

Lessons from Nanoscience:  
A Lecture Note Series

Vol. **1**

# Lessons from **Nanoelectronics** A New Perspective on Transport

**Supriyo Datta**

*Purdue University, USA*

 **World Scientific**

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