SERIES PARALLEL RESISTOR COMBINATIONS

UP TO NOW WE HAVE STUDIED CIRCUITS THAT CAN BE ANALYZED WITH ONE APPLICATION OF KVL(SINGLE LOOP) OR KCL(SINGLE NODE-PAIR)

WE HAVE ALSO SEEN THAT IN SOME SITUATIONS IT IS ADVANTAGEOUS TO COMBINE RESISTORS TO SIMPLIFY THE ANALYSIS OF A CIRCUIT

NOW WE EXAMINE SOME MORE COMPLEX CIRCUITS WHERE WE CAN SIMPLIFY THE ANALYSIS USING THE TECHNIQUE OF COMBINING RESISTORS…

… PLUS THE USE OF OHM'S LAW

SERIES COMBINATIONS

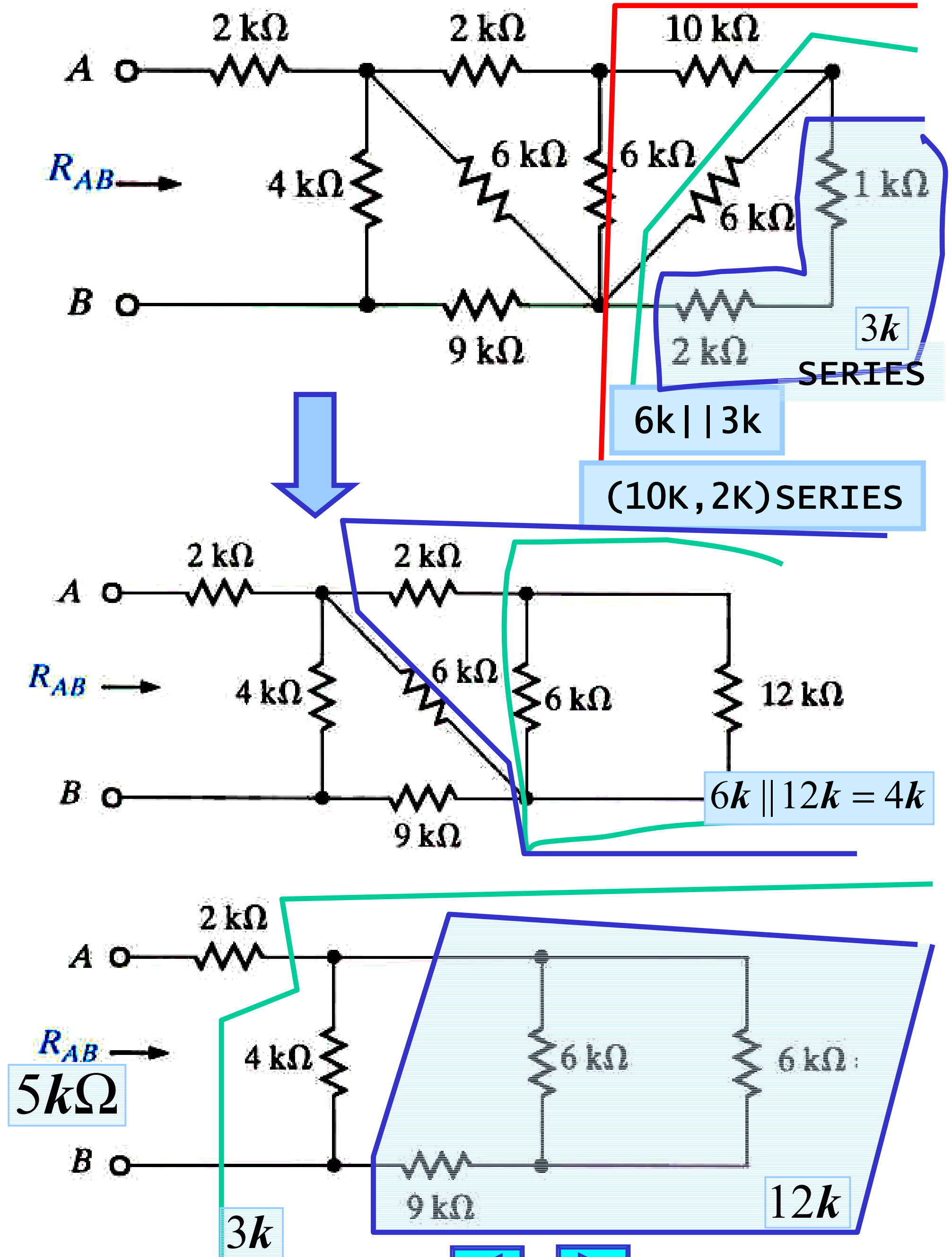
$$R_S = R_1 + R_2 + \cdots + R_N$$

PARALLEL COMBINATION

