Resistor-Inductor (RL) Circuits

Session 3b for Basic Electricity
A Fairfield University E-Course
Powered by LearnLinc

Module: Basic Electronics
(AC Circuits and Impedance: two parts)

- Text: "Electricity One-Seven," Harry Mileaf, Prentice-Hall, 1996, ISBN 0-13-889585-6 (Covers much more material than this section)
- References:
  - "Digital Mini Test: Principles of Electricity Lessons One and Two," SNET Home Study Coordinator, (203) 771-5400
  - Electronics Tutorial (Thanks to Alex Pounds)
  - Electronics Tutorial (Thanks to Mark Sokos)
  - Basic Math Tutorial (Thanks to George Mason University)
  - Vector Math Tutorial (Thanks to California Polytec at atom.physics.calpoly.edu)
- Alternating Current and Impedance
  - 5 on-line sessions plus one lab
- Resonance and Filters
  - 5 on-line sessions plus one lab

Section 3:
AC, Inductors and Capacitors

- 0BJECTIVES: This section introduces AC voltage / current and their effects on circuit components (resistors, inductors, transformers and capacitors). The concept of impedance and the use of the vector analogy for computations is also introduced.

Section 3 Schedule:

Session 3a	– 05/13	Sine Waves, Magnitude, Phase and Vectors (again)	Text 4.1 – 4.24
3a continued	– 05/20	Complete 3a	
Session 3b	**– 05/22**	**R-L Circuits (no class on 05/27)**	**Text 4.25 – 4.54**
3b continued	– 05/29	Complete 3b	
Session 3c	– 06/03	R-C Circuits	Text 4.55 – 4.76
Session 3d	– 06/05	Series LC Circuits Series RLC Circuits	Text 4.77 – 4.88, 4.89 – 4.113
(lab - 06/08, Sat.)			
Session 3e (Quiz 3 due 06/17)	– 06/10	Parallel LC Circuits Parallel RLC Circuits	Text 4.114 – 4.122, 4.123 – 4.146
Session 3f	– 06/10	Review (Discuss Quiz 3)	

Session 3a (Vectors) Review

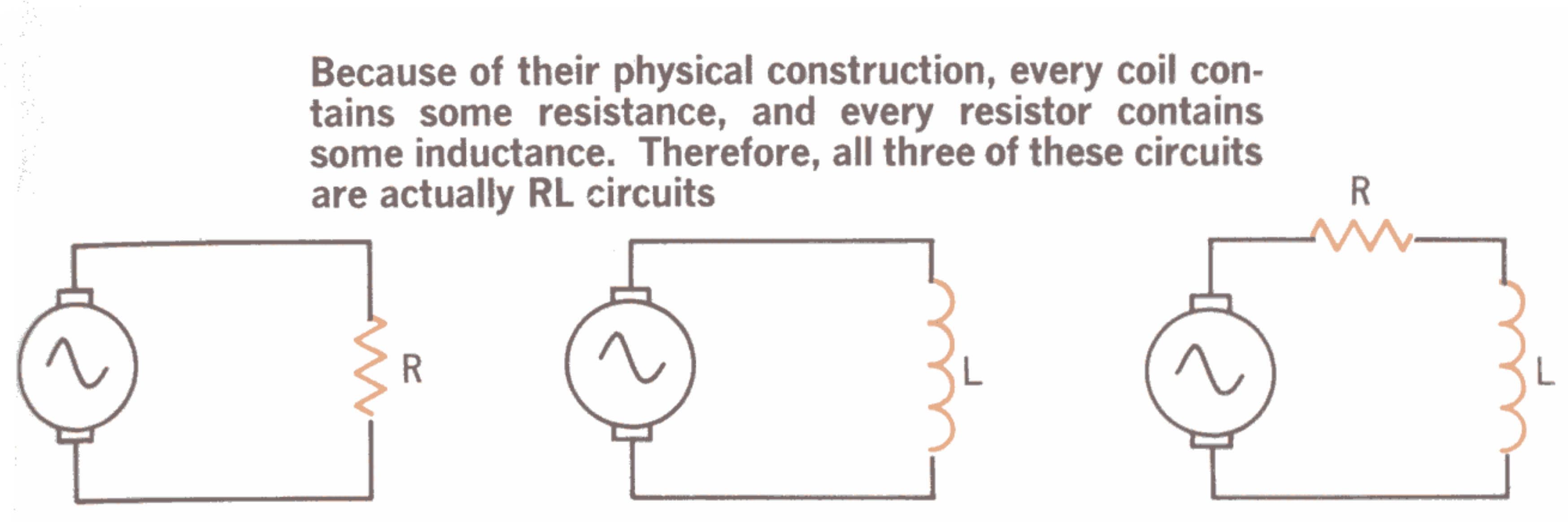
- Sine Waves
  - $\sin(2\pi ft + \theta)$
  - $\cos(2\pi ft + \theta)$
- Strength
  - Peak Value, Amplitude and Magnitude
  - Equivalent Value (RMS)
- Frequency: Hertz (Hz) – cycles/second
- Phase (Degrees or Radians)
- Period: time for one Cycle
- Wavelength (λ): distance traveled in one cycle

Session 3a (Vectors) Review

- Vector Analogy (frequency is not shown, corresponds to rotation)
  - Vector length corresponds to signal amplitude
  - Vector angle corresponds to phase (need a reference phase)
  - $\sin(2\pi ft)$: vertical vector (points up)
  - $\cos(2\pi ft)$: horizontal vector (points right)
- Vector Addition
  - Head-to-Tail
  - Parallelogram
- Vector Components (θ is the angle w.r.t. the horizontal axis)
  - Horizontal component: $A*\cos(\theta)$
  - Vertical component : $A*\sin(\theta)$
- You can add vectors by adding their corresponding components!

