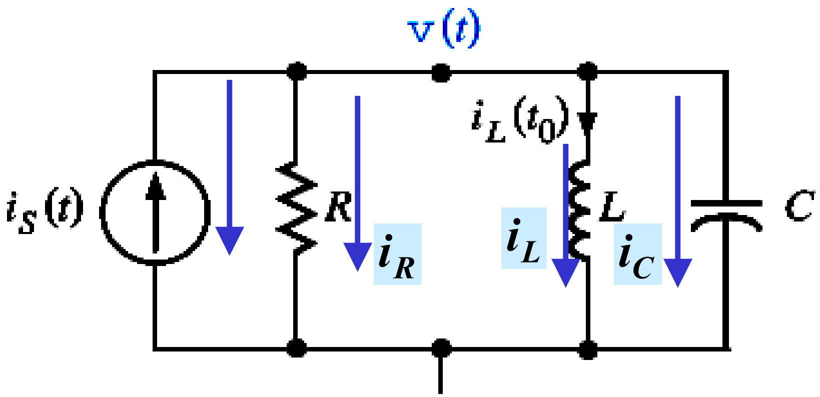
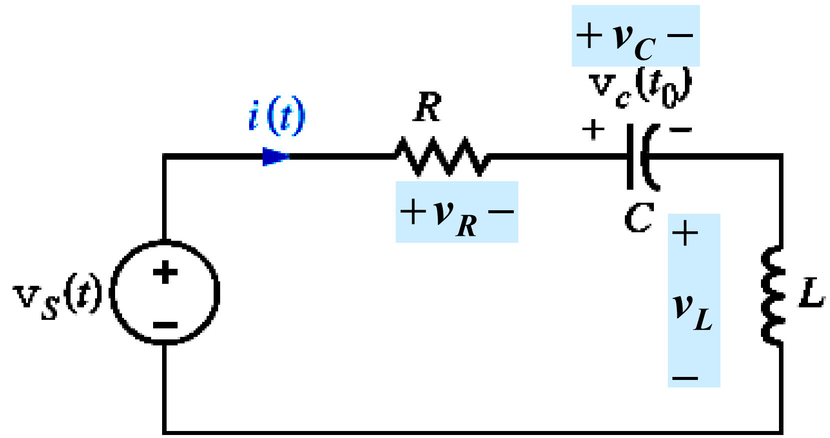
SECOND-ORDER CIRCUITS

THE BASIC CIRCUIT EQUATION





Single Node-pair: Use KCL

$$-i_S + i_R + i_L + i_C = 0$$

$$i_R = \frac{v(t)}{R}; \quad i_L = \frac{1}{L}\int_{t_0}^{t} v(x)dx + i_L(t_0); \quad i_C = C\frac{dv}{dt}(t)$$

$$\frac{v}{R} + \frac{1}{L}\int_{t_0}^{t} v(x)dx + i_L(t_0) + C\frac{dv}{dt}(t) = i_S$$

Differentiating

$$C\frac{d^2v}{dt^2} + \frac{1}{R}\frac{dv}{dt} + \frac{v}{L} = \frac{di_S}{dt}$$

Single Loop: Use KVL

$$-v_S + v_R + v_C + v_L = 0$$

$$v_R = Ri; \; v_C = \frac{1}{C}\int_{t_0}^{t} i(x)dx + v_C(t_0); \; v_L = L\frac{di}{dt}(t)$$

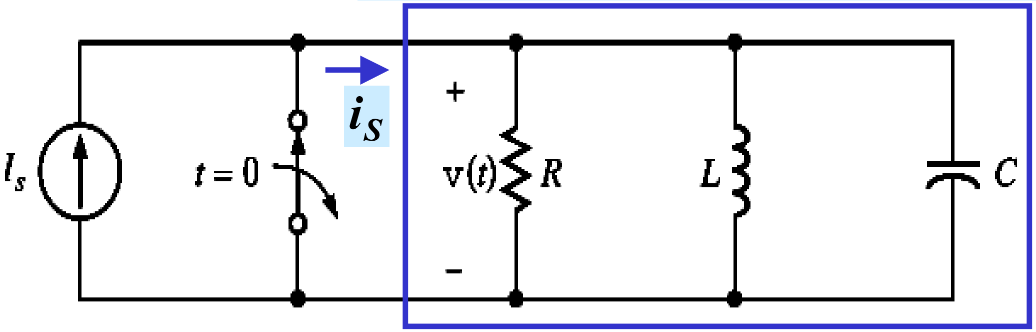
$$Ri + \frac{1}{C}\int_{t_0}^{t} i(x)dx + v_C(t_0) + L\frac{di}{dt}(t) = v_S$$

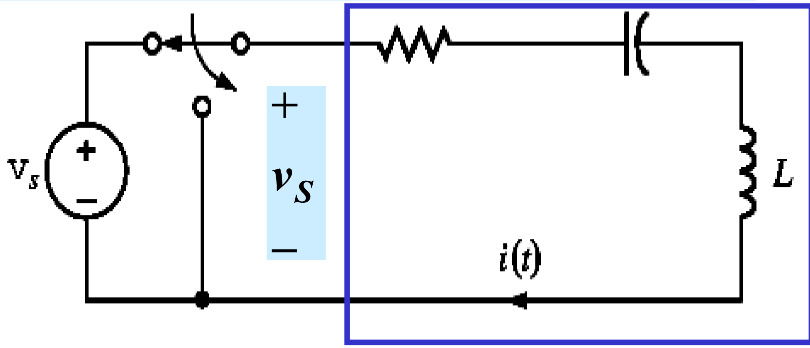
$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = \frac{dv_S}{dt}$$

GEAUX

LEARNING BY DOING

WRITE THE DIFFERENTIAL EQUATION FOR $v(t), i(t)$, RESPECTIVELY





$$i_S(t) = \begin{cases} 0 & t<0 \\ I_S & t>0 \end{cases}$$

$$\frac{di_S}{dt}(t) = 0;\ t>0$$

$$v_S(t) = \begin{cases} V_S & t<0 \\ 0 & t>0 \end{cases}$$

$$\frac{dv_S}{dt}(t) = 0;\ t>0$$

MODEL FOR RLC PARALLEL

$$C\frac{d^2v}{dt^2} + \frac{1}{R}\frac{dv}{dt} + \frac{v}{L} = \frac{di_S}{dt}$$

$$C\frac{d^2v}{dt^2} + \frac{1}{R}\frac{dv}{dt} + \frac{v}{L} = 0$$

MODEL FOR RLC SERIES

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = \frac{dv_S}{dt}$$

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = 0$$

GEAUX

THE RESPONSE EQUATION

WE STUDY THE SOLUTIONS FOR THE EQUATION

$$\frac{d^2x}{dt^2}(t) + a_1\frac{dx}{dt}(t) + a_2x(t) = f(t)$$

KNOWN: $x(t) = x_p(t) + x_c(t)$

x_p particular solution

x_c complementary solution

THE COMPLEMENTARY SOLUTION SATISIFES

$$\frac{d^2x_c}{dt^2}(t) + a_1\frac{dx_c}{dt}(t) + a_2x_c(t) = 0$$

IF THE FORCING FUNCTION IS A CONSTANT

$f(t) = A \Rightarrow x_p = \dfrac{A}{a_2}$ is a particular solution

PROOF: $x_p = \dfrac{A}{a_2} \Rightarrow \dfrac{dx_p}{dt} = \dfrac{d^2x_p}{dt^2} = 0 \Rightarrow a_2x_p = A$

