Alexander-Sadiku
Fundamentals of Electric Circuits

Chapter 11
AC Power Analysis

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Chapter 11

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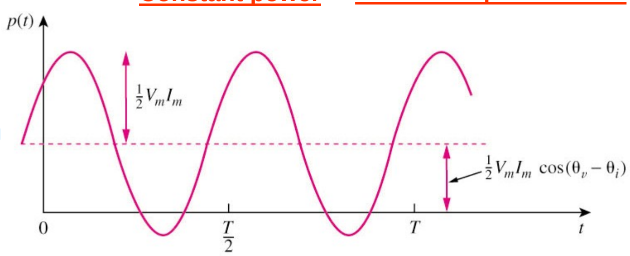
11.1 Instantaneous and Average Power (1)

The instantaneously power, p(t)

$$p(t) = v(t)\,i(t) = V_m I_m \cos(\omega t + \theta_v)\cos(\omega t + \theta_i)$$

$$= \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i) + \frac{1}{2}V_m I_m \cos(2\omega t + \theta_v + \theta_i)$$

Constant power   Sinusoidal power at 2ωt

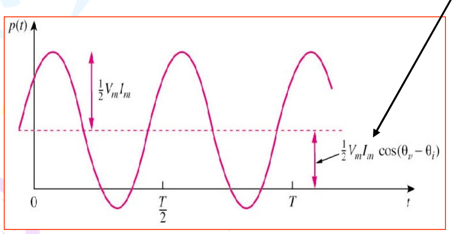


p(t) > 0: power is absorbed by the circuit; p(t) < 0: power is absorbed by the source.

11.1 Instantaneous and Average Power (2)

The average power, P, *is the average of the instantaneous power over one period.*

$$P = \frac{1}{T}\int_0^T p(t)\,dt = \frac{1}{2}V_m I_m \cos(\theta_v - \theta_i)$$



1. **P is not time dependent.**
2. **When $\theta_v = \theta_i$, it is a purely resistive load case.**
3. **When $\theta_v - \theta_i = \pm 90$o, it is a purely reactive load case.**
4. **P = 0 means that the circuit absorbs no average power.**

11.1 Instantaneous and Average Power (3)

Example 1

Calculate the instantaneous power and average power absorbed by a passive linear network if:

$$v(t) = 80\cos(10t + 20°)$$
$$i(t) = 15\sin(10t + 60°)$$

Answer: $385.7 + 600\cos(20t - 10°)\mathrm{W}, 387.5\mathrm{W}$

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11.1 Instantaneous and Average Power (4)

Example 2

A current $\mathbf{I} = 10\angle 30°$ flows through an impedance $\mathbf{Z} = 20\angle -22°\Omega$. Find the average power delivered to the impedance.

Answer: 927.2W

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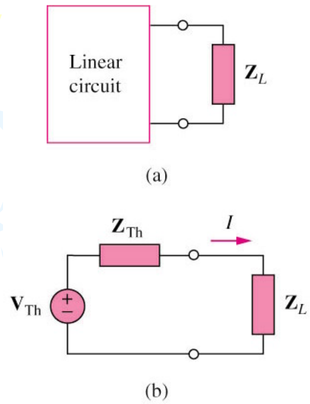
11.2 Maximum Average Power Transfer (1)



$$Z_{TH} = R_{TH} + jX_{TH}$$

$$Z_L = R_L + jX_L$$

The maximum average power can be transferred to the load if

$X_L = -X_{TH}$ **and** $R_L = R_{TH}$

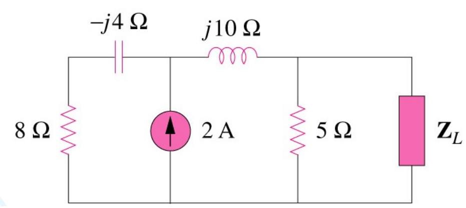
$$P_{max} = \frac{|V_{TH}|^2}{8R_{TH}}$$

If the load is purely real, then $R_L = \sqrt{R_{TH}^2 + X_{TH}^2} = |Z_{TH}|$

11.2 Maximum Average Power Transfer (2)

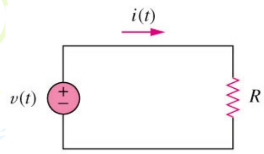
Example 3

For the circuit shown below, find the load impedance Z_L that absorbs the maximum average power. Calculate that maximum average power.

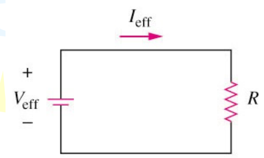


Answer: 3.415   j0.7317Ω, 1.429W

11.3 Effective or RMS Value (1)



(a)



(b)

The total power dissipated by R is given by:

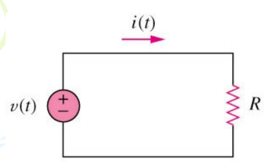
$$P = \frac{1}{T}\int_0^T i^2 R dt = \frac{R}{T}\int_0^T i^2 dt = I_{rms}^2 R$$

Hence, I_{eff} is equal to: $\boxed{I_{eff} = \sqrt{\frac{1}{T}\int_0^T i^2\, dt} = I_{rms}}$

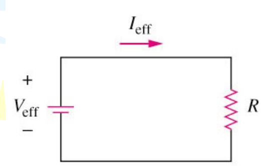
The rms value is a constant itself which depending on the shape of the function i(t).