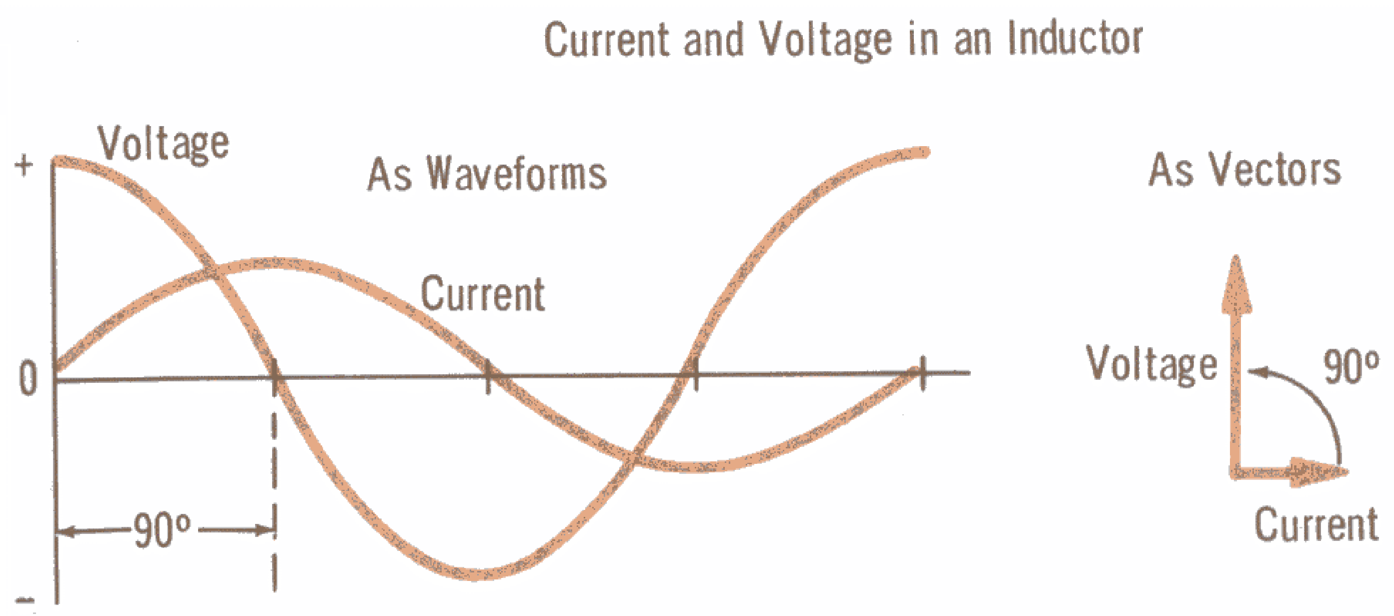
R-L Circuits

- 10:1 Ratio
  - If the effect of resistance (reactance) is 10 time larger than reactance (resistance), the smaller can be generally ignored in computations



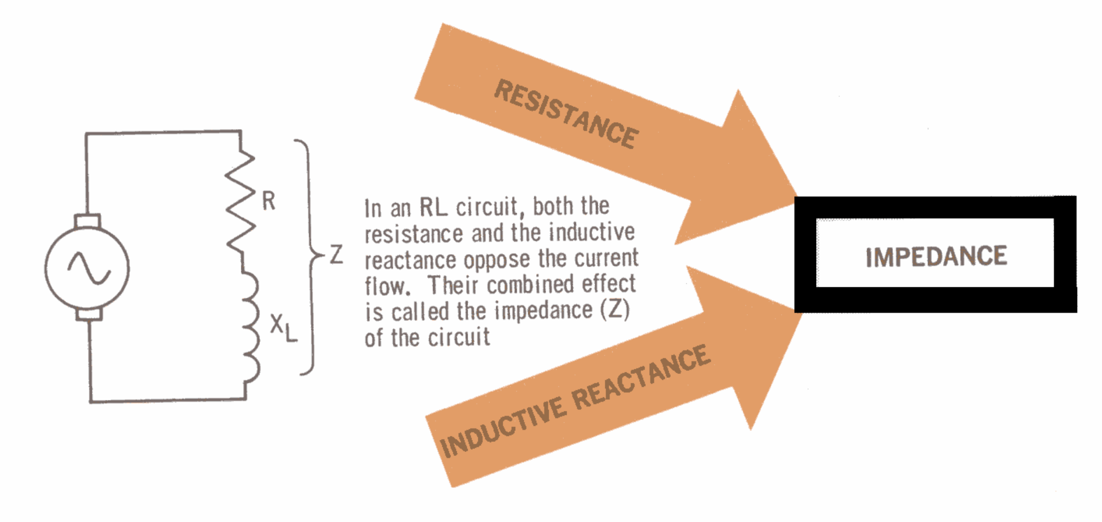
Current "Lags" Voltage

- Purely inductive circuit
  - Current lags voltage by 90º (Eli)



Series R-L Impedance

- Resistor
  - "resists current with an "in-phase" voltage drop (resistance)
- Inductor
  - Resists current with an "out-of-phase" voltage drop (reactance)



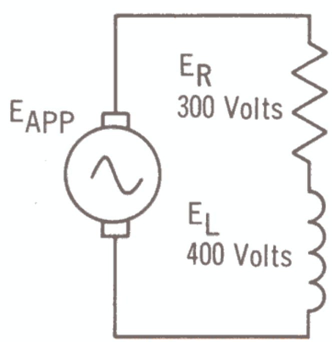
The combined effect (superposition again) is to have the current "lag" the voltage by a phase somewhere between 0 and 90 degrees. This "effective resistance" is called the impedance.

