POLYPHASE CIRCUITS

LEARNING GOALS

Three Phase Circuits
Advantages of polyphase circuits

Three Phase Connections
Basic configurations for three phase circuits

Source/Load Connections
Delta-Y connections

Power Relationships
Study power delivered by three phase circuits

Power Factor Correction
Improving power factor for three phase circuits

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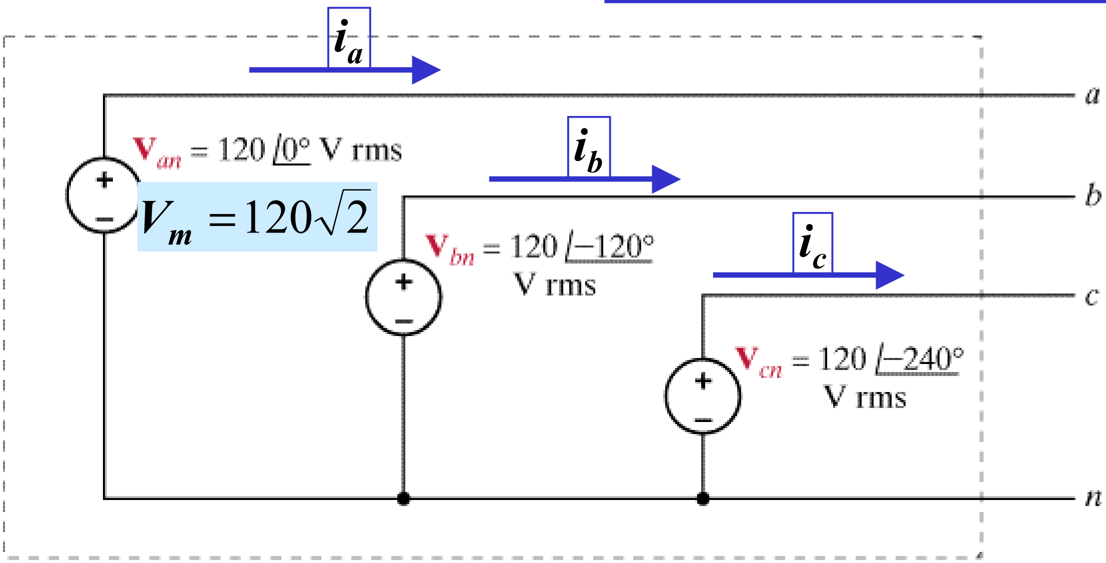
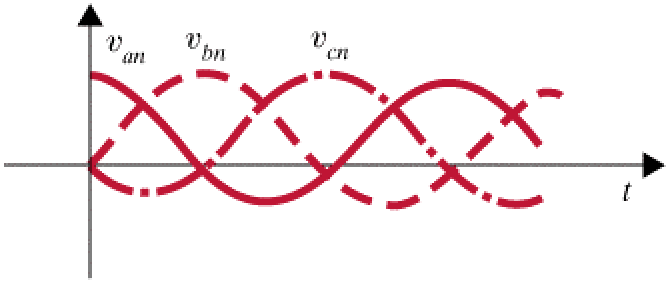
THREE PHASE CIRCUITS





Instantaneous Phase Voltages

$$v_{an}(t) = V_m \cos(\omega t)(V)$$

$$v_{bn}(t) = V_m \cos(\omega t - 120°)(V)$$

$$v_c(t) = V_m \cos(\omega t - 240°)(V)$$

Balanced Phase Currents

$$i_a(t) = I_m \cos(\omega t - \theta)$$

$$i_b(t) = I_m \cos(\omega t - \theta - 120°)$$

$$i_c(t) = I_m \cos(\omega t - \theta - 240°)$$

Instantaneous power

$$p(t) = v_{an}(t)i_a(t) + v_{bn}(t)i_b(t) + v_{cn}(t)i_c(t)$$

Theorem
For a balanced three phase circuit the instantaneous power is constant

$$p(t) = 3\frac{V_m I_m}{2}\cos\theta\,(W)$$

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Proof of Theorem

For a balanced three phase circuit the instantaneous power is constant

$$p(t) = 3\frac{V_m I_m}{2}\cos\theta\,(W)$$

$\cos(120) = -0.5$

Lemma

$$\cos\phi + \cos(\phi - 120) + \cos(\phi + 120) = 0$$

Proof

$$\cos\phi = \cos\phi$$

$$\cos(\phi - 120) = \cos\phi\cos(120) + \sin\phi\sin(120)$$

$$\cos(\phi + 120) = \cos\phi\cos(120) - \sin\phi\sin(120)$$

$$\boxed{\cos\phi + \cos(\phi - 120) + \cos(\phi + 120) = 0}$$

Instantaneous power

$$p(t) = v_{an}(t)i_a(t) + v_{bn}(t)i_b(t) + v_{cn}(t)i_c(t)$$

$$p(t) = V_m I_m \begin{bmatrix} \cos\omega t\cos(\omega t - \theta) \\ +\cos(\omega t - 120)\cos(\omega t - 120 - \theta) \\ +\cos(\omega t - 240)\cos(\omega t - 240 - \theta) \end{bmatrix}$$