FOR ANY FORCING FUNCTION $f(t) = A$

$$x(t) = \frac{A}{a_2} + x_c(t)$$

GEAUX

THE HOMOGENEOUS EQUATION

$$\frac{d^2x}{dt^2}(t)+a_1\frac{dx}{dt}(t)+a_2x(t)=0$$

NORMALIZED FORM

$$\frac{d^2x}{dt^2}(t)+2\varsigma\omega_n\frac{dx}{dt}(t)+\omega_n^2x(t)=0$$

ω_n (undamped) natural frequency

ς damping ratio

CHARACTERISTIC EQUATION

$$s^2+2\varsigma\omega_n s+\omega_n^2=0$$

$$a_2=\omega_n^2 \Rightarrow \omega_n=\sqrt{a_2}$$

$$a_1=2\varsigma\omega_n \Rightarrow \varsigma=\frac{a_1}{2\sqrt{a_2}}$$

LEARNING BY DOING

DETERMINE THE CHARACTERISTIC EQUATION, DAMPING RATIO AND NATURAL FREQUENCY

$$4\frac{d^2x}{dt^2}(t)+8\frac{dx}{dt}(t)+16x(t)=0$$

COEFFICIENT OF SECOND DERIVATIVE MUST BE ONE

$$\frac{d^2x}{dt^2}(t)+2\frac{dx}{dt}(t)+4x(t)=0$$

CHARACTERISTIC EQUATION

$$s^2+2s+4=0$$

DAMPING RATIO, NATURAL FREQUENCY

$$\frac{d^2x}{dt^2}(t)+2\frac{dx}{dt}(t)+4x(t)=0$$

$2\varsigma\omega_n$ $\omega_n^2 \Rightarrow \omega_n=2$

$\Downarrow$

$\varsigma=0.5$

GEAUX

ANALYSIS OF THE HOMOGENEOUS EQUATION

NORMALIZED FORM

$$\frac{d^2x}{dt^2}(t)+2\varsigma\omega_n\frac{dx}{dt}(t)+\omega_n^2x(t)=0$$

$x(t)=Ke^{st}$ is a solution iff

$$s^2+2\varsigma\omega_n s+\omega_n^2=0$$

Iff *s* is solution of the *characteristic equation*

PROOF: $\frac{dx}{dt}(t)=sKe^{st}$; $\frac{d^2x}{dt^2}=s^2Ke^{st}$

$$\frac{d^2x}{dt^2}(t)+2\varsigma\omega_n\frac{dx}{dt}(t)+\omega_n^2x(t)=(s^2+2\varsigma\omega_n s+\omega_n^2)Ke^{st}$$

CHARACTERISTIC EQUATION

$$s^2+2\varsigma\omega_n s+\omega_n^2=0$$

$$(s+\varsigma\omega_n)^2+(\omega_n^2-\varsigma^2\omega_n^2)=0$$

$$s=-\varsigma\omega_n\pm\sqrt{\varsigma^2\omega_n^2-\omega_n^2}$$

$$s=-\varsigma\omega_n\pm\omega_n\sqrt{\varsigma^2-1}$$

(modes of the system)

CASE 1: $\varsigma>1$ (real and distinct roots)

$$x(t)=K_1e^{s_1t}+K_2e^{s_2t}$$

CASE 2: $\varsigma<1$ (complex conjugate roots)

$$x(t)=K_1e^{s_1t}+K_2e^{s_2t}$$

$$s=-\varsigma\omega_n\pm j\omega_n\sqrt{1-\varsigma^2}$$

$$s=-\sigma\pm j\omega_d$$

$x(t)$ real $\Rightarrow K_2=K_1^*$

$\omega_d=$ damped oscillation frequency

$\sigma=$ damping factor

$$x(t)=e^{-\sigma t}(A_1\cos\omega_d t+A_2\sin\omega_d t)$$

HINT: $e^{st}=e^{-(\varsigma\omega_n\pm j\omega_d)t}=e^{-\varsigma\omega_n t}e^{\mp j\omega_d t}$

$$e^{\mp j\omega_d t}=\cos\omega_d t\mp j\sin\omega_d t$$

ASSUME $K_1=(A_1+jA_2)/2$

$$\left.\begin{array}{l}K_2=K_1^*\\ s=-\sigma\pm j\omega_d\end{array}\right\}\Rightarrow x(t)=2\,\mathrm{Re}\left[K_1e^{-(\sigma+j\omega_d)t}\right]$$

CASE 3: $\varsigma=1$ (real and equal roots)

$$s=-\varsigma\omega_n$$

$$x(t)=(B_1+B_2t)e^{-\varsigma\omega_n t}$$

HINT: te^{st} is solution iff

$$(s^2+2\varsigma\omega_n s+\omega_n^2=0)\ AND\ (2s+2\varsigma\omega_n=0)$$

GEAUX

LEARNING EXTENSIONS

DETERMINE THE GENERAL FORM OF THE SOLUTION

$$\frac{d^2x}{dt^2}(t)+4\frac{dx}{dt}(t)+4x(t)=0$$

CHARACTERISTIC EQUATION

$$s^2+4s+4=0$$

$$\omega_n^2=4\Rightarrow\omega_n=2 \qquad 2\varsigma\omega_n=4\Rightarrow\varsigma=1$$

$$s^2+4s+4=0\Rightarrow(s+2)^2=0$$

Roots are real and equal

this is a critically damped (case 3) system

$$x(t)=(B_1+B_2t)e^{st}$$

$$x(t)=(B_1+B_2t)e^{-2t}$$

$$4\frac{d^2x}{dt^2}(t)+8\frac{dx}{dt}(t)+16x(t)=0$$

Divide by coefficient of second derivative

$$\frac{d^2x}{dt^2}(t)+2\frac{dx}{dt}(t)+4x(t)=0$$

$$\omega_n^2=4\Rightarrow\omega_n=2 \qquad 2\varsigma\omega_n=2\Rightarrow\varsigma=0.5$$

$$s^2+2s+4=(s+1)^2+3=0\Rightarrow s=-1\pm j\sqrt{3}$$

ω_d

Roots are complex conjugate

underdamped (case 2) system

$$\sigma=\varsigma\omega_n=1;\quad \omega_d=\omega_n\sqrt{1-\varsigma^2}=2\sqrt{1-0.25}=\sqrt{3}$$

$$x(t)=e^{-\sigma t}\left(A_1\cos\omega_d t+A_2\sin\omega_d t\right)$$

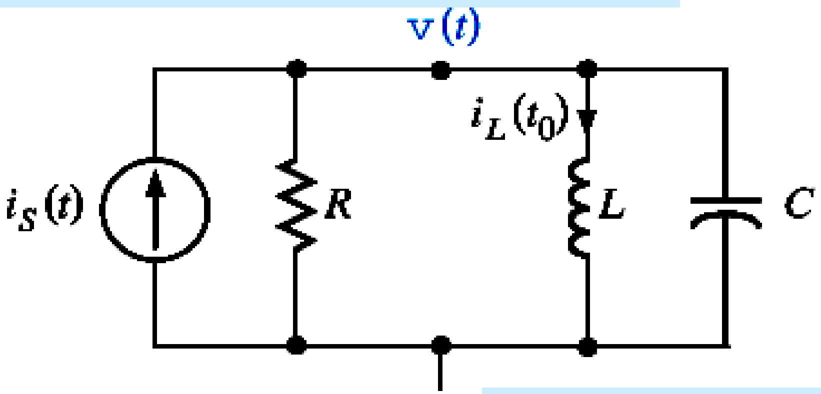
$$x(t)=e^{-t}\left(A_1\cos\sqrt{3}t+A_2\sin\sqrt{3}t\right)$$