Series RL Voltage

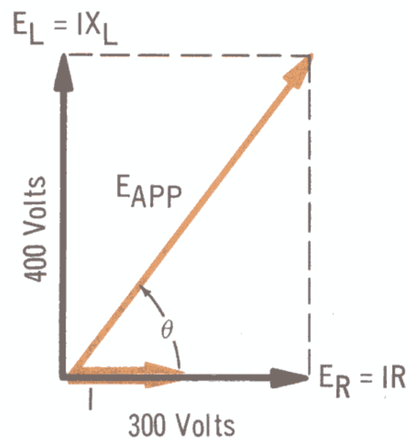
- The phase of the current is used as the reference here for convenience.

The relationship between the applied voltage and the voltage drops in a series RL circuit is such that the applied voltage equals the VECTOR SUM of the voltage drops

Circuit



Vector Representation



Calculation

$$E_{APP} = \sqrt{E_R^2 + E_L^2}$$

$$= \sqrt{(300)^2 + (400)^2}$$

$$= 500 \text{ Volts}$$

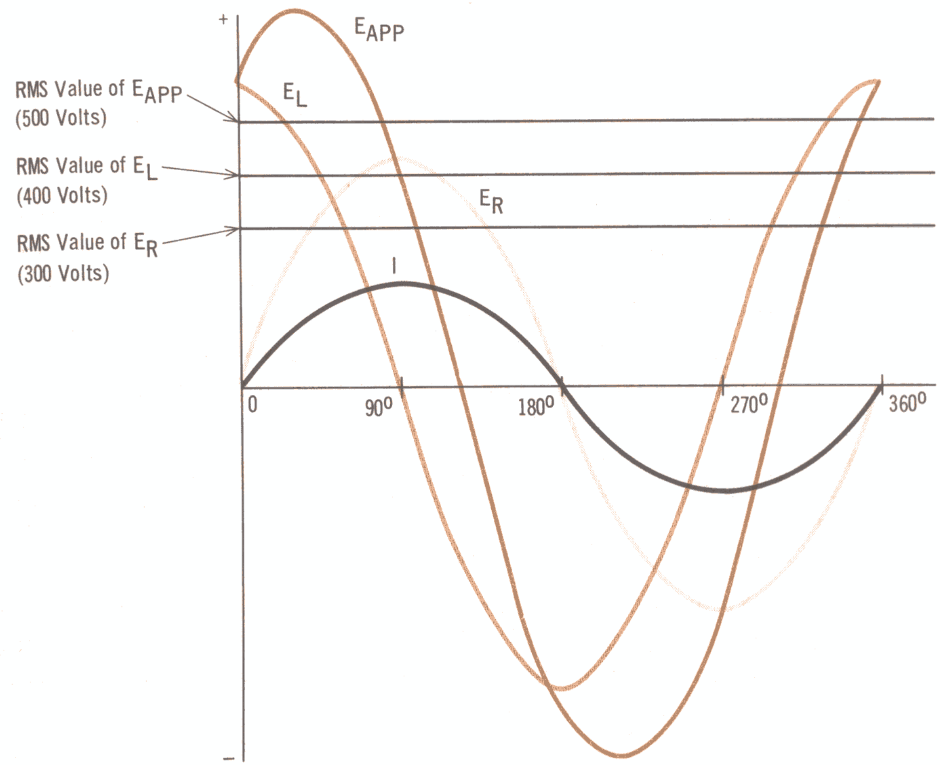
$$\tan \theta = E_L / E_R$$

$$= 400/300 = 1.33$$

$$\theta = 53.1°$$

Voltage Waveforms

