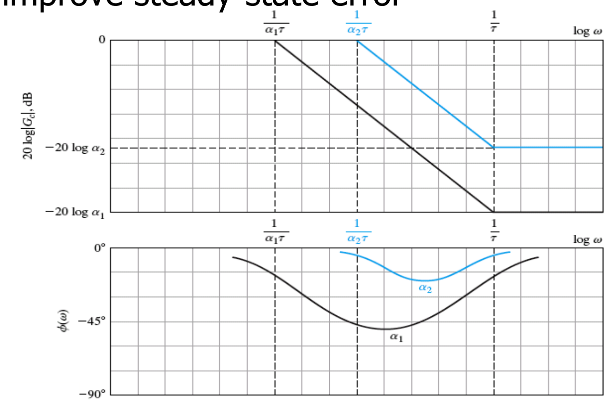
Review: Lead Control Design



- 5. Determine the frequency where the uncompensated magnitude curve 20 log|G(jω)| is equal to -10 log α dB. This frequency is ω_m, and is the new crossover frequency.
- 6. Calculate the pole $p = \omega_m\sqrt{\alpha}$, and zero $z = p/\alpha$.
- 7. Draw the compensated frequency response, check the resulting phase margin. Raise the gain of the amplified to account for the attenuation 1/α.

Phase-Lag Controller

- Lag controllers provide attenuation and improve steady-state error



Phase-Lag Control

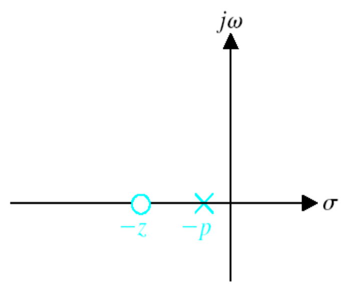
$$G_c(s) = \frac{K(s+z)}{(s+p)}$$

When $|p| < |z|$

$$G_c(j\omega) = K\frac{1 + j\omega\tau}{1 + j\omega\alpha\tau}$$

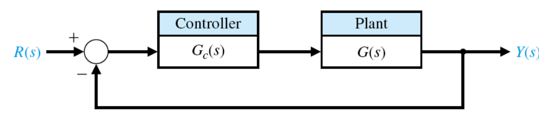
where $z = 1/\tau$ and $p = 1/(\alpha\tau)$

$$\phi(j\omega) = \tan^{-1}(\omega\tau) - \tan^{-1}(\omega\alpha\tau)$$



α > 1

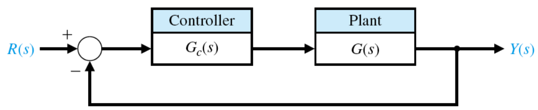
Phase-Lag Controller Design



$$G_c(j\omega) = \frac{1 + j\omega\tau}{1 + j\omega\alpha\tau}$$

- 1. Pick gain of uncompensated system G(s) so that error constants are satisfied
- 2.Evaluate the phase margin of the uncompensated system G(s).
- 3. Determine the frequency where the phase margin requirement would be satisfied if the magnitude curve passed the 0 dB line at this frequency, ω_c'. Include a margin of safety of at least 5 degrees due to an anticipated 5^0 lag from the controller.