LEARNING EXTENSIONS

Form of the solution

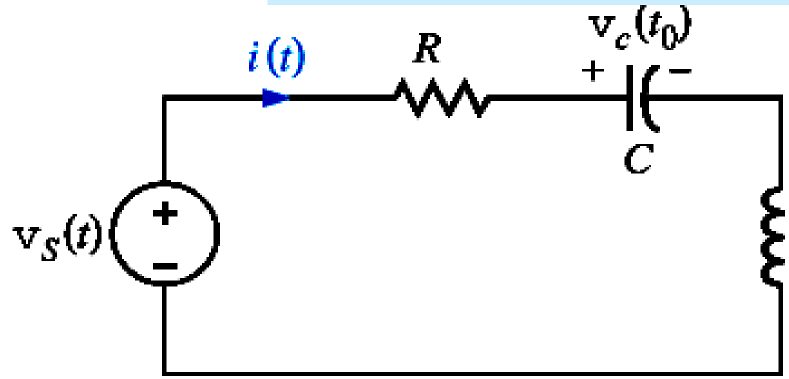
RLC PARALLEL CIRCUIT WITH

$R = 1\Omega, L = 2H, C = 2F$



RLC SERIES CIRCUIT WITH

$R = 2\Omega; L = 1H, C = 0.5F, 1F, 2F$



HOMOGENEOUS EQUATION

$$C\frac{d^2v}{dt^2} + \frac{1}{R}\frac{dv}{dt} + \frac{v}{L} = 0$$

$$2\frac{d^2v}{dt^2} + \frac{dv}{dt} + \frac{v}{2} = 0$$

$$\frac{d^2v}{dt^2} + \frac{1}{2}\frac{dv}{dt} + \frac{v}{4} = 0$$

$$s^2 + \frac{s}{2} + \frac{1}{4} = (s + \frac{1}{4})^2 + \frac{3}{16} = 0$$

$$\omega_n = \frac{1}{2}; \varsigma\omega_n = \frac{1}{4} \Rightarrow \varsigma = \frac{1}{2}$$

$$\sigma = \frac{1}{4}$$

$$\omega_d = \omega_n\sqrt{1-\varsigma^2} = \frac{1}{2}\sqrt{1-\frac{1}{4}} = \frac{\sqrt{3}}{4}$$

$$v_c(t) = e^{-\frac{t}{4}}\left(A_1\cos\frac{\sqrt{3}}{4}t + A_2\sin\frac{\sqrt{3}}{4}t\right)$$

Classify the responses for the given values of C

$$L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{C} = 0 \quad :/L \text{ \& replace values}$$

$$\frac{d^2i}{dt^2} + 2\frac{di}{dt} + \frac{i}{C} = 0$$

$$\omega_n = \frac{1}{\sqrt{C}};\ 2\varsigma\omega_n = 2 \Rightarrow \varsigma = \sqrt{C}$$

C=0.5	**underdamped**
C=1.0	**critically damped**
C=2.0	**overdamped**

$$\text{discriminant} = 4 - \frac{4}{C}$$

GEAUX

THE NETWORK RESPONSE

DETERMINING THE CONSTANTS

NORMALIZED FORM

$$\frac{d^2x}{dt^2}(t) + 2\varsigma\omega_n \frac{dx}{dt}(t) + \omega_n^2 x(t) = A$$

$$x(t) = \frac{A}{\omega_n^2} + K_1 e^{s_1 t} + K_2 e^{s_2 t}$$

$$x(0+) - \frac{A}{\omega_n^2} = K_1 + K_2$$

$$\frac{dx}{dt}(0+) = s_1 K_1 + s_2 K_2$$

$$x(t) = \frac{A}{\omega_n^2} + e^{-\varsigma\omega_n t}\left(A_1 \cos\omega_d t + A_2 \sin\omega_d t\right)$$

$$x(0+) - \frac{A}{\omega_n^2} = A_1$$

$$\frac{dx}{dt}(0+) = -\varsigma\omega_n A_1 + \omega_d A_2$$

$$x(t) = \frac{A}{\omega_n^2} + \left(B_1 + B_2 t\right)e^{-\varsigma\omega_n t}$$