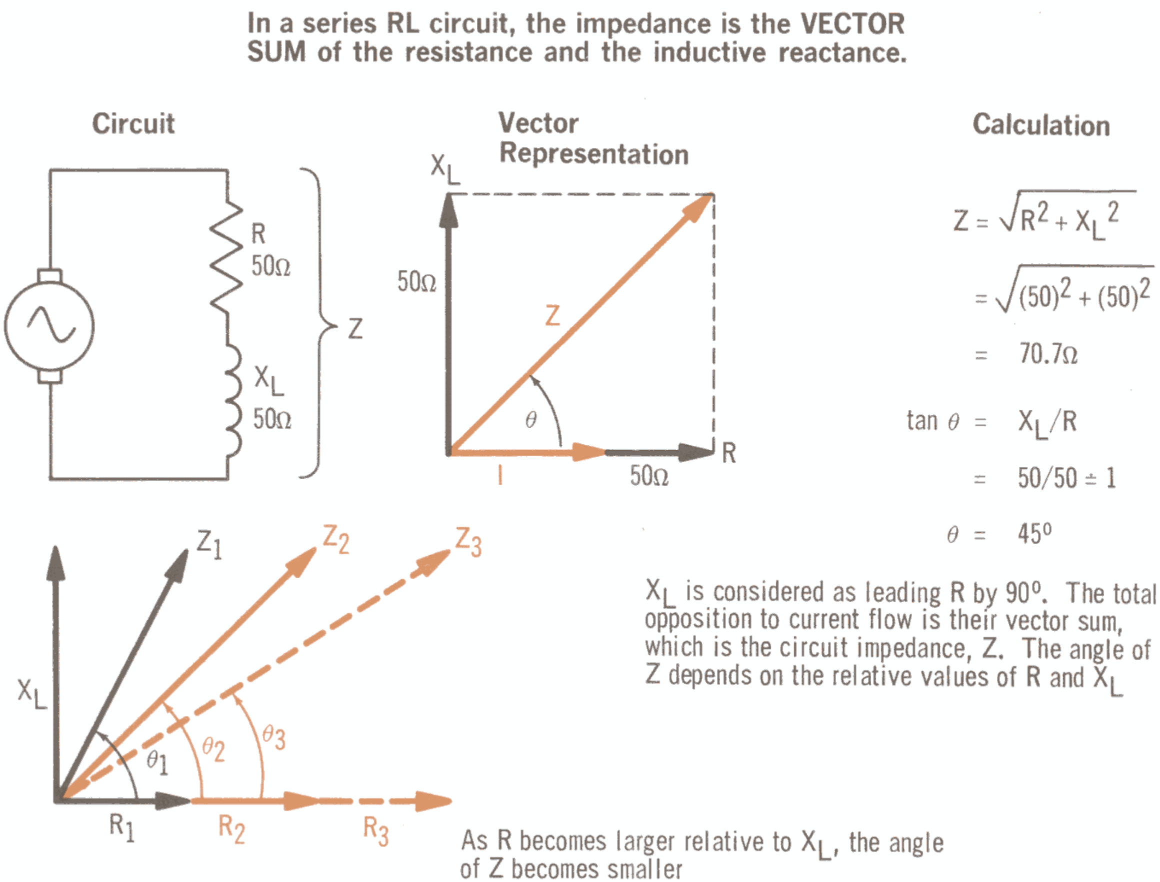
- The voltages are out of phase
  - Instantaneous waveform values add
  - Use vector addition!



Every point on the applied voltage waveform (E_{APP}) is the algebraic sum of the instantaneous values of the E_R and E_L waveforms. The rms value of the applied voltage waveform is equal to the vector sum of the rms values of the E_R and E_L waveforms

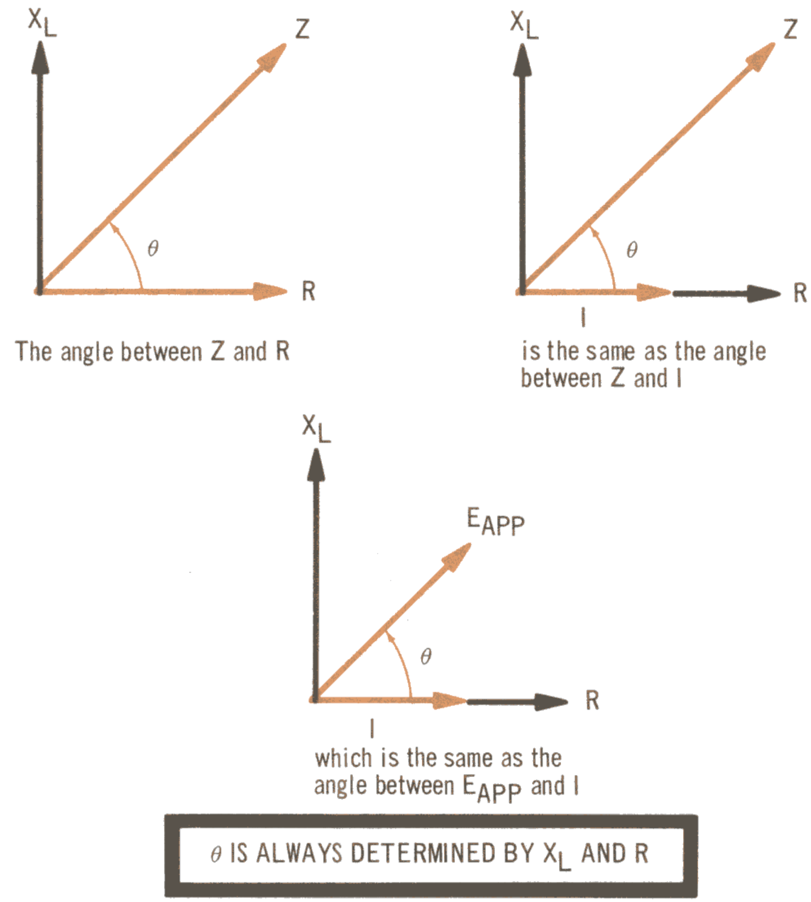
Series R-L Impedance

- Ohm's Law for AC
  - Add resistance and reactance as vectors
  - The series impedance (Z) results
  - E = I * Z or V = I * Z (and its variants)