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \cdots + \frac{1}{R_N}$$

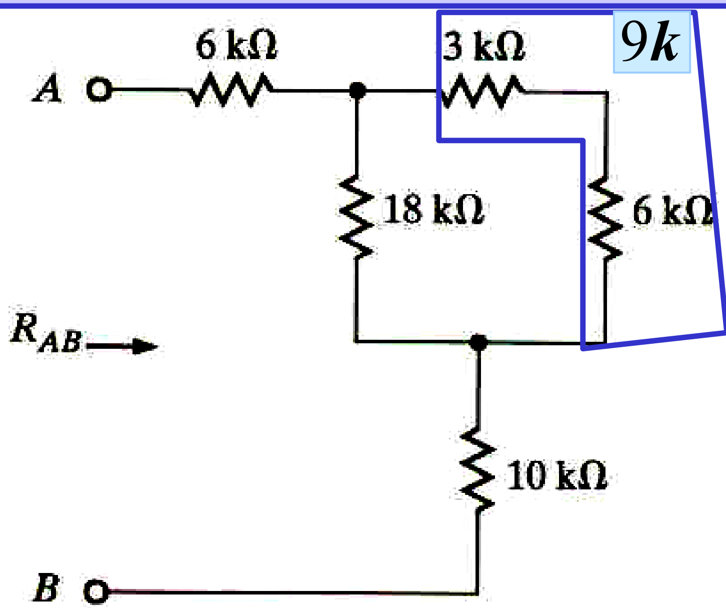
$$G_p = G_1 + G_2 + \ldots + G_N$$

FIRST WE PRACTICE COMBINING RESISTORS

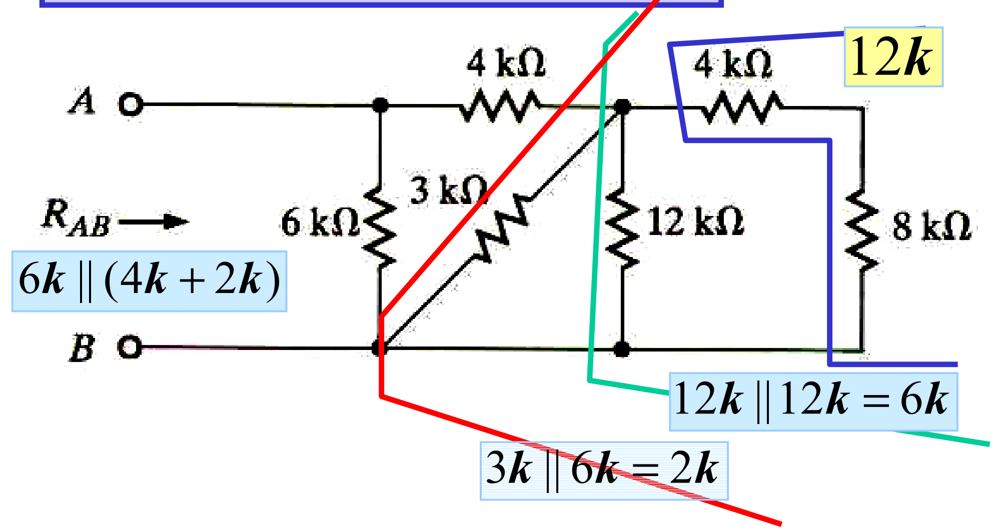


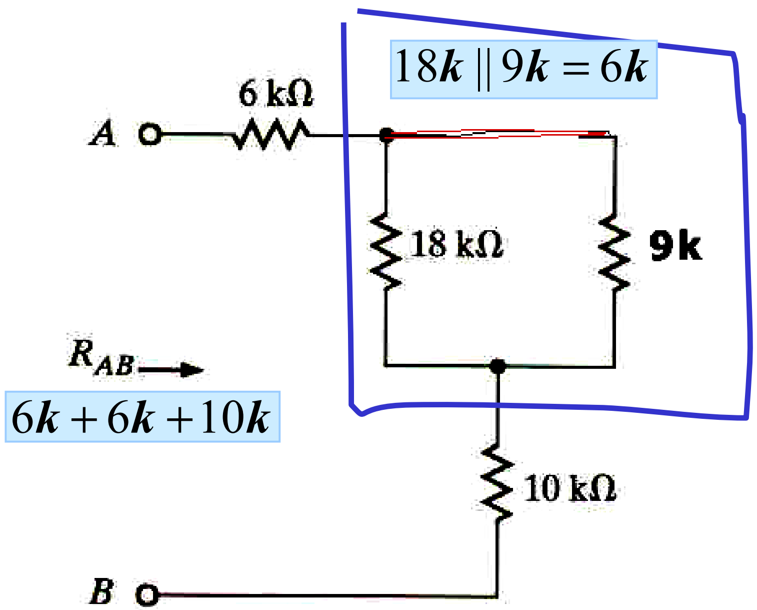
GEAUX

EXAMPLES COMBINATION SERIES-PARALLEL



AN EXAMPLE WITHOUT REDRAWING



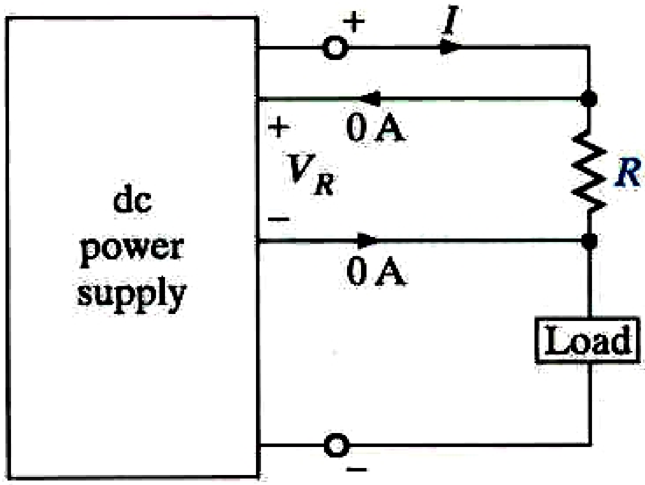


RESISTORS ARE IN SERIES IF THEY CARRY EXACTLY THE SAME CURRENT

RESISTORS ARE IN PARALLEL IF THEY ARE CONNECTED EXACTLY BETWEEN THE SAME TWO NODES

GEAUX

AN "INVERSE SERIES PARALLEL COMBINATION"