The effective of a periodic current is the dc current that delivers the same average power to a resistor as the periodic current.

11.3 Effective or RMS Value (2)



(a)



(b)

The rms value of a sinusoid $i(t) = I_m\cos(\omega t)$ is given by:

$$\boxed{I_{rms}^2 = \frac{I_m}{\sqrt{2}}}$$

The average power can be written in terms of the rms values:

$$\boxed{I_{eff} = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i) = V_{rms} I_{rms} \cos(\theta_v - \theta_i)}$$

Note: If you express amplitude of a phasor source(s) in rms, then all the answer as a result of this phasor source(s) must also be in rms value.

11.4 Apparent Power and Power Factor (1)

Apparent Power, S, is the product of the r.m.s. values of voltage and current.

It is measured in volt-amperes or VA to distinguish it from the average or real power which is measured in watts.

$$\boxed{P = V_{rms}\, I_{rms} \cos(\theta_v - \theta_i) = \underbrace{S}_{\text{Apparent Power, S}} \underbrace{\cos(\theta_v - \theta_i)}_{\text{Power Factor, pf}}}$$

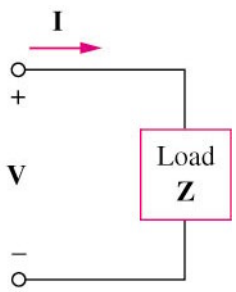
Power factor is the cosine of the phase difference between the voltage and current. It is also the cosine of the angle of the load impedance.

11.4 Apparent Power and Power Factor (2)

Purely resistive load (R)	$\theta_v - \theta_i = 0$, Pf = 1	P/S = 1, all power are consumed
Purely reactive load (L or C)	$\theta_v - \theta_i = \pm 90^o$, pf = 0	P = 0, no real power consumption
Resistive and reactive load (R and L/C)	$\theta_v - \theta_i > 0$ $\theta_v - \theta_i < 0$	• Lagging - inductive load • Leading - capacitive load

11.5 Complex Power (1)

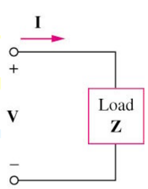
Complex power **S** is the product of the voltage and the complex conjugate of the current:



$$\mathbf{V} = V_m \angle \theta_v \qquad \mathbf{I} = I_m \angle \theta_i$$

$$\frac{1}{2} \mathrm{V}\, \mathrm{I}^* = V_{rms}\, I_{rms} \angle \theta_v - \theta_i$$

11.5 Complex Power (2)



$$S = \frac{1}{2} \mathrm{V}\, \mathrm{I}^* = V_{rms}\, I_{rms} \angle \theta_v - \theta_i$$

$$\Rightarrow S = \underbrace{V_{rms}\, I_{rms} \cos(\theta_v - \theta_i)}_{P} + j\, \underbrace{V_{rms}\, I_{rms} \sin(\theta_v - \theta_i)}_{Q}$$

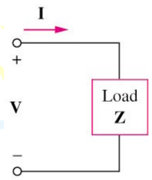
***S* = P + j Q**

P: is the average power in watts delivered to a load and it is the only useful power.

Q: is the reactive power exchange between the source and the reactive part of the load. It is measured in VAR.

- Q = 0 for *resistive loads* (unity pf).
- Q < 0 for *capacitive loads* (leading pf).
- Q > 0 for *inductive loads* (lagging pf).

11.5 Complex Power (3)



$$\Rightarrow S = \underbrace{V_{rms}\, I_{rms} \cos(\theta_v - \theta_i)}_{P} + j\, \underbrace{V_{rms}\, I_{rms} \sin(\theta_v - \theta_i)}_{Q}$$

S = P + j Q

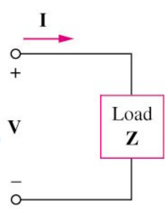
Apparent Power, $S = |\mathbf{S}| = V_{rms} * I_{rms} = \sqrt{P^2 + Q^2}$

Real power, $P = \mathrm{Re}(\mathbf{S}) = S \cos(\theta_v \quad \theta_i)$

Reactive Power, $Q = \mathrm{Im}(\mathbf{S}) = S \sin(\theta_v \quad \theta_i)$

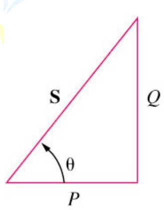
Power factor, $pf = P/S = \cos(\theta_v \quad \theta_i)$