Phase-Lag Controller Design



$$G_c(j\omega) = \frac{1 + j\omega\tau}{1 + j\omega\alpha\tau}$$

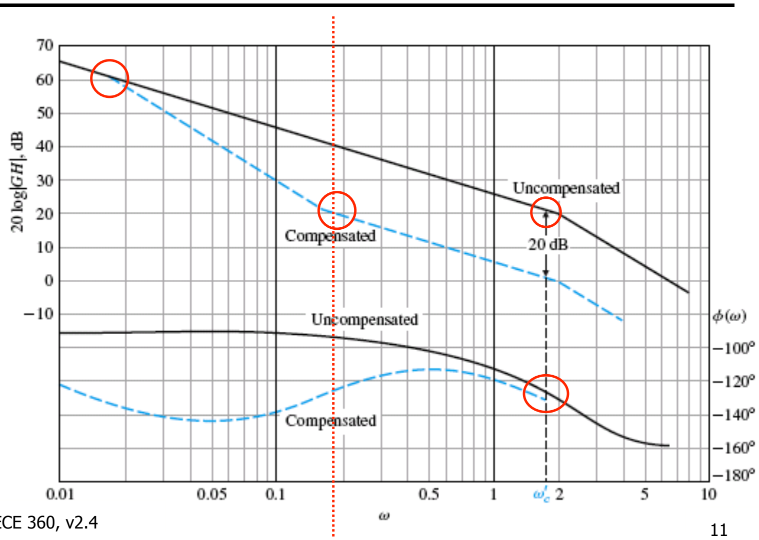
- 4. Place the zero of the compensator one decade below the new crossover frequency (gives τ)
- 5. Measure the necessary attenuation at ω_c' to bring magnitude curve to 0 dB at this frequency
- 6. Calculate α by noting that the attenuation introduced by the phase-lag network is -20log α at ω_c'.
- 7. Calculate pole location as $\omega_p=1/(\alpha\tau)$

Example: Phase Lag Design

- Given the plant transfer function: $G(j\omega) = \frac{K}{j\omega(j\omega+2)} = \frac{K/2}{j\omega(0.5j\omega+1)}$
- and transient requirements: $K_v = K/2$

  Specifications: $K_v = 20$ and $M_\phi = 45^\circ$
- Now plot G(jω).
  - Phase margin of G(jω) is 20^0
  - Plan for an additional 5^0 margin due to the lag controller.
  - For a compensated phase margin of 45^0, locate frequency where the phase is -130 degrees.
  - This is the new crossover frequency $\omega_c' = 1.5$

Example: Phase Lag Design



Example: Phase Lag Design

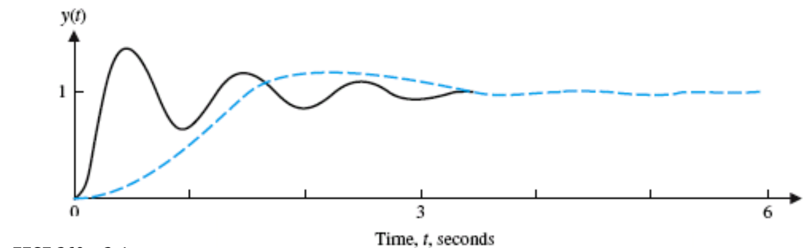
- Now note that a gain increase of 20 dB is required to make $\omega_c' = 1.5$ the new crossover frequency

  $$20\log\alpha = 20 \Rightarrow \alpha = 10$$
- Therefore the zero is one decade below the crossover, or $z = \frac{\omega_c'}{10} = 0.15$
- And therefore the pole is at $p = \frac{z}{\alpha} = 0.015$
- So the compensated system is

  $$G_c(s)G(s) = \frac{20(6.66s+1)}{s(0.5s+1)(66.6s+1)}$$

Example: Phase Lag Design

- Step response for uncompensated (solid) and compensated (dashed) systems
- Overshoot is approximately 25%; peak time is approximately 2 seconds.
- Slower response due to reduced bandwidth