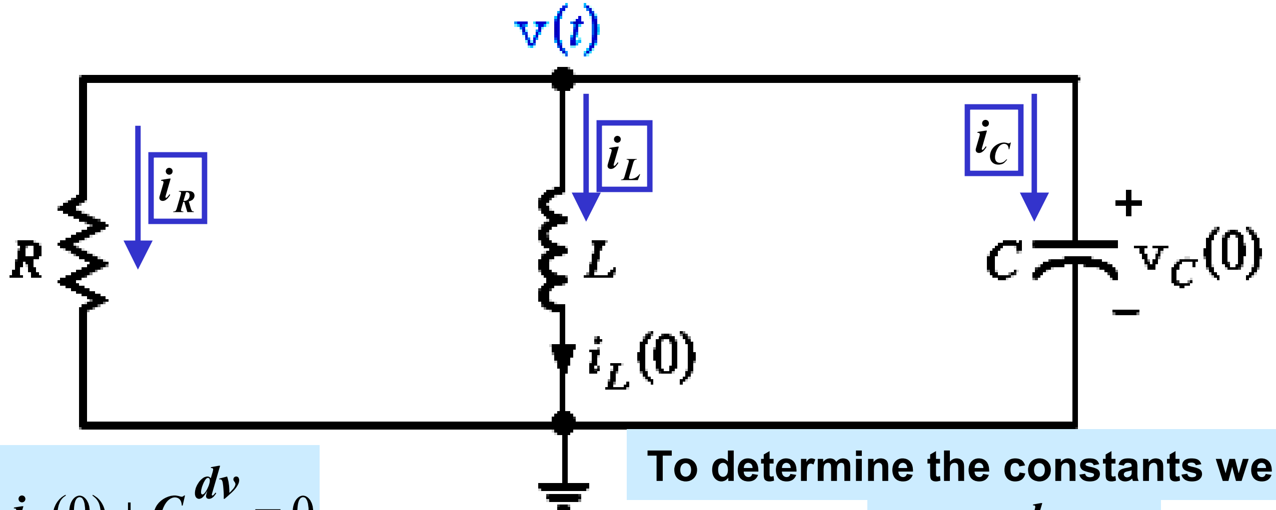
$$x(0+) - \frac{A}{\omega_n^2} = B_1$$

$$\frac{dx}{dt}(0+) = -\varsigma\omega_n B_1 + B_2$$

LEARNING EXAMPLE

$R = 2\Omega, L = 5H, C = \frac{1}{5}F$

$i_L(0) = -1A,\ v_C(0) = 4V$



$i_R + i_L + i_C = 0$

$$\frac{v}{R} + \frac{1}{L}\int_0^t v(x)dx + i_L(0) + C\frac{dv}{dt} = 0$$

$$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{1}{LC}v = 0$$

STEP 1 MODEL

CHARACTERISTIC EQUATION

STEP 2

$s^2 + 2.5s + 1 = 0 \Rightarrow \omega_n = 1;\ \varsigma = 1.5$

$$s = \frac{-2.5 \pm \sqrt{(2.5)^2 - 4}}{2} = \frac{-2.5 \pm 1.5}{2}$$

STEP 3 ROOTS

$v(t) = K_1 e^{-2t} + K_2 e^{-0.5t}$

STEP 4 FORM OF SOLUTION

STEP 5: FIND CONSTANTS

To determine the constants we need

$v(0+);\ \frac{dv}{dt}(0+)$

IF NOT GIVEN FIND $v_C(0), i_L(0)$

$v(0+) = v_C(0+) = v_C(0) = 4V$

KCL AT $t = 0+$

ANALYZE CIRCUIT AT t=0+

$$\frac{v_C(0+)}{R} + i_L(0+) + C\frac{dv}{dt}(0+) = 0$$

$$\frac{dv}{dt}(0+) = -\frac{4}{2(1/5)} - \frac{(-1)}{(1/5)} = -5$$

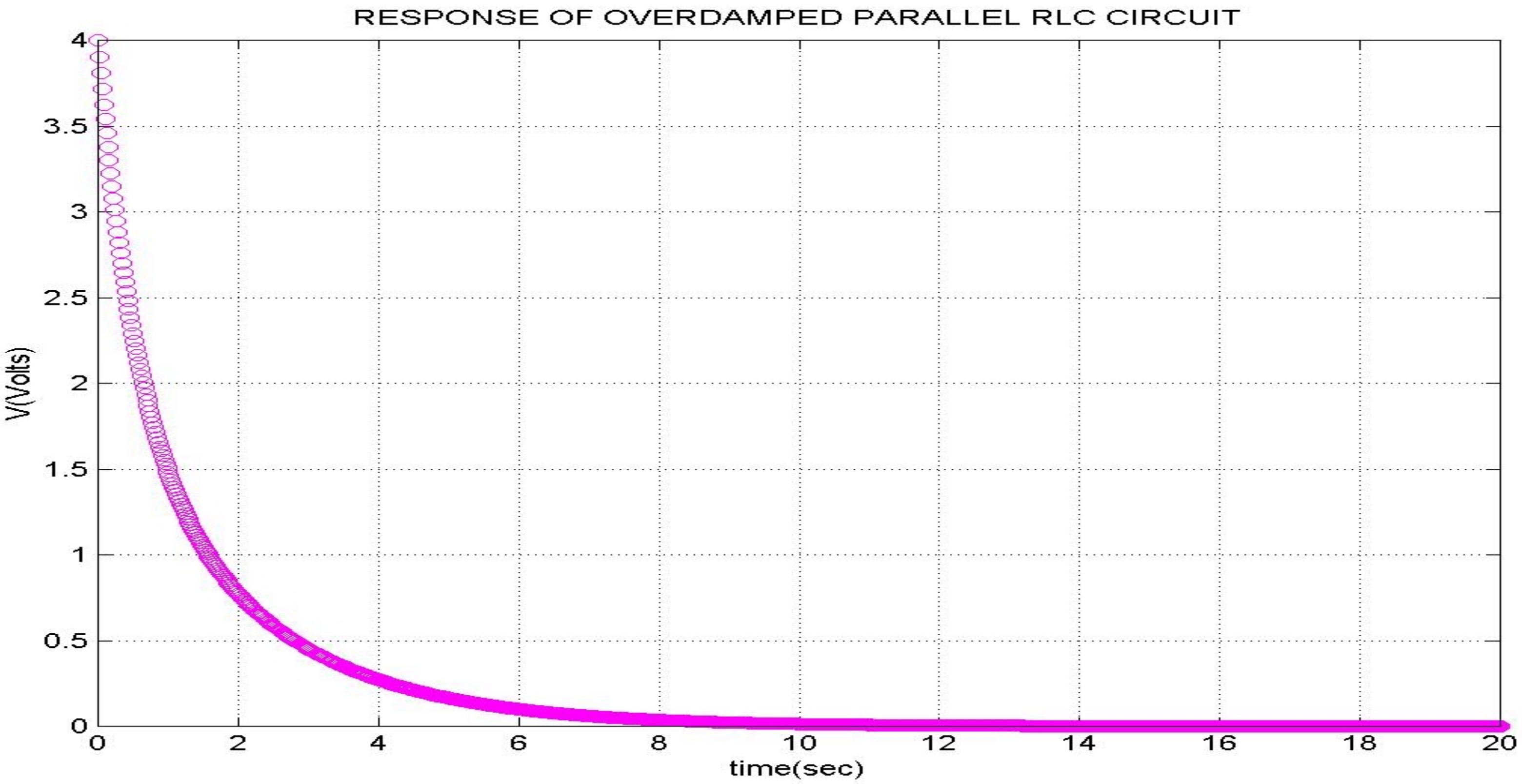
$$\left.\begin{aligned} K_1 + K_2 &= 4 \\ -2K_1 - 0.5K_2 &= -5 \end{aligned}\right\} \Rightarrow K_1 = 2;\ K_2 = 2$$

$v(t) = 2e^{-2t} + 2e^{-0.5t};\ t > 0$

GEAUX

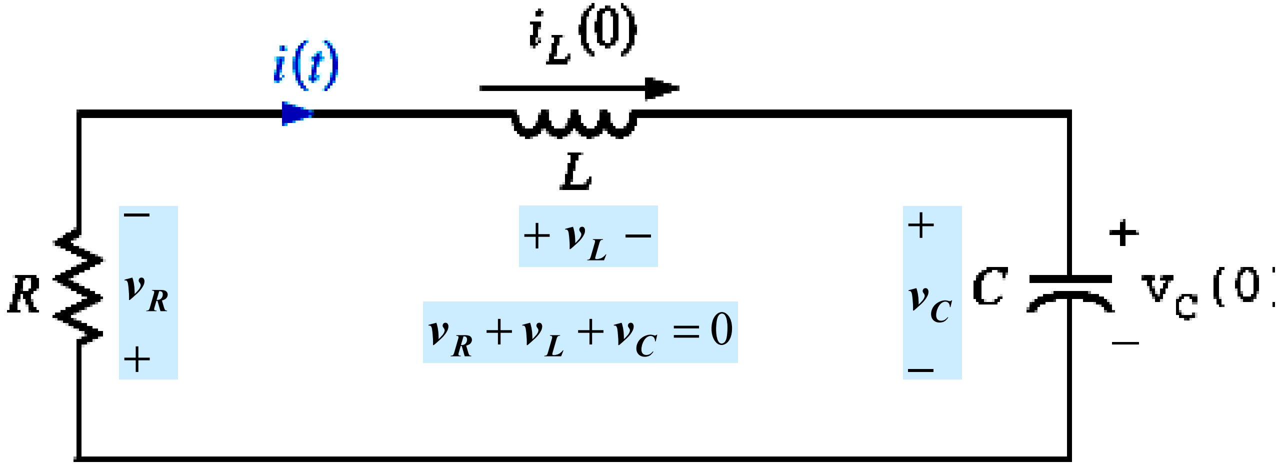
USING MATLAB TO VISUALIZE THE RESPONSE

```
%script6p7.m
%plots the response in Example 6.7
%v(t)=2exp(-2t)+2exp(-0.5t); t>0
t=linspace(0,20,1000);
v=2*exp(-2*t)+2*exp(-0.5*t);
plot(t,v,'mo'), grid, xlabel('time(sec)'), ylabel('V(Volts)')
title('RESPONSE OF OVERDAMPED PARALLEL RLC CIRCUIT')
```