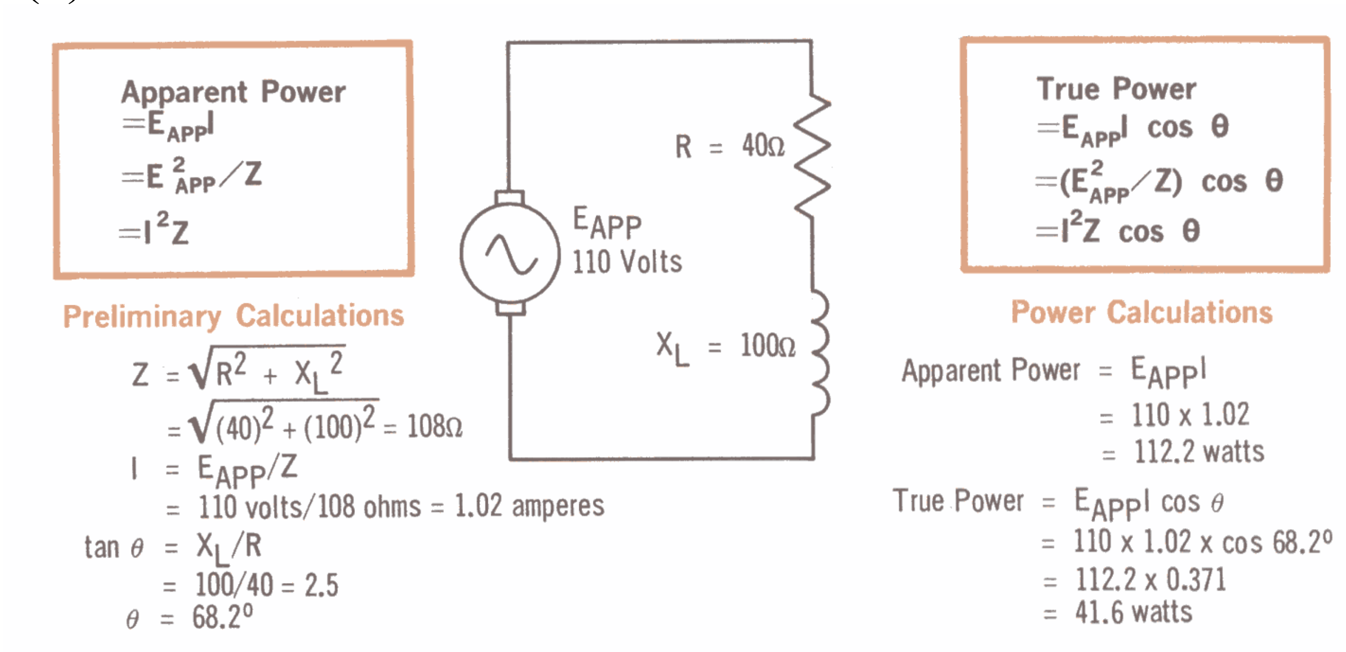
Impedance and Current

- Current is still the reference
- The phase between Z and R is the same as between $V_{applied}$ and I



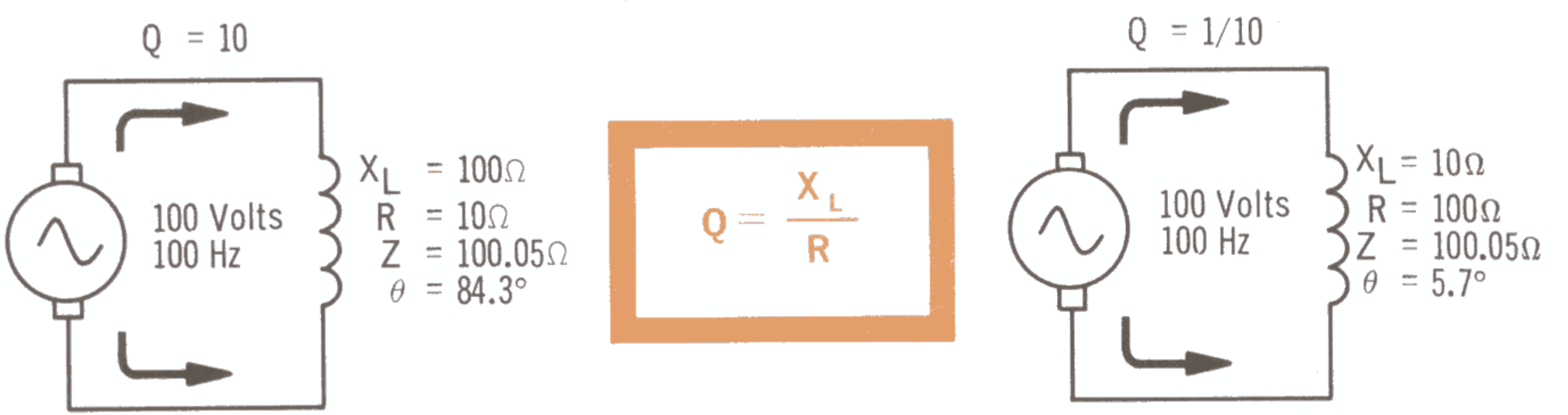
Series R-L Power

- Apparent power has magnitude (in volt-amps, called watts but different) and phase
- True power (watts) is dissipated in the resistance
- $P_{true} = P_{apparent} * \cos(\theta)$
- $\cos(\theta)$ is the "Power Factor"