SIMPLE CASE

V_R MUST BE 600mV WHEN $I = 3A$

ONLY 0.1Ω RESISTORS ARE AVAILABLE

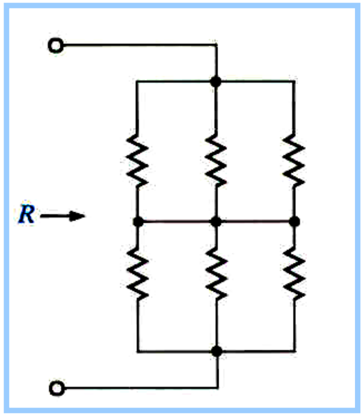
REQUIRED $R = \frac{.6V}{3A} = 0.2\Omega \Rightarrow$ $R = 0.1\Omega + 0.1\Omega$

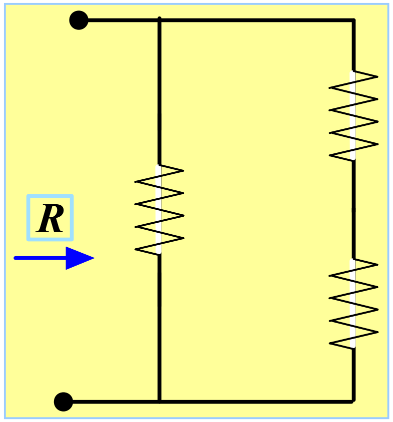
NOT SO SIMPLE CASE

V_R MUST BE 600mV WHEN $I = 9A$

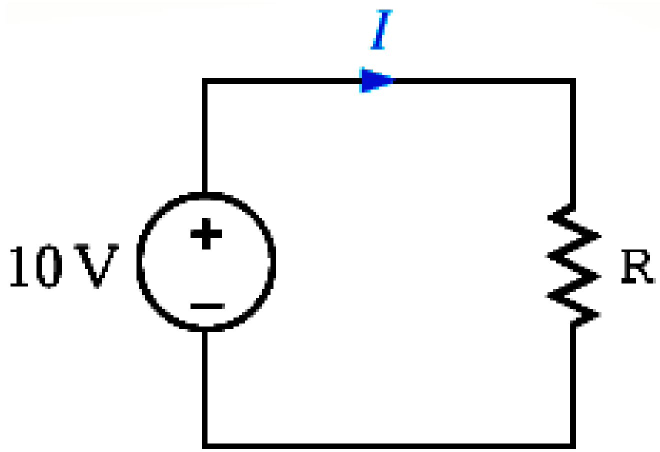
ONLY 0.1Ω RESISTORS ARE AVAILABLE

REQUIRED $R = \frac{.6V}{9A} = 0.0667\Omega \Rightarrow$





EFFECT OF RESISTOR TOLERANCE



NOMINAL RESISTOR VALUE: 2.7kΩ

RESISTOR TOLERANCE: 10%

RANGES FOR CURRENT AND POWER?

NOMINAL CURRENT: $\bar{I} = \frac{10}{2.7} = 3.704\,mA$

NOMINAL POWER: $\bar{P} = \frac{(10)^2}{2.7} = 37.04\,mW$

MINIMUM CURRENT: $I_{min} = \frac{10}{1.1 \times 2.7} = 3.367\,mA$

MAXIMUM CURRENT: $I_{max} = \frac{10}{0.9 \times 2.7} = 4.115\,mA$

MINIMUM POWER(VI_{min}): $33.67\,mW$

MAXIMUM POWER: $41.15\,mW$

THE RANGES FOR CURRENT AND POWER ARE DETERMINED BY THE TOLERANCE BUT THE PERCENTAGE OF CHANGE MAY BE DIFFERENT FROM THE PERCENTAGE OF TOLERANCE. THE RANGES MAY NOT EVEN BE SYMMETRIC

GEAUX

CIRCUIT WITH SERIES-PARALLEL RESISTOR COMBINATIONS

THE COMBINATION OF COMPONENTS CAN REDUCE THE COMPLEXITY OF A CIRCUIT AND RENDER IT SUITABLE FOR ANALYSIS USING THE BASIC TOOLS DEVELOPED SO FAR.

COMBINING RESISTORS IN SERIES ELIMINATES ONE NODE FROM THE CIRCUIT.
COMBINING RESISTORS IN PARALLEL ELIMINATES ONE LOOP FROM THE CIRCUIT

GENERAL STRATEGY:

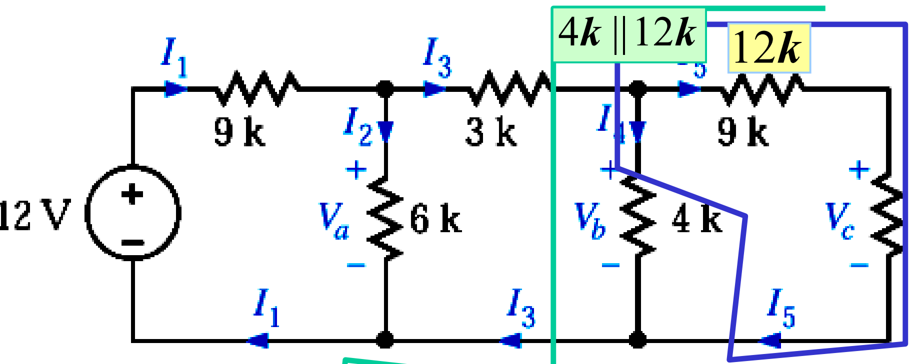
- REDUCE COMPLEXITY UNTIL THE CIRCUIT BECOMES SIMPLE ENOUGH TO ANALYZE.
- USE DATA FROM SIMPLIFIED CIRCUIT TO COMPUTE DESIRED VARIABLES IN ORIGINAL CIRCUIT - HENCE ONE MUST KEEP TRACK OF ANY RELATIONSHIP BETWEEN VARIABLES