11.5 Complex Power (4)

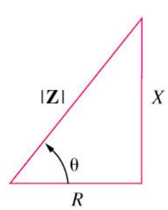


$$\Rightarrow S = \underbrace{V_{rms}\, I_{rms} \cos(\theta_v - \theta_i)}_{P} + j\, \underbrace{V_{rms}\, I_{rms} \sin(\theta_v - \theta_i)}_{Q}$$

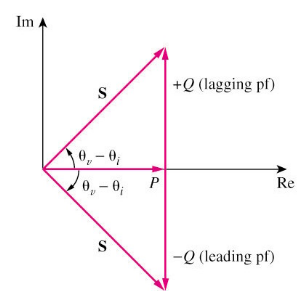
S = P + j Q



Power Triangle



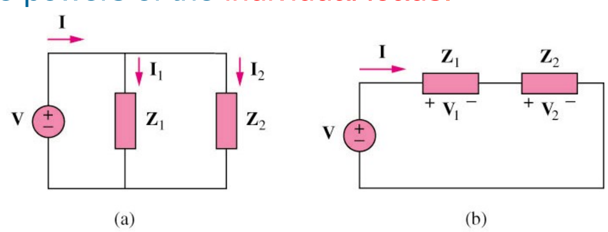
Impedance Triangle



Power Factor

11.6 Conservation of AC Power (1)

The complex real, and reactive powers of the sources **equal** the respective **sums of** the complex, real, and reactive powers of the individual loads.



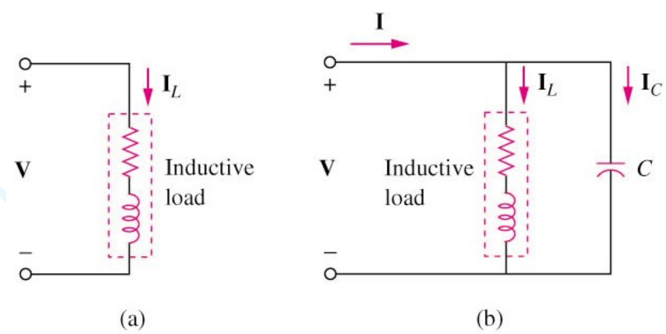
For parallel connection:

$$\overline{S} = \frac{1}{2}\overline{V}\,\overline{I}^* = \frac{1}{2}\overline{V}(\overline{I_1}^* + \overline{I_2}^*) = \frac{1}{2}\overline{V}\,\overline{I_1}^* + \frac{1}{2}\overline{V}\,\overline{I_2}^* = \overline{S_1} + \overline{S_2}$$

The same results can be obtained for a series connection.

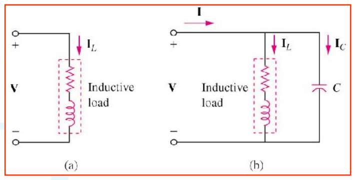
11.7 Power Factor Correction (1)

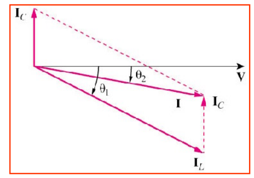
Power factor correction is the process of increasing the power factor without altering the voltage or current to the original load.

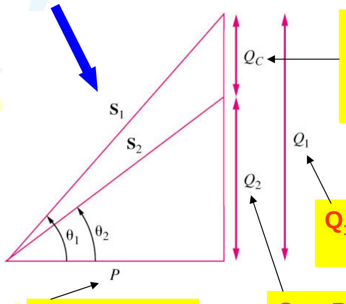


Power factor correction is necessary for economic reason.

11.7 Power Factor Correction (2)







$Q_c = Q_1 - Q_2$
$= P\,(\tan\theta_1 - \tan\theta_2)$
$= \omega C V_{2rms}$

$Q_1 = S_1 \sin\theta_1$
$= P \tan\theta_1$

$P = S_1 \cos\theta_1$

$Q_2 = P \tan\theta_2$

$$C = \frac{Q_c}{\omega V_{rms}^2} = \frac{P\,(\tan\theta_1 - \tan\theta_2)}{\omega\, V_{rms}^2}$$

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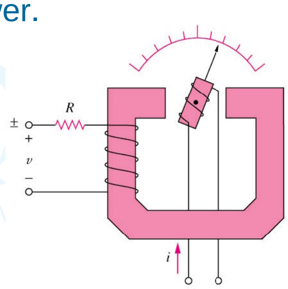
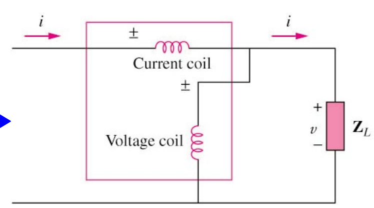
11.8 Power Measurement (1)

The wattmeter is the instrument for measuring the average power.



The basic structure



Equivalent Circuit with load

If $v(t) = V_m \cos(\omega t + \theta_v)$ and $i(t) = I_m \cos(\omega t + \theta_i)$

$$P = |V_{rms}||I_{rms}| \cos(\theta_v - \theta_i) = \tfrac{1}{2} V_m\, I_m \cos(\theta_v - \theta_i)$$

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