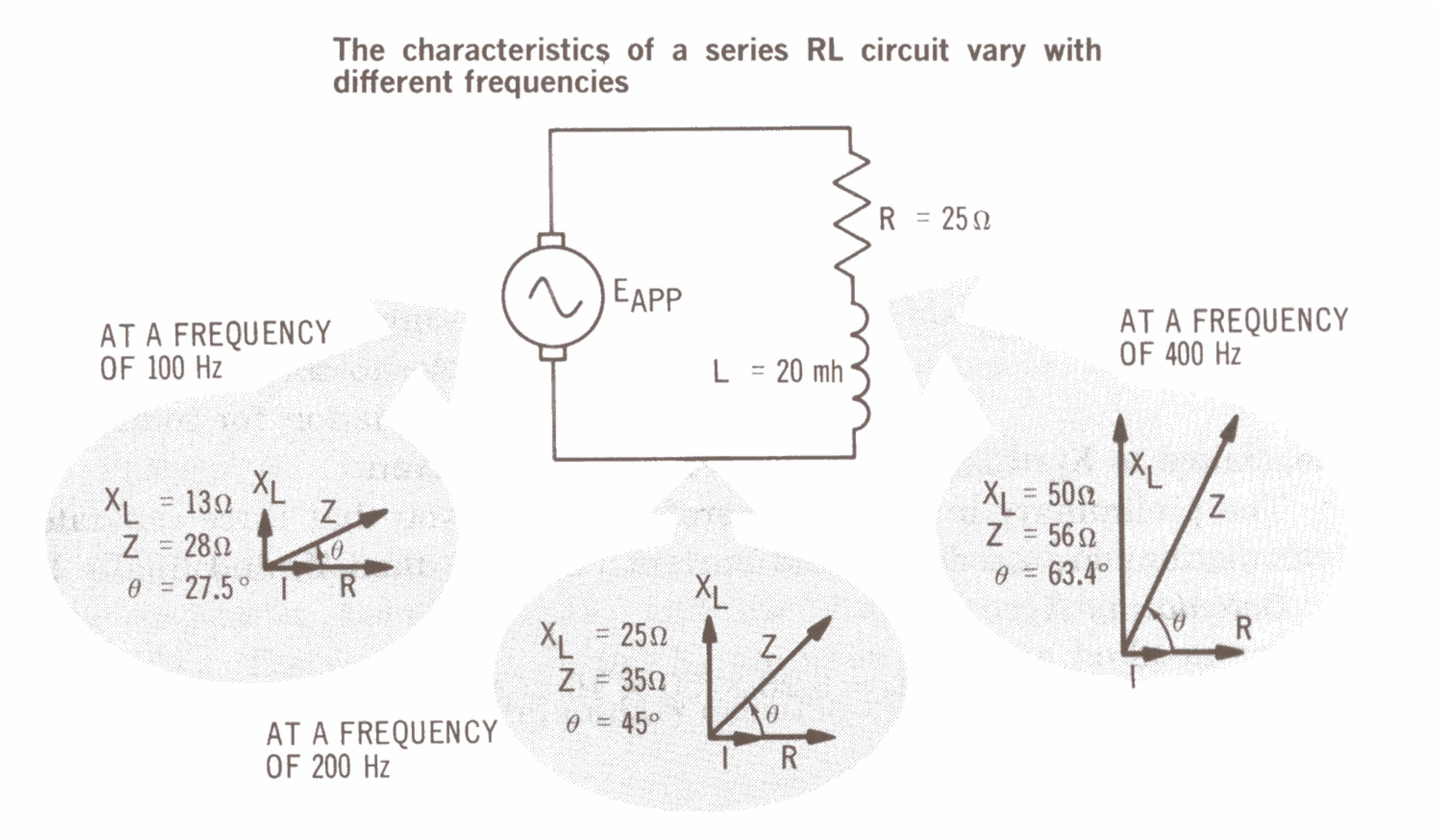
“Q” of a Coil

- Q here stands for quality
  - High Q means relatively low losses in the inductor



The high-Q coil produces a greater phase angle, and so is a better inductor

Effects of Frequency In Series R-L Circuits



Example Impedance Calculation

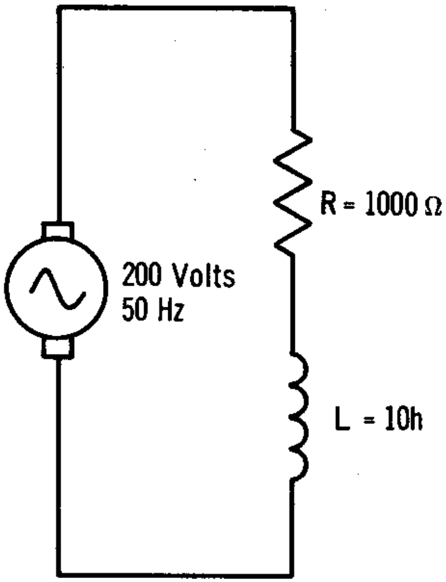
- $X_L = 2\pi * f * L = 6.28*50*10$

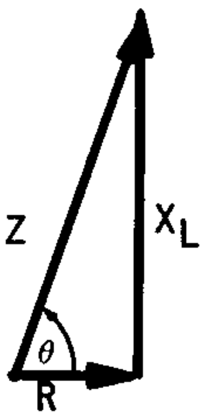
  or $X_L = 3140$ (at 90º)
- $|Z| = (R^2 + X_L{}^2)^{1/2} = 3295$
- $\angle Z = \tan^{-1}(X_L / R)$