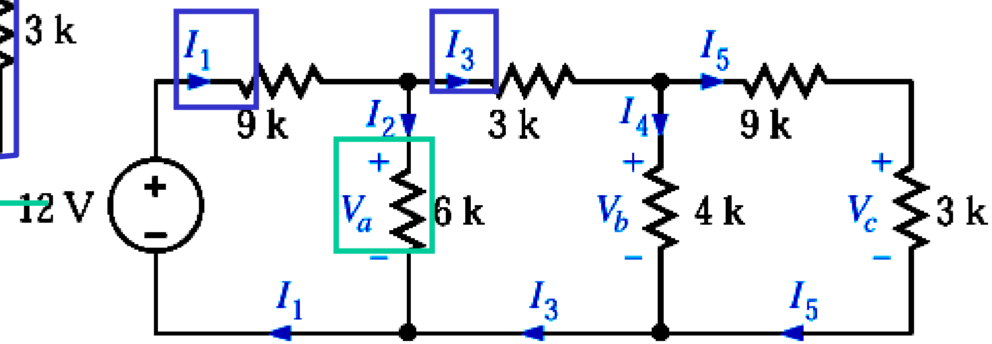
GEAUX

We wish to find all the currents and voltages labeled in the ladder network shown

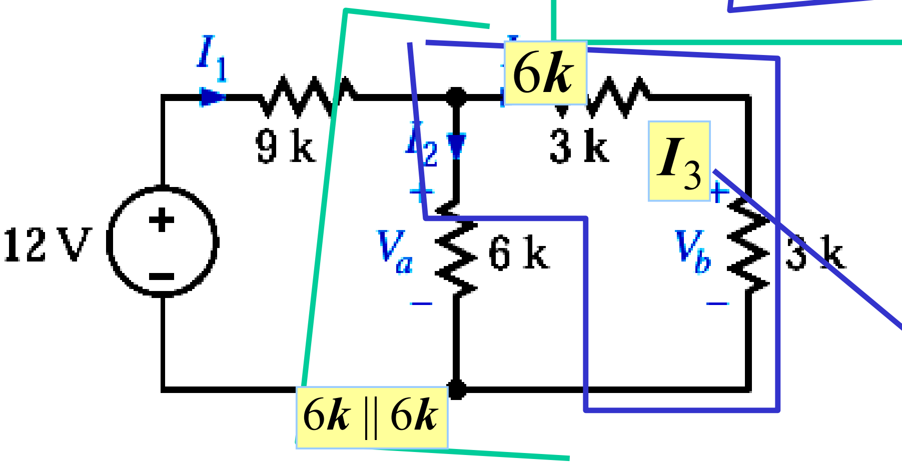
FIRST REDUCE IT TO A SINGLE LOOP CIRCUIT

SECOND: "BACKTRACK" USING KVL, KCL OHM'S





KCL : $I_1 - I_2 - I_3 = 0$



OHM'S : $I_2 = \dfrac{V_a}{6k}$

OHM'S : $V_b = 3k * I_3$

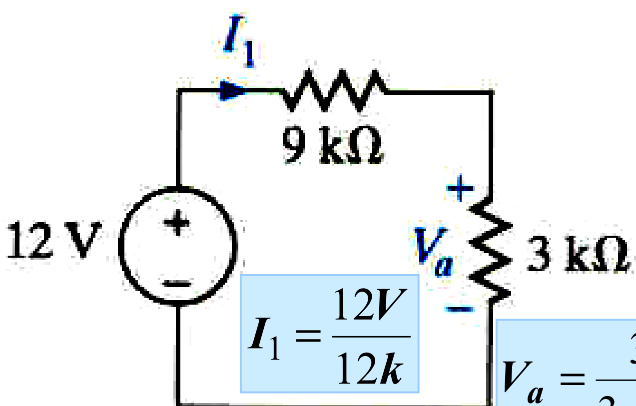
...OTHER OPTIONS...

$$I_4 = \frac{12}{4+12} I_3$$

$$V_b = 4k * I_4$$

KCL : $I_5 + I_4 - I_3 = 0$

OHM'S : $V_C = 3k * I_5$



$$I_1 = \frac{12V}{12k}$$

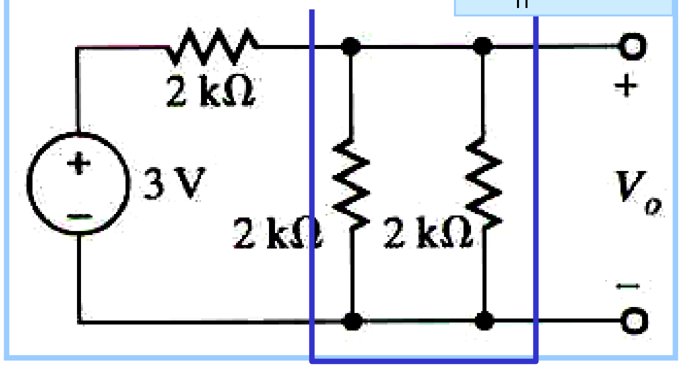
$$V_a = \frac{3}{3+9}(12)$$

GEAUX

LEARNING BY DOING

Find V_o in the following network:

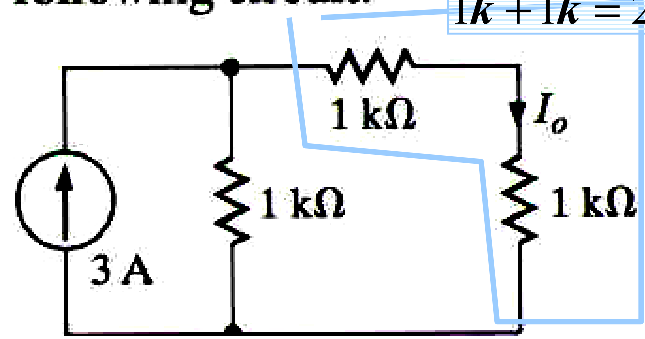
$2k \parallel 2k = 1k$



VOLTAGE DIVIDER : $V_o = \frac{1k}{1k + 2k}(3V) = 1V$

Find I_o in the following circuit:

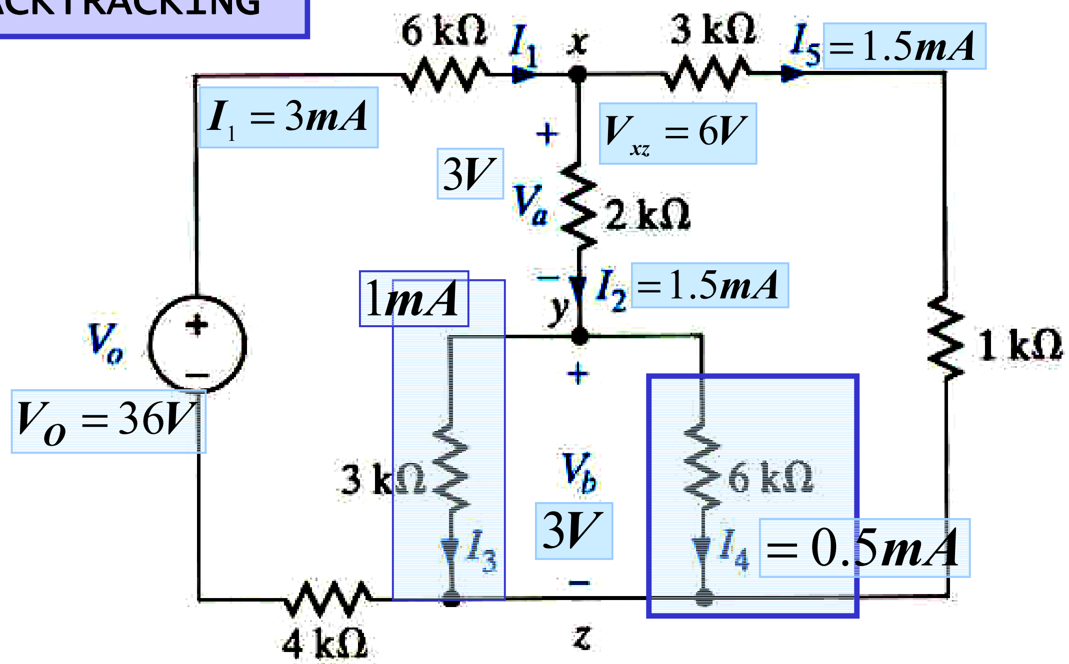
$1k + 1k = 2k$



CURRENT DIVIDER : $I_o = \frac{1k}{1k + 2k}(3A) = 1A$

GEAUX

AN EXAMPLE OF "BACKTRACKING"



$I_4 = \frac{1}{2}$ mA, let us find the source voltage V_o.

A STRATEGY. ALWAYS ASK: "WHAT ELSE CAN I COMPUTE?"