Example 2: Phase Lag

- Consider the system with plant transfer function

$$G(s) = \frac{K}{s(s+10)^2} = \frac{K_v}{s(s/10+1)^2}$$

- And for desired steady-state error and transient response, we need to have

$$K_v = 20, \quad \zeta = .707$$

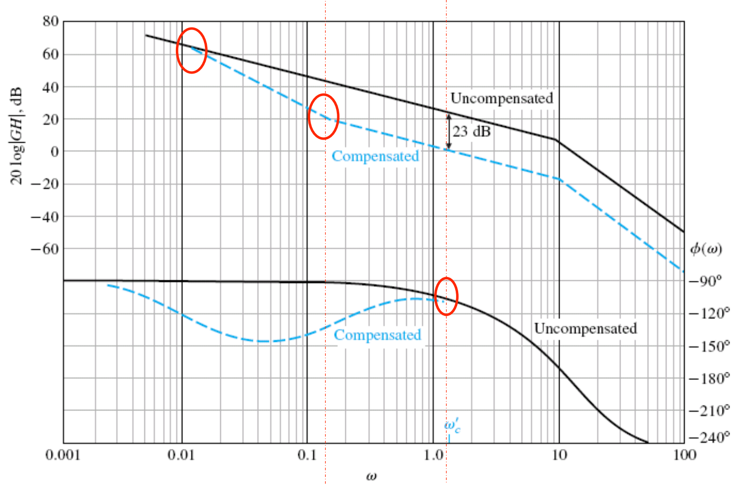
A compensator must be designed in order to meet both requirements.

- This means the phase lag should be

$$\zeta \approx 0.01 M_\phi, \quad M_\phi = 65^\circ$$

- So now evaluate the frequency response G(jω) to find the gain value where the phase margin is 70^0.

Example 2: Phase Lag



Example 2: Phase Lag

- The phase of G(jω) is -110^0 at $\omega_c' = 1.5$, the new crossover frequency
- At this frequency, the compensator will need to contribute 23 dB, therefore the pole/zero ratio is

$$23 = 20\log\alpha, \quad \alpha = 14.2$$

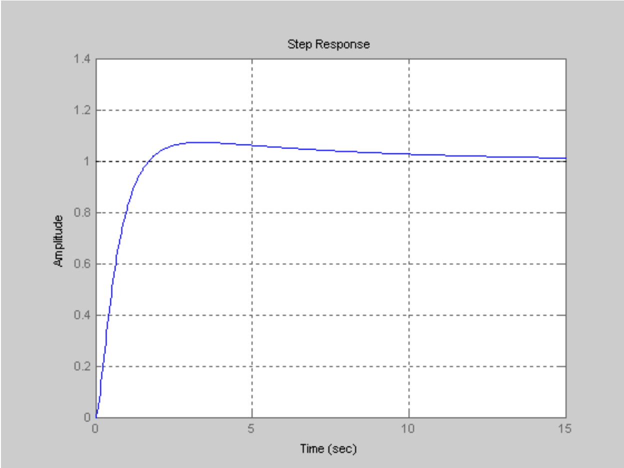
- Since the zero in $G_c(s)$ is one decade below ω_c',

$$z = \frac{\omega_c'}{10} = 0.15 \quad \text{and} \quad p = \frac{z}{\alpha} = \frac{0.15}{14.2}$$

- The compensated system is

$$G_c(s)G(s) = \frac{20(6.66s+1)}{s(0.1s+1)^2(94.6s+1)}$$

Example 2: Phase Lag



Summary: Phase Lead

- Adds phase near crossover
- Increases bandwidth
- Increases high frequency gain
- Improves dynamic response
- Requires additional amplifier gain
- Increases susceptibility to noise
- Applicable when fast response is desired
- Not applicable when phase decreases rapidly near crossover frequency