  $= \tan^{-1}(3140 / 1000)$

  $= \tan^{-1}(3.14)$

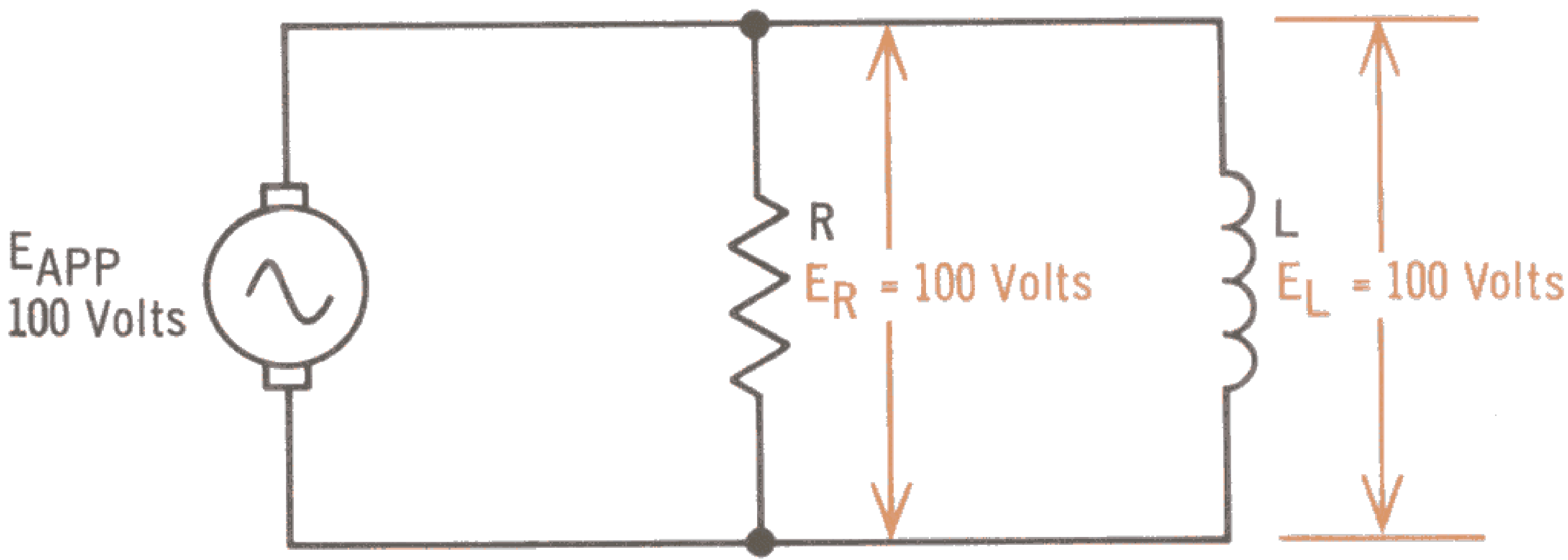
  $= 72.3º$





Parallel R-L Circuits

- Voltages are equal across all branches

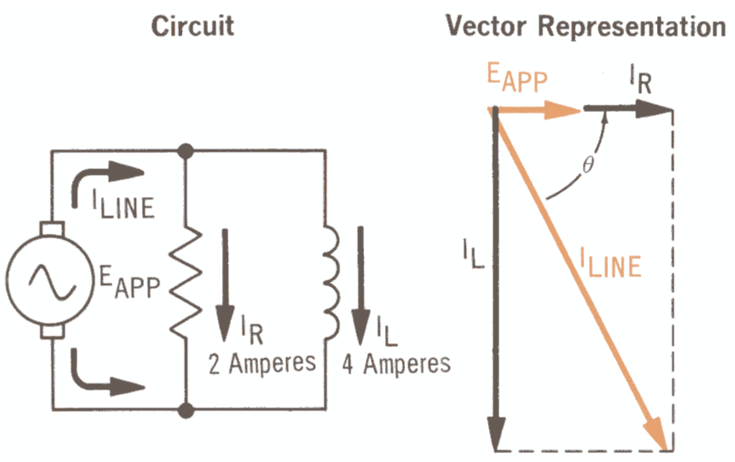


Branch and Total (Line) Current

- Branch currents add (as vectors) for the total (line) current (Kirchoff's current law)

$$|I_T| = (2^2 + 4^2)^{½} = (4 + 16)^{½}$$
$$= (20)^{½} = 2* (5)^{½}$$
$$= 2*2.236 \approx 4.5$$

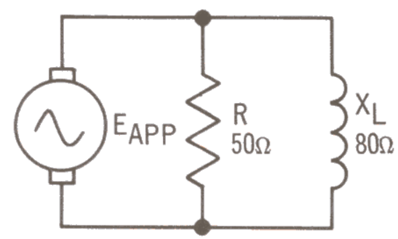
$$\angle I_T = \tan^{-1}(I_L / I_R)$$
$$= \tan^{-1}(-4 / 2)$$
$$= \tan^{-1}(-2)$$
$$= -63.4^{o} \text{ (-1.11 radians)}$$



Parallel R-L Impedance

- Parallel impedances add as inverses, but they are vectors.

Circuit



In a Parallel RL Circuit:

$$Z = \frac{R \times X_L}{\sqrt{R^2 + X_L^{\,2}}}$$

$$(1/Z) = (1/R) + (1/X_L)$$

$$= 1/50 + 1/(100\angle 90^{o}) = 0.02 + 0.01 \angle{-90^{o}}$$

$$= (.0005)^{1/2} \angle{-\tan^{-1}(1/2)} = (.0005)^{1/2} \angle{-\tan^{-1}(1/2)}$$