$V_b = 6k * I_4$

$I_3 = \frac{V_b}{3k}$

$I_2 = I_3 + I_4$

$V_a = 2k * I_2$

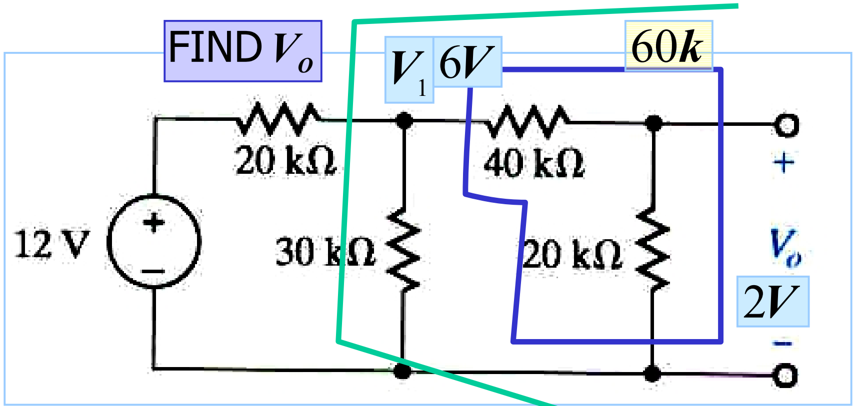
$V_{xz} = V_a + V_b$

$I_5 = \frac{V_{xz}}{4k}$

$I_1 = I_2 + I_5$

$V_O = 6k * I_1 + V_{xz} + 4k * I_1$

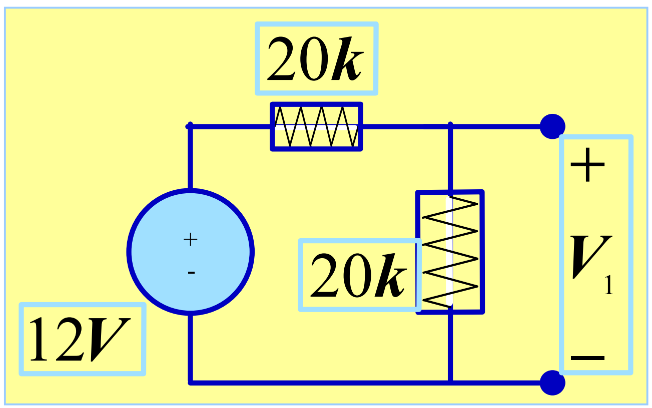
GEAUX



$30k \| 60k = 20k$

STRATEGY : FIND V_1

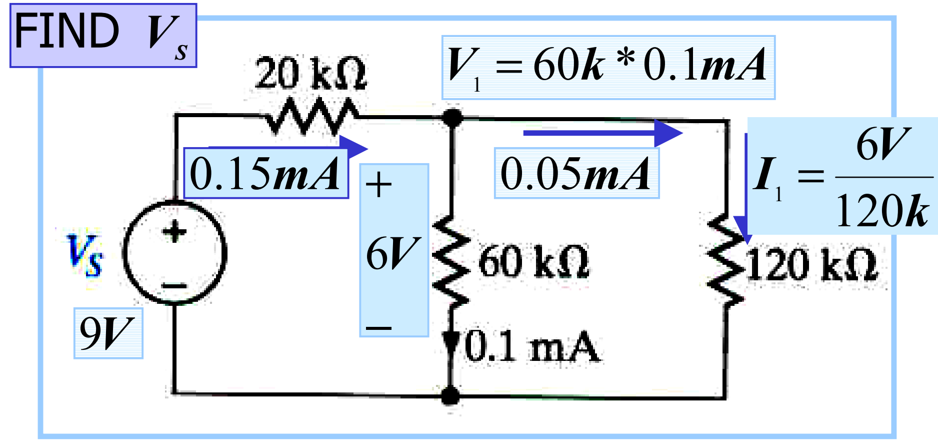
USE VOLTAGE DIVIDER



$$= \frac{20k}{20k + 20k}(12) = 6V$$

VOLTAGE DIVIDER

$$V_o = \frac{20k}{20k + 40k} V_1$$



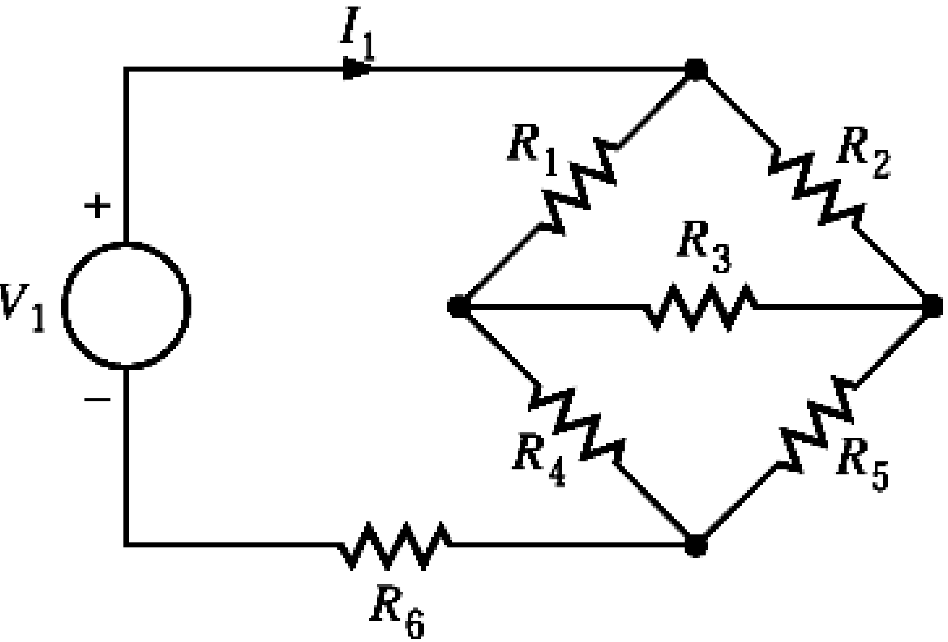
THIS IS AN INVERSE PROBLEM
WHAT CAN BE COMPUTED?