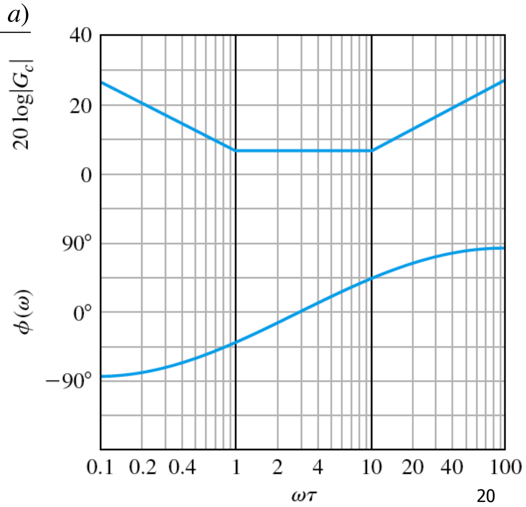
Summary: Phase Lag

- Increases error constant (gain at ω=1) while maintaining phase margin
- Decreases system bandwidth
- Suppresses high-frequency noise
- Reduces steady-state error
- Slows down response
- Applicable when error constants are specified
- Not applicable when no low-frequency range exists where desired phase margin exists

PID Controller

$$G_c(s) = \frac{K_I(\tau s + 1)(\frac{\tau}{\alpha} s + a)}{s}$$

- Frequency response of notch (PID) controller
- Phase gain of 180^0



Basics of PID control

- Basics of PID Tuning
- PID Autotuning

"The PID controller is the backbone of industrial control systems" Leva, Cox and Ruona (2001)

K. Astrom and T. Hagglund, *PID Controllers: Theory, Design and Tuning,* 2nd Edition, ISA Press, 1995.

A. Leva, C. Cox and A. Ruano, *Hands-on PID Autotuning, A Guide to Better Utilisation*, IFAC Professional Brief, 2001. Available at http://www.ifac-control.org/

Example: Lead vs. Lag control

- Consider the system with open-loop transfer function

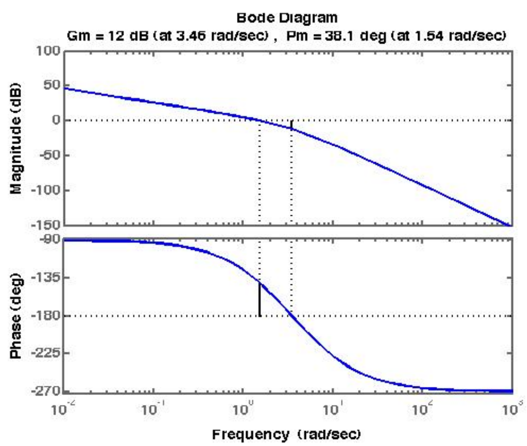
$$G(s) = \frac{2}{s(s/2+1)(s/6+1)}$$

- Design a controller $G_c(s)$ such that
  - Steady-state error to a ramp input is less than 0.05
  - Phase margin is between 40^0 and 50^0
  - Gain crossover frequency ω_c > 1 rad/s