$$\cos\alpha\cos\beta = \frac{1}{2}[\cos(\alpha - \beta) + \cos(\alpha + \beta)]$$

$$p(t) = V_m I_m \begin{bmatrix} 3\cos\theta + \cos(2\omega t - \theta) \\ +\cos(2\omega t - 240 - \theta) \\ +\cos(2\omega t - 480 - \theta) \end{bmatrix}$$

$$\phi = \omega t - \theta$$

$$\cos(\phi - 240) = \cos(\phi + 120)$$

$$\cos(\phi - 480) = \cos(\phi - 120)$$

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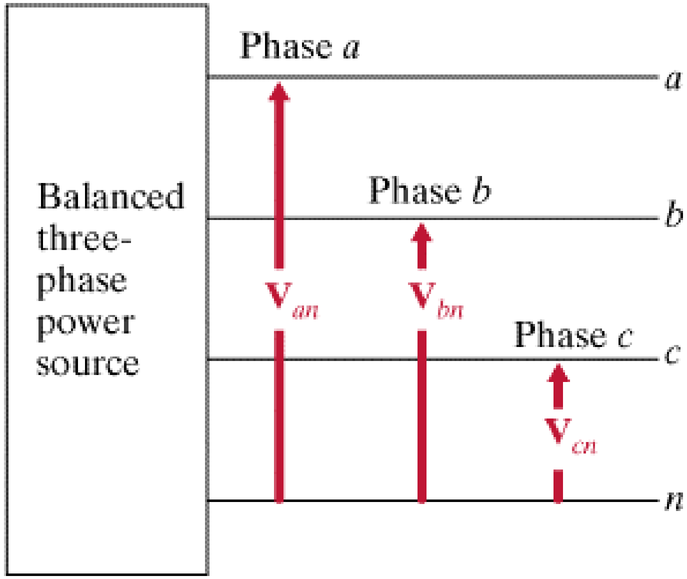
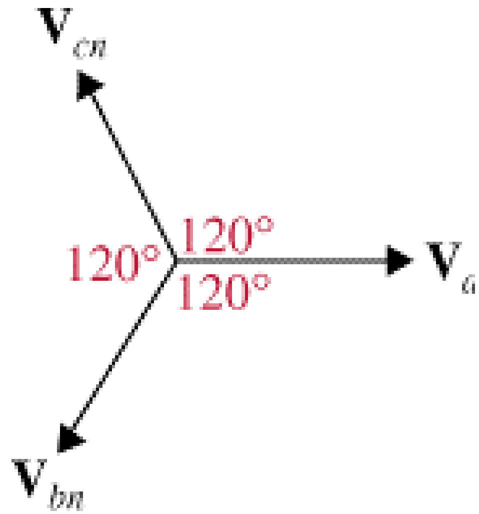
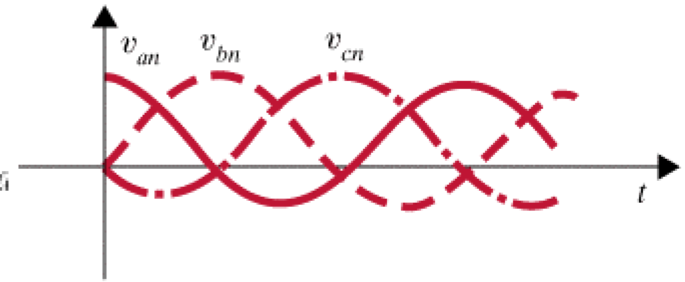
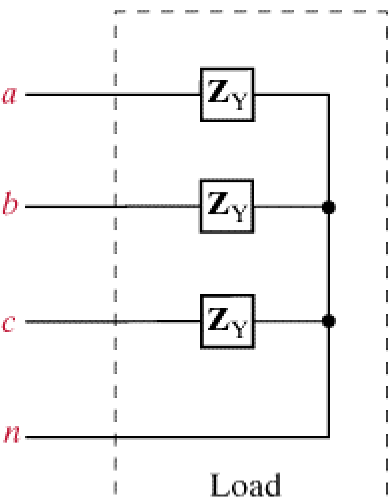
THREE-PHASE CONNECTIONS