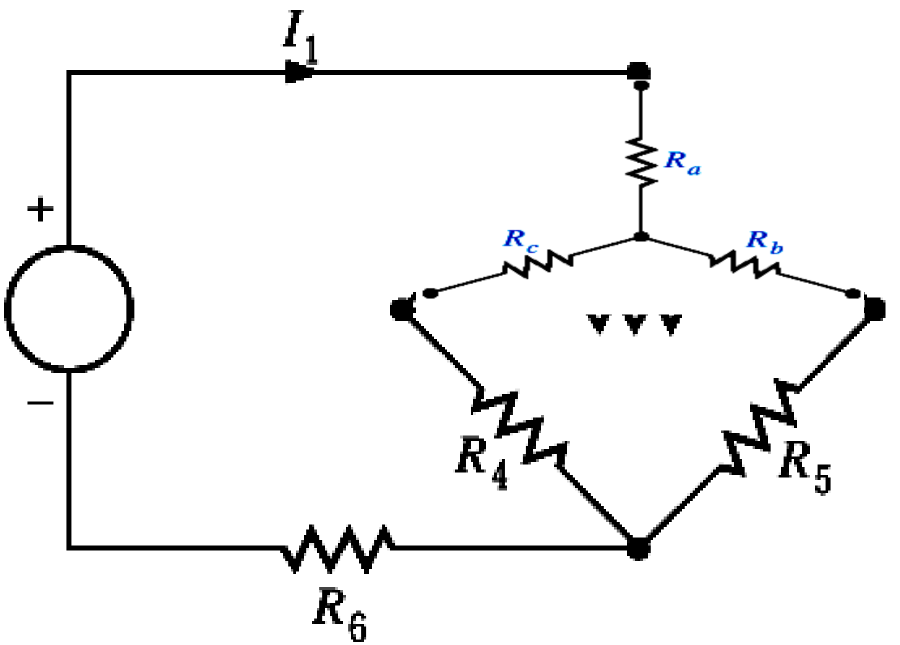
$$V_s = 20k * 0.15mA + 6V$$

SERIES
PARALLEL

STOP

$Y-\Delta$ TRANSFORMATIONS

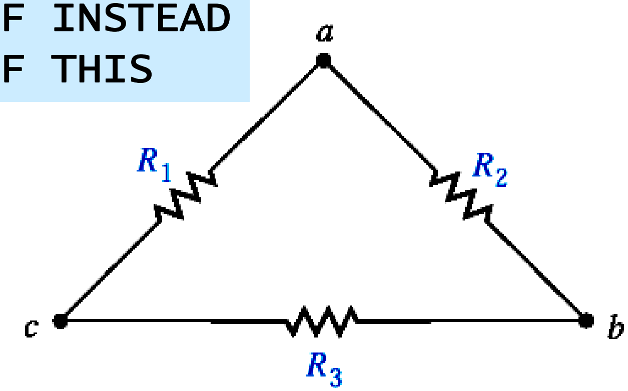




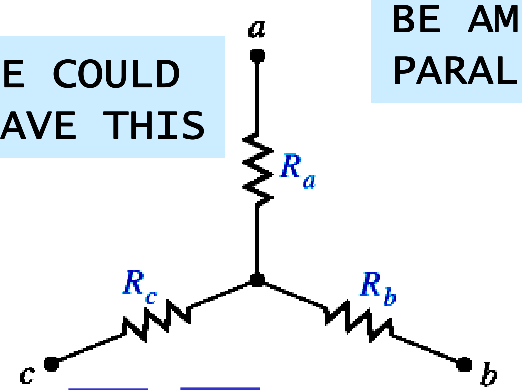
THIS CIRCUIT HAS NO RESISTOR IN SERIES OR PARALLEL