LEARNING EXAMPLE

$R = 6\Omega, L = 1H, C = 0.04F$ $\quad i_L(0) = 4A;\ v_C(0) = -4V$



$v_R + v_L + v_C = 0$

NO SWITCHING OR DISCONTINUITY AT t=0. USE t=0 OR t=0+

$$Ri(t) + L\frac{di}{dt}(t) + \frac{1}{C}\int_0^t i(x)dx + v_C(0) = 0$$

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt}(t) + \frac{1}{LC}i(t) = 0 \quad \textbf{model}$$

$$\frac{d^2i}{dt^2} + 6\frac{di}{dt}(t) + 25i(t) = 0$$

Ch. Eq.: $s^2 + 6s + 25 = 0$

$$\omega_n^2 = 25 \Rightarrow \omega_n = 5$$
$$2\varsigma\omega_n = 6 \Rightarrow \varsigma = 0.6$$

roots: $s = \frac{-6 \pm \sqrt{36-100}}{2} = -3 \pm j4 \quad \omega_d$

Form: $i(t) = e^{-3t}(A_1 \cos 4t + A_2 \sin 4t)$

$i(0) = i_L(0) = 4A \Rightarrow A_1 = 4$

TO COMPUTE $\frac{di}{dt}(0+)$ $\quad v_L(t) = L\frac{di}{dt}(t)$

$$L\frac{di}{dt}(0) = -Ri(0) - v_C(0) \Rightarrow \frac{di}{dt}(0+) = -20$$

$$\frac{di}{dt}(t) = -3i(t) + e^{-3t}(-4A_1 \sin 4t + 4A_2 \cos 4t)$$

$@t = 0:\ -20 = -3\times(4) + 4A_2 \Rightarrow A_2 = -2$

$$i(t) = e^{-3t}(4\cos 4t - 2\sin 4t)[A];\ t > 0$$

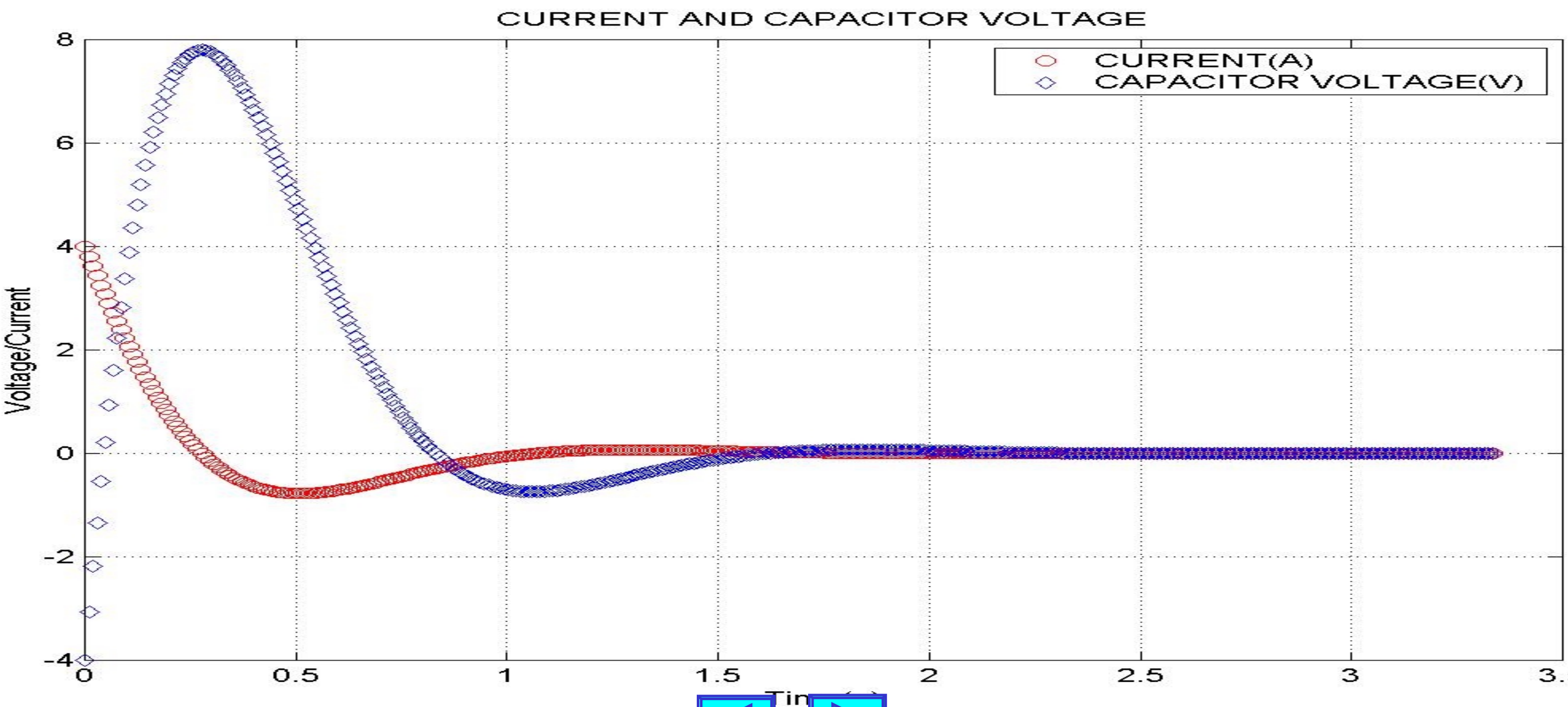
$$v_C(t) = -Ri(t) - L\frac{di}{dt}(t) = v_C(0) + \frac{1}{C}\int_0^t i(x)dx$$

$$v_C(t) = e^{-3t}(-4\cos 4t + 22\sin 4t)[V];\ t > 0$$

GEAUX

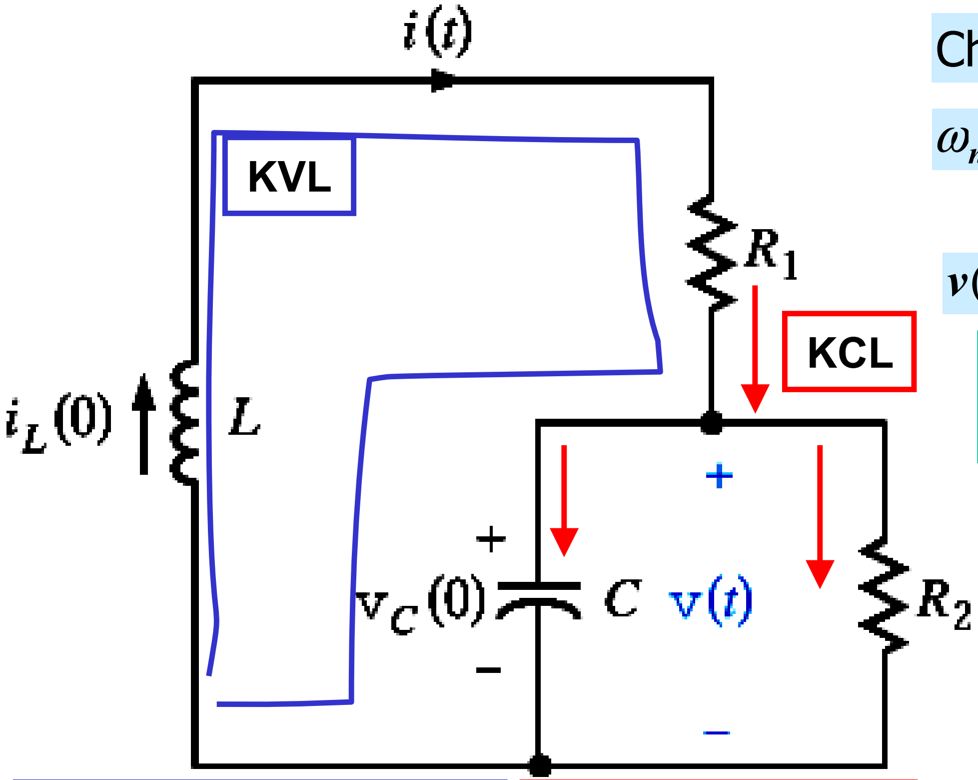
USING MATLAB TO VISUALIZE THE RESPONSE

```
%script6p8.m
%displays the function i(t)=exp(-3t)(4cos(4t)-2sin(4t))
% and the function vc(t)=exp(-3t)(-4cos(4t)+22sin(4t))
% use a simle algorithm to estimate display time
tau=1/3;
tend=10*tau;
t=linspace(0,tend,350);
it=exp(-3*t).*(4*cos(4*t)-2*sin(4*t));
vc=exp(-3*t).*(-4*cos(4*t)+22*sin(4*t));
plot(t,it,'ro',t,vc,'bd'),grid,xlabel('Time(s)'),ylabel('Voltage/Current')
title('CURRENT AND CAPACITOR VOLTAGE')
legend('CURRENT(A)','CAPACITOR VOLTAGE(V)')
```