Example

$$G(s) = \frac{2}{s(s/2+1)(s/6+1)}$$

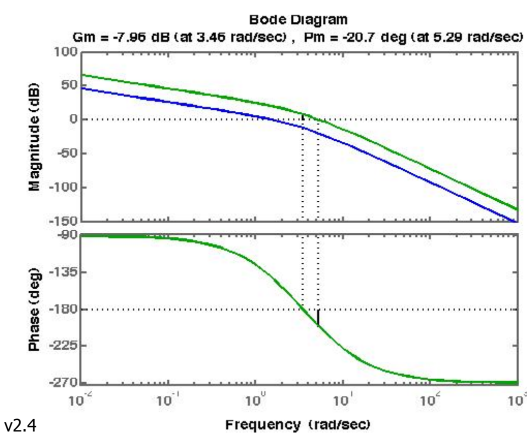
- Bode diagram of the plant



Example

$$G(s) = \frac{2}{s(s/2+1)(s/6+1)}$$

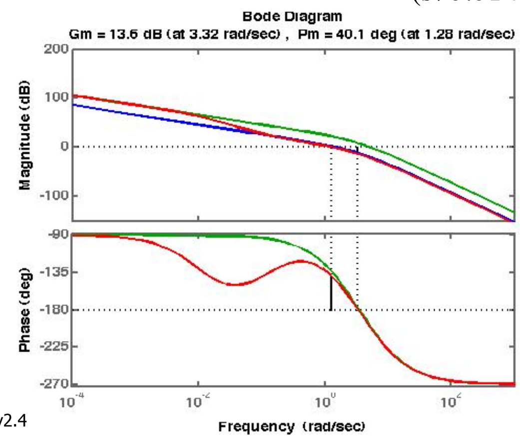
- Proportional gain: $G_c(s)=K$



Example

$$G(s) = \frac{2}{s(s/2+1)(s/6+1)}$$

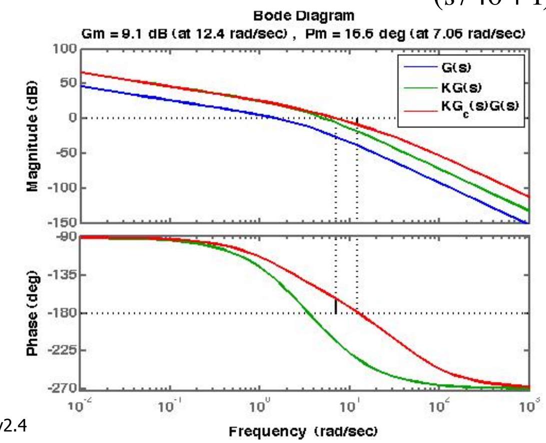
- Phase-lag control $G_c(s) = \frac{10(s/0.13+1)}{(s/0.01+1)}$



Example

$$G(s) = \frac{2}{s(s/2+1)(s/6+1)}$$

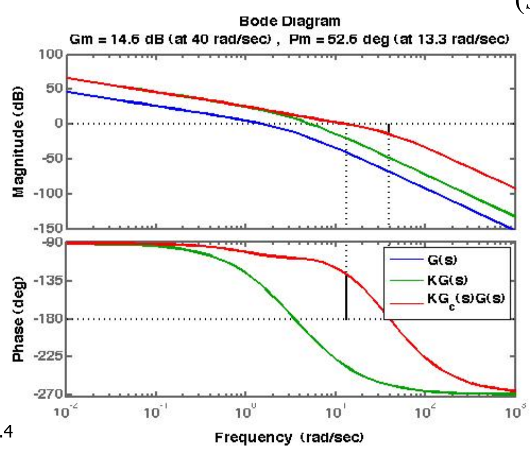
- Phase-lead control $G_c(s) = \frac{10(s/4+1)}{(s/40+1)}$



Example

$$G(s) = \frac{2}{s(s/2+1)(s/6+1)}$$

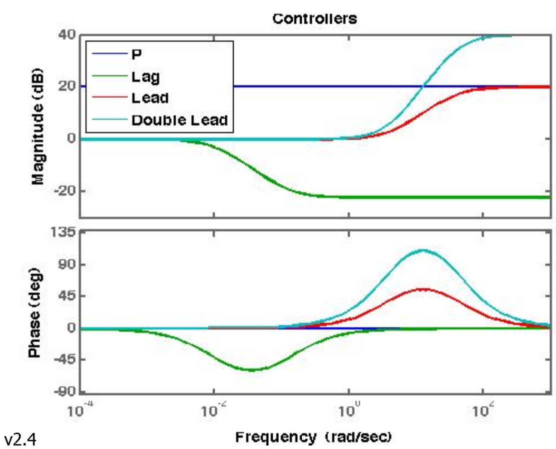
- Cascade phase-lead control $G_c(s) = \frac{10(s/4+1)^2}{(s/40+1)^2}$



Example

$$G(s) = \frac{2}{s(s/2+1)(s/6+1)}$$

- Bode diagrams of the controllers:



Summary

- Introduction to Bode plots
  - Sketching Bode plots
  - Sketching lead, lag, and PID

- Relative Stability with Bode plots
  - Phase margin
  - Gain margin

- Control design with Bode plots
  - Lead, lag and guidelines for PID