THEN THE CIRCUIT WOULD BECOME LIKE THIS AND BE AMENABLE TO SERIES PARALLEL TRANSFORMATIONS

IF INSTEAD OF THIS

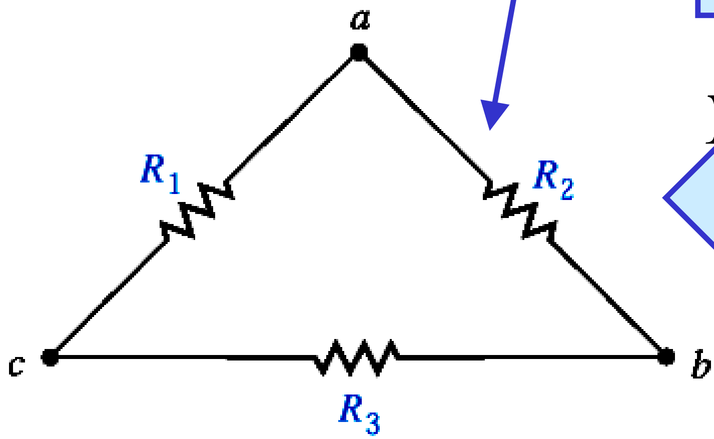


WE COULD HAVE THIS



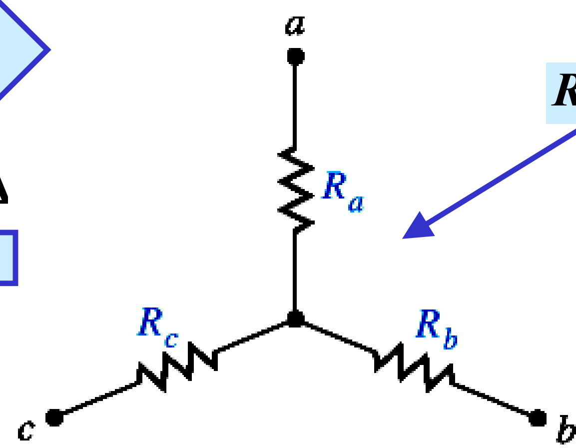
GEAUX

$$R_{ab} = R_2 \parallel (R_1 + R_3)$$



$\Delta \rightarrow Y$

$Y \rightarrow \Delta$



$$R_{ab} = R_a + R_b$$

$$R_a + R_b = \frac{R_2(R_1 + R_3)}{R_1 + R_2 + R_3}$$

$$R_b + R_c = \frac{R_3(R_1 + R_2)}{R_1 + R_2 + R_3}$$

$$R_c + R_a = \frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3}$$

SUBTRACT THE FIRST TWO THEN ADD TO THE THIRD TO GET Ra

$$R_a = \frac{R_1 R_2}{R_1 + R_2 + R_3}$$

$$R_b = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$

$$R_c = \frac{R_3 R_1}{R_1 + R_2 + R_3}$$

$\Delta \rightarrow Y$

$$\frac{R_a}{R_b} = \frac{R_1}{R_3} \Rightarrow R_3 = \frac{R_b R_1}{R_a}$$

$$\frac{R_b}{R_c} = \frac{R_2}{R_1} \Rightarrow R_2 = \frac{R_b R_1}{R_c}$$

REPLACE IN THE THIRD AND SOLVE FOR R1

$$R_1 = \frac{R_a R_b + R_b R_c + R_c R_a}{R_b}$$

$$R_2 = \frac{R_a R_b + R_b R_c + R_c R_a}{R_c}$$

$$R_3 = \frac{R_a R_b + R_b R_c + R_c R_a}{R_a}$$

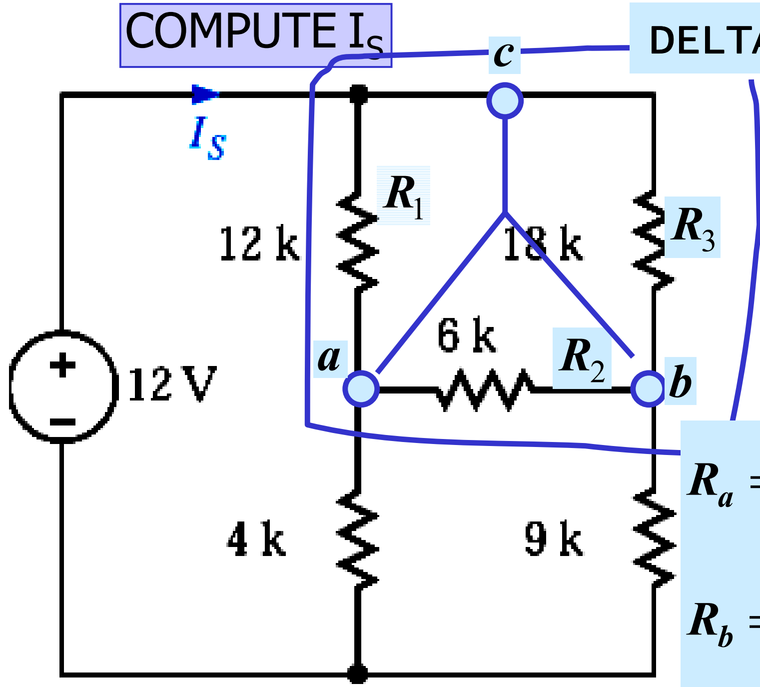
$Y - \Delta$

GEAUX

LEARNING EXAMPLE: APPLICATION OF WYE-DELTA TRANSFORMATION

COMPUTE I_S

DELTA CONNECTION

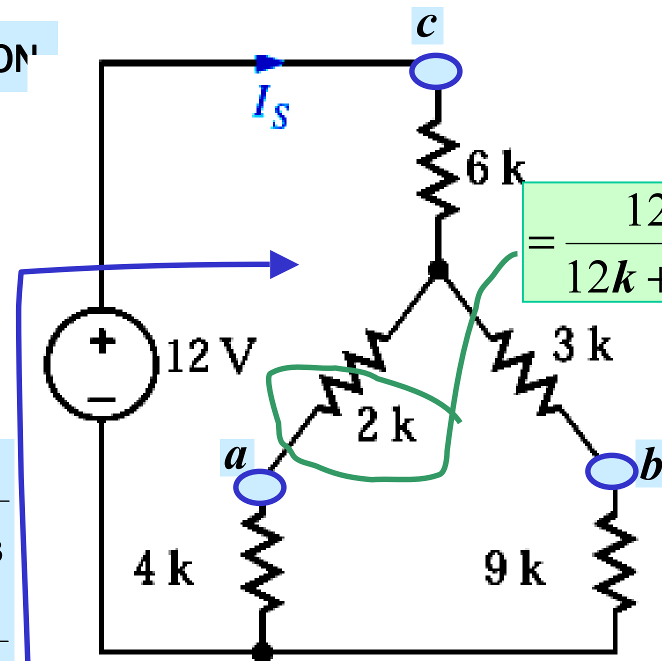


$$R_a = \frac{R_1 R_2}{R_1 + R_2 + R_3}$$

$$R_b = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$

$$R_c = \frac{R_3 R_1}{R_1 + R_2 + R_3}$$

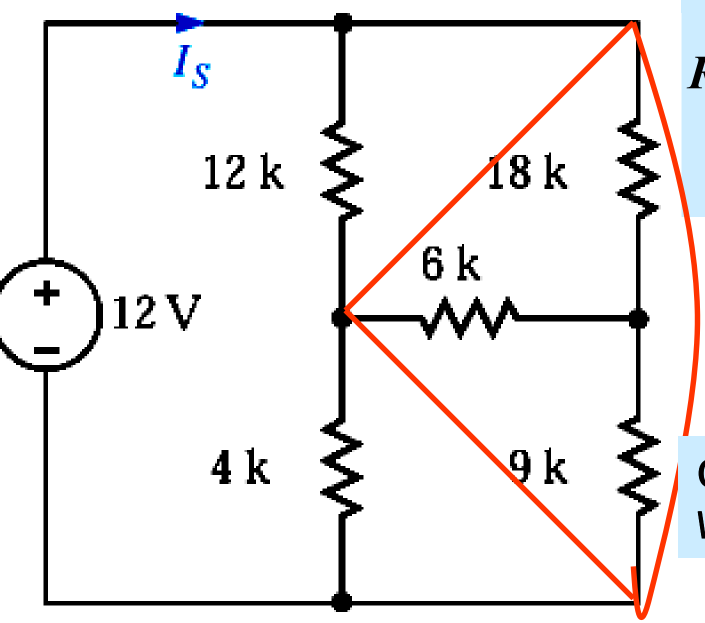
$$\Delta \rightarrow Y$$



$$= \frac{12k \times 6k}{12k + 6k + 18k}$$

$$R_{EQ} = 6k + (3k + 9k) \,||\, (2k + 6k) = 10k$$

$$I_S = \frac{12V}{12k} = 1.2mA$$



ONE COULD ALSO USE A
WYE - DELTA TRANSFORMATION ...

WYE DELTA

STOP