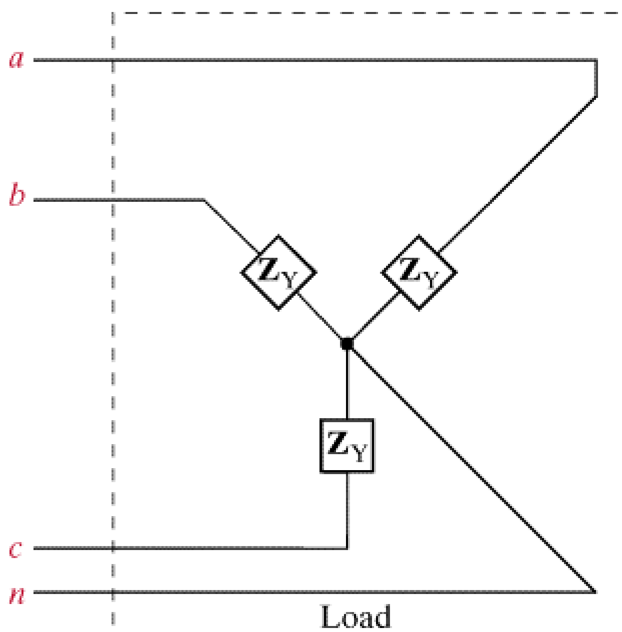






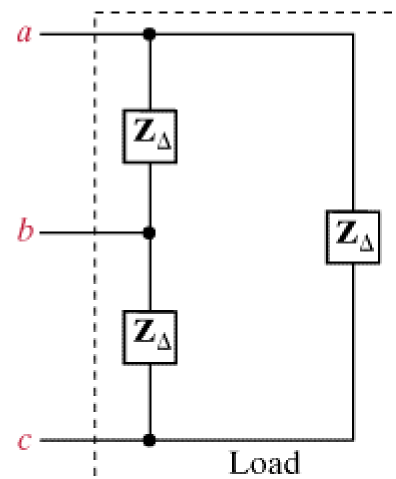
Positive sequence a-b-c

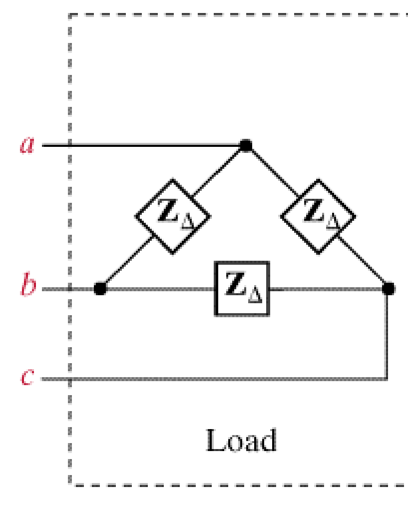




(b)

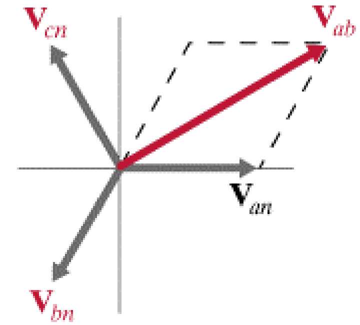
Y-connected loads





Delta connected loads

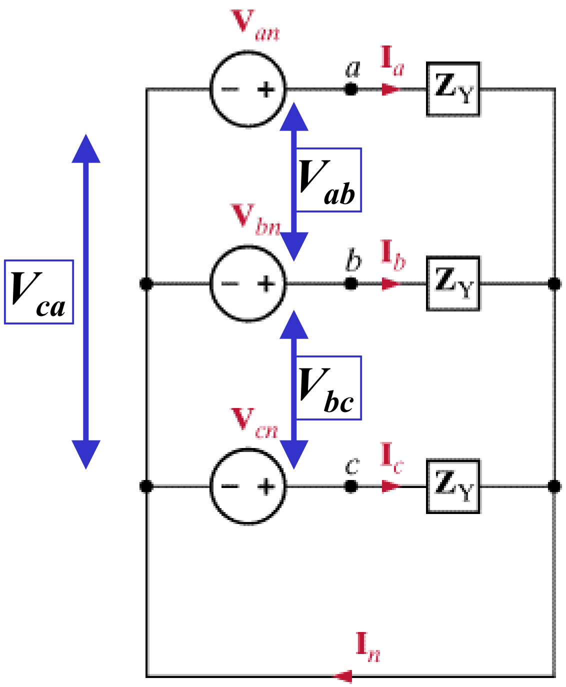
SOURCE/LOAD CONNECTIONS



(a)