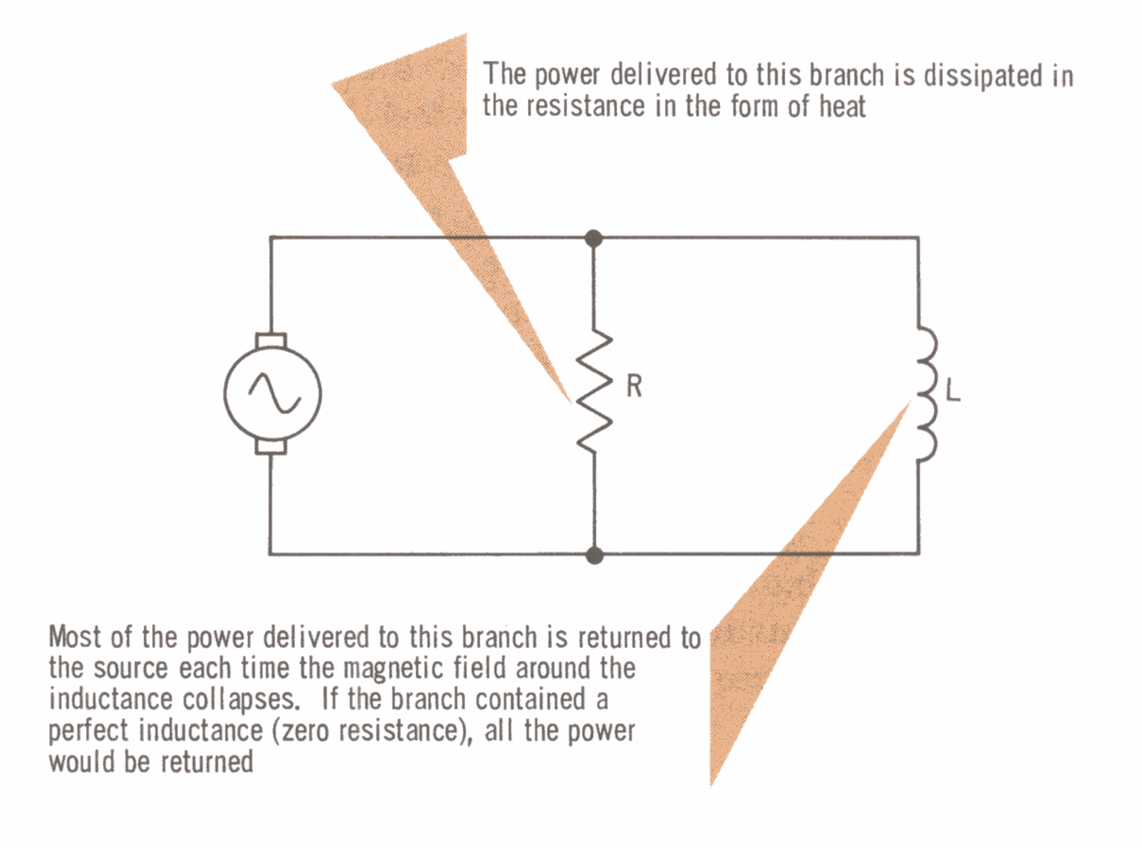
$$= (5)^{1/2} * 10^{-2} \angle{-\tan^{-1}(1/2)} = 0.0236 \angle{-26.6^{o}} \ (-0.464 \text{ r})$$

$$Z = (1/0.0236) \angle 26.6^{o} \ (0.464 \text{ r})$$

$$= 044.7 \angle 26.6^{o} \text{ Ohms (note: inductive impedance)}$$

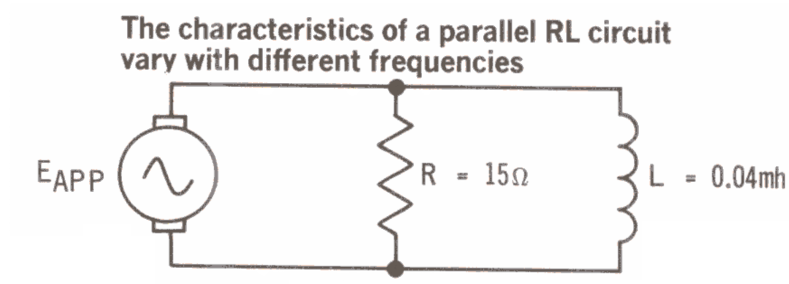
Power in Parallel R-L Circuits

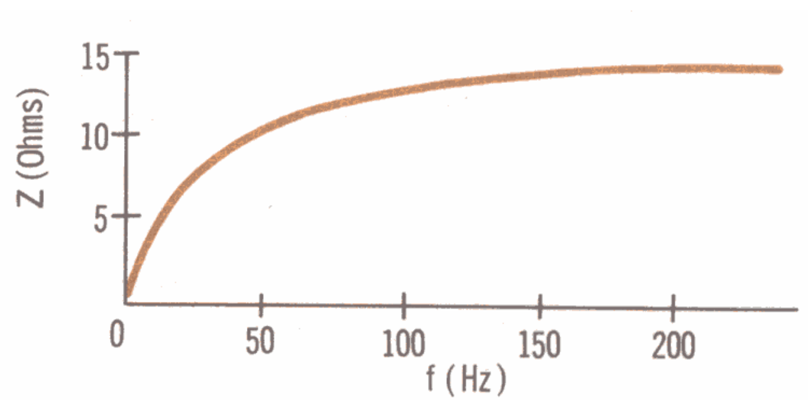
- True power is still only dissipated in the resistor
- Power is temporarily stored in the inductor and returned to the circuit



Effects of Frequency in Parallel R-L Circuits

- At high frequencies
  - Inductor is an open circuit
  - Impedance is the resistance (15 Ohms here)
- At DC (f = 0)
  - Inductor is a short circuit
  - Impedance is 0 (∠90º)





Section 3 Schedule:

Session 3a	– 05/13	Sine Waves, Magnitude, Phase and Vectors (again)	Text 4.1 – 4.24
3a continued	– 05/20	Complete 3a	
Session 3b	– 05/22	R-L Circuits (no class on 05/27)	Text 4.25 – 4.54
3b continued	**– 05/29**	Complete 3b	
Session 3c	– 06/03	R-C Circuits	Text 4.55 – 4.76
Session 3d	– 06/05	Series LC Circuits Series RLC Circuits	Text 4.77 – 4.88, 4.89 – 4.113
(lab - 06/08, Sat.)			
Session 3e	– 06/10	Parallel LC Circuits	Text 4.114 – 4.122,
(Quiz 3 due 06/17)		Parallel RLC Circuits	4.123 – 4.146
Session 3f	– 06/10	Review (Discuss Quiz 3)	