LEARNING EXAMPLE

$R_1 = 10\Omega,\ R_2 = 8\Omega,\ C = 1F,\ L = 2H$ $\quad v_C(0) = 1V,\ i_L(0) = 0.5A$



$$L\frac{di}{dt}(t) + R_1 i(t) + v(t) = 0$$

$$i(t) = \frac{v(t)}{R_2} + C\frac{dv}{dt}(t)$$

$$L\left(\frac{1}{R_2}\frac{dv}{dt}(t) + C\frac{d^2v}{dt^2}\right) + R_1\left(\frac{v(t)}{R_2} + C\frac{dv}{dt}(t)\right) + v(t) = 0$$

$$\frac{d^2v}{dt^2}(t) + \left(\frac{1}{R_2C} + \frac{R_1}{L}\right)\frac{dv}{dt}(t) + \frac{R_1 + R_2}{R_2LC}v(t) = 0$$

$$\frac{d^2v}{dt^2}(t) + 6\frac{dv}{dt}(t) + 9v(t) = 0$$

Ch. Eq.: $s^2 + 6s + 9 = 0$

Ch. Eq.: $s^2 + 6s + 9 = 0 = (s+3)^2$

$\omega_n = 3,\ 2\varsigma\omega_n = 6 \Rightarrow \varsigma = 1$ $\quad v(t) = e^{-3t}(B_1 + B_2 t)$

$v(0+) = v_c(0+) = 1V$

NO SWITCHING OR DISCONTINUITY AT t=0. USE t=0 OR t=0+

KCL AT $t = 0+$

$$i(0) = i_L(0) = \frac{v(0)}{R_2} + C\frac{dv}{dt}(0) \Rightarrow \frac{dv}{dt}(0) = 3$$

$v(0) = 1 = B_1$

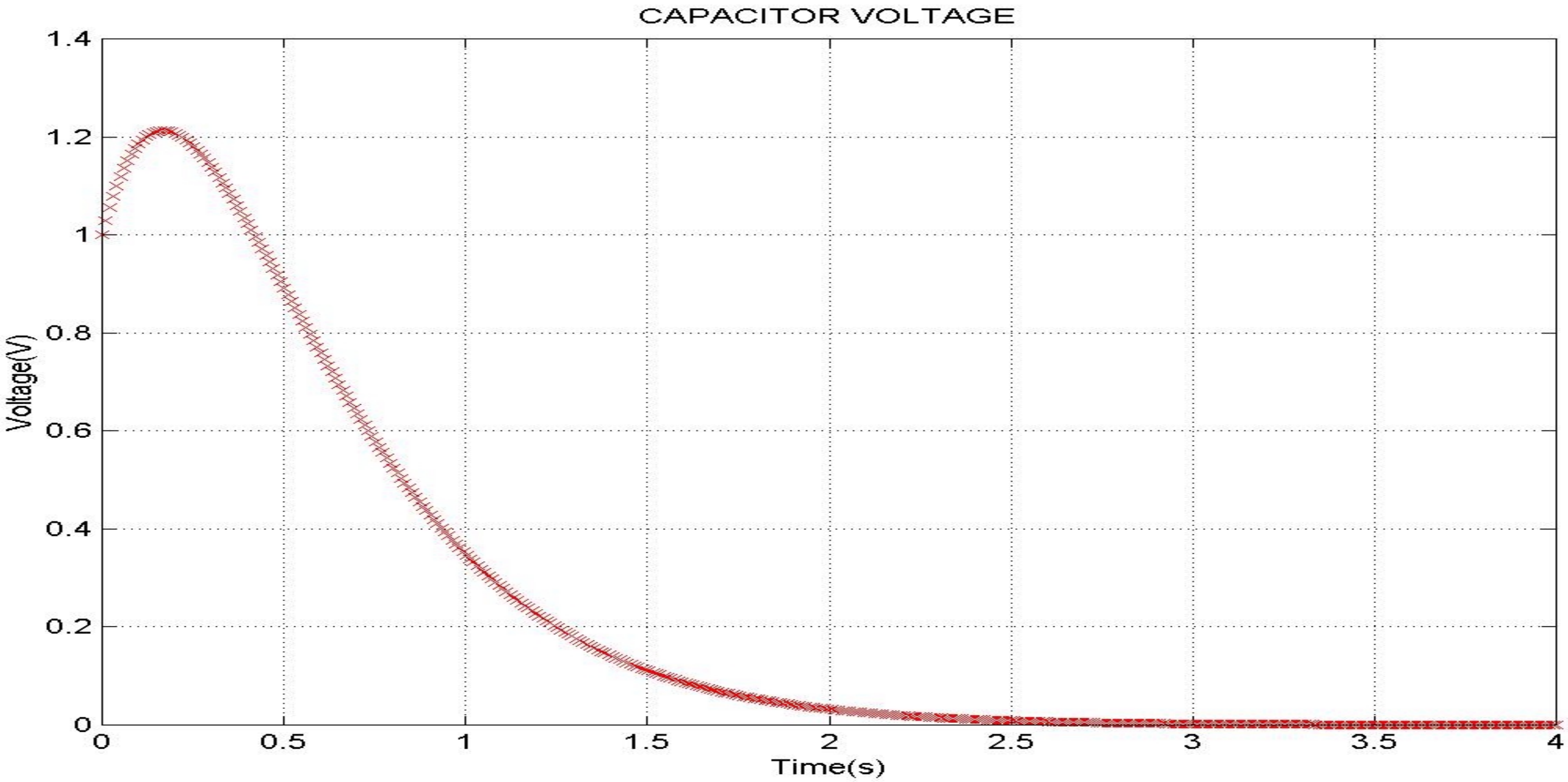
$$\frac{dv}{dt}(0) = -3v(0) + B_2 = 3 \Rightarrow B_2 = 6$$

$$v(t) = e^{-3t}(1 + 6t);\ \ t > 0$$

GEAUX

USING MATLAB TO VISUALIZE RESPONSE

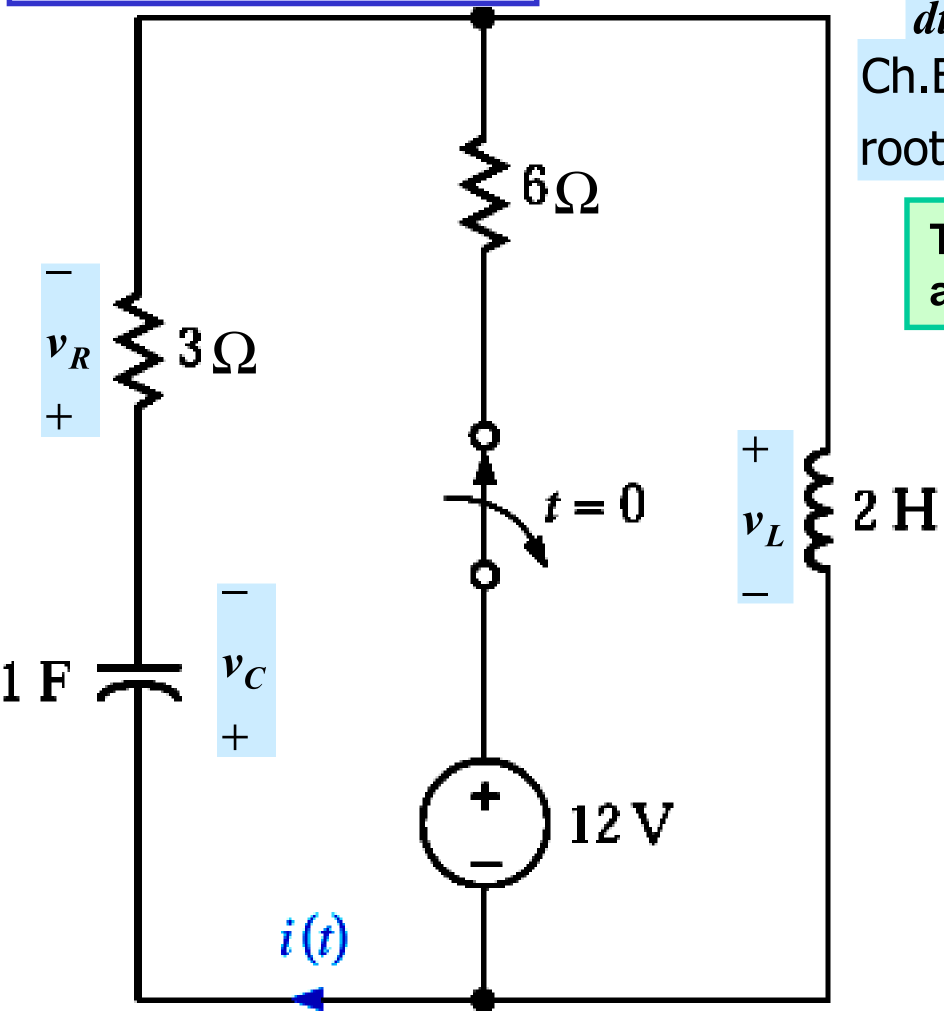
```
%script6p9.m
%displays the function v(t)=exp(-3t)(1+6t)
tau=1/3;
tend=ceil(10*tau);
t=linspace(0,tend,400);
vt=exp(-3*t).*(1+6*t);
plot(t,vt,'rx'),grid, xlabel('Time(s)'), ylabel('Voltage(V)')
title('CAPACITOR VOLTAGE')
```



GEAUX

LEARNING EXTENSION

FIND $i(t),\ t>0$



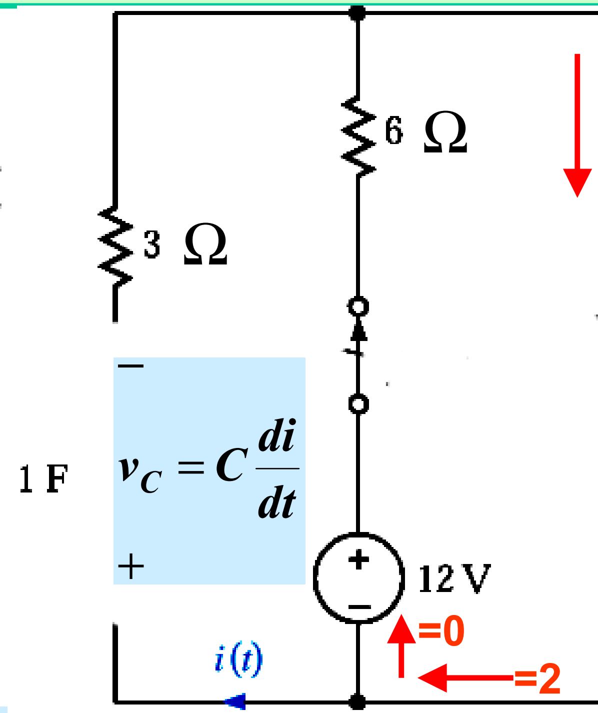
$$\frac{d^2i}{dt^2}(t)+\frac{3}{2}\frac{di}{dt}(t)+\frac{1}{2}i(t)=0$$

Ch.Eq.: $s^2+1.5s+0.5=0$

roots: $s=-1,-0.5$

$$i(t)=K_1e^{-t}+K_2e^{-\frac{t}{2}};\ t>0$$

To find initial conditions use steady state analysis for t<0



$i_L(0)=2A$

$v_C(0)=0V$

$v_C=C\frac{di}{dt}$

$\frac{di}{dt}(0+)=0$

And analyze circuit at t=0+

$i(0+)=2A$

KCL at $t=0+$

Once the switch opens the circuit is RLC series

$$3i(t)+2\frac{di}{dt}(t)+v_C(0)+\int_0^t i(x)dx=0$$

$$2=K_1+K_2$$

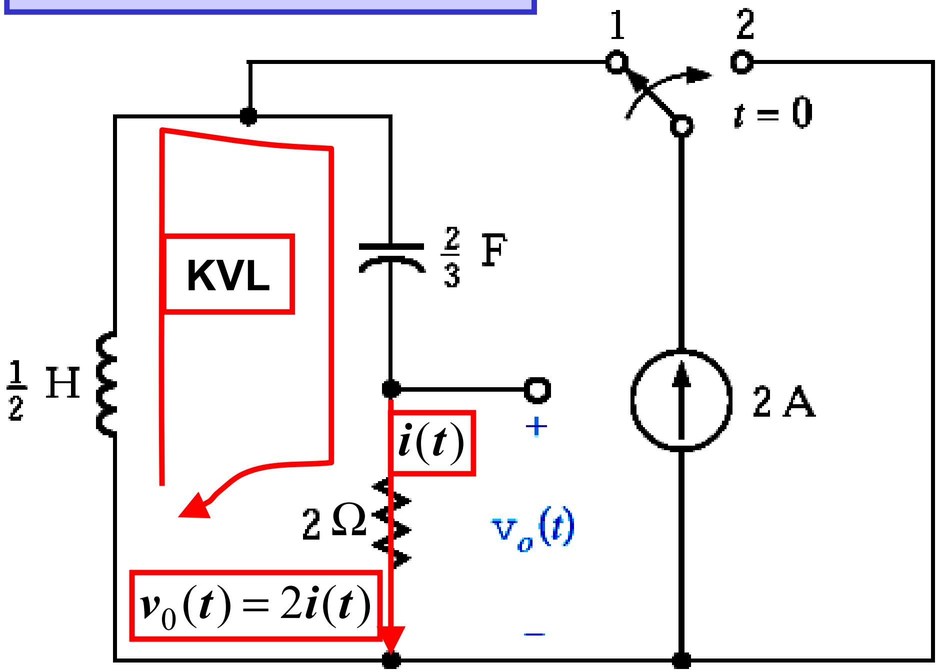
$$0=-K_1-\frac{1}{2}K_2$$

$$i(t)=-2e^{-t}+4e^{-\frac{t}{2}};\ t>0$$

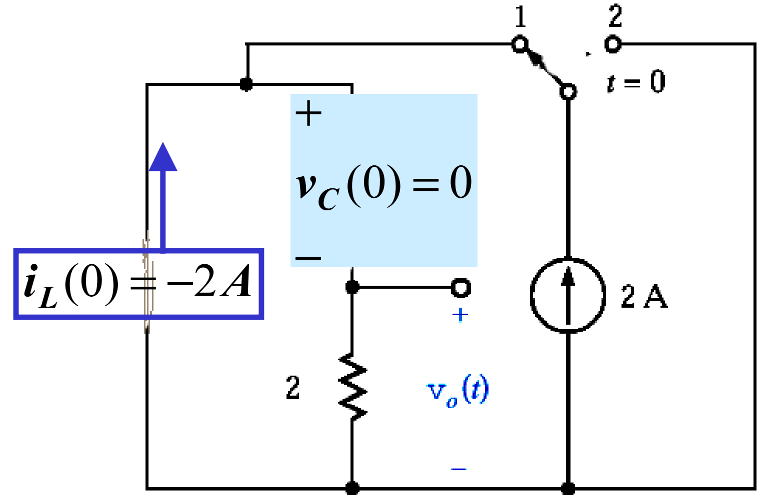
GEAUX

LEARNING EXTENSION

FIND $v_0(t),\ t>0$



To find initial conditions we use steady state analysis for t<0



And analyze circuit at t=0+

$i(0+) = -2A$

$$v_C(0+) = C\frac{di}{dt}(0+) = 0$$

For t>0 the circuit is RLC series

$$\frac{1}{2}\frac{di}{dt}(t) + \frac{1}{2/3}\int_0^t i(x)dx + v_C(0) + 2i(t) = 0$$

$$\frac{d^2i}{dt^2}(t) + 4\frac{di}{dt}(t) + 3i(t) = 0$$

Ch. Eq. : $s^2 + 4s + 3 = 0$

roots : $s = -1, -3$

$i(t) = K_1 e^{-t} + K_2 e^{-3t};\ t>0$

$$i(0+) = 0 \Rightarrow K_1 + K_2 = -2$$

$$\frac{di}{dt}(0+) = 0 \Rightarrow -K_1 - 3K_2 = 0$$

$K_2 = 1$

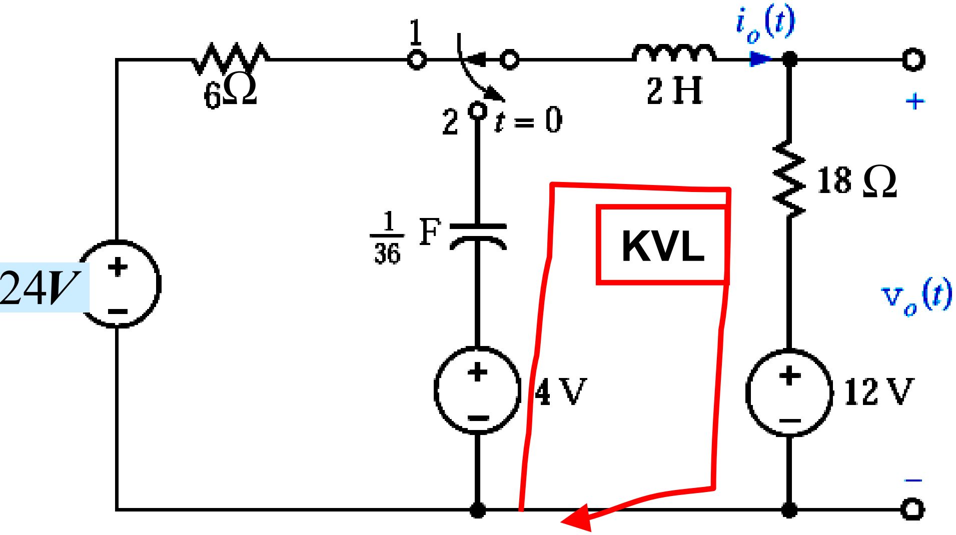
$K_1 = -3$

$$\therefore i(t) = -3e^{-t} + e^{-3t};\ t>0$$

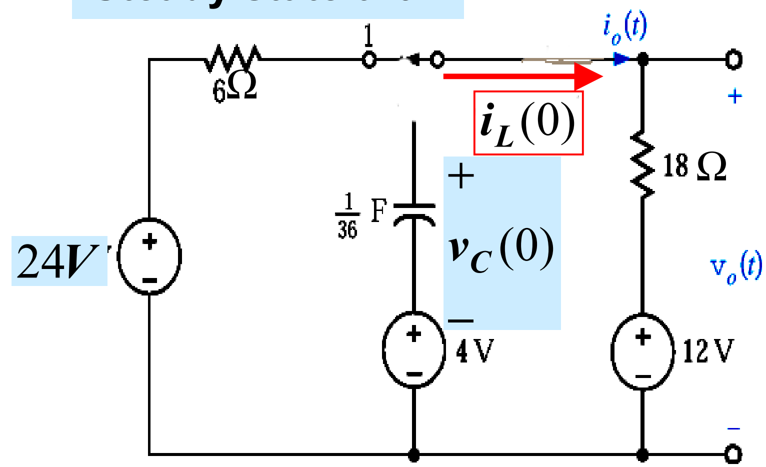
$$v_0(t) = 2\left(-3e^{-t} + e^{-3t}\right);\ t>0$$

GEAUX

LEARNING EXTENSION DETERMINE $i_0(t), v_0(t);\ t>0$ $v_0(t)=18i_0(t)+12(V)$



Steady state t<0



$v_C(0)=0$ $i_L(0)=0.5A$

$$-4+\frac{1}{1/36}\int_0^t i(x)dx+v_C(0)+2\frac{di}{dt}(t)+18i(t)+12=0$$

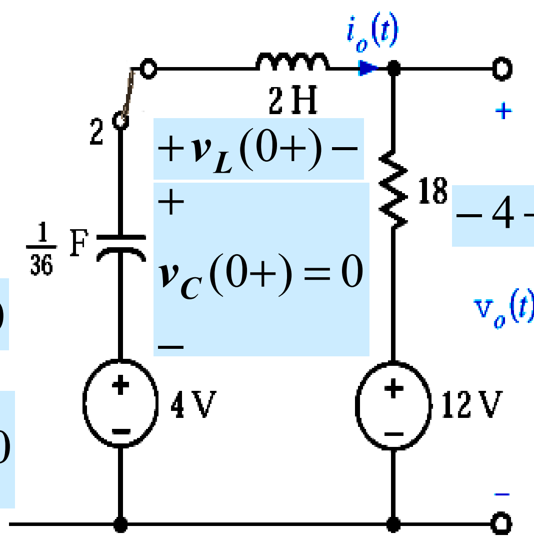
$$\frac{d^2i}{dt^2}(t)+9\frac{di}{dt}(t)+18i(t)=0$$

Ch. Eq.: $s^2+9s+18=0$

roots: $s=-3,-6$

$$i_0(t)=K_1e^{-3t}+K_2e^{-6t};\ t>0$$

$$i_0(t)=-\frac{11}{6}e^{-3t}+\frac{14}{6}e^{-6t};\ t>0$$



Analysis at t=0+

$$i_0(0+)=i_L(0+)=0.5(A)$$

$$v_L(0+)=L\frac{di_L}{dt}(0+)=L\frac{di_0}{dt}(0+)$$

$$-4+v_L(0+)+18i_L(0+)+12=0 \quad v_L(0+)=-17$$

$$\frac{di_0}{dt}(0+)=-17/2=-3K_1-6K_2$$

$$i_0(0+)=0.5=K_1+K_2$$

$$K_1=-\frac{11}{6};\quad K_2=\frac{14}{6}$$

Second Order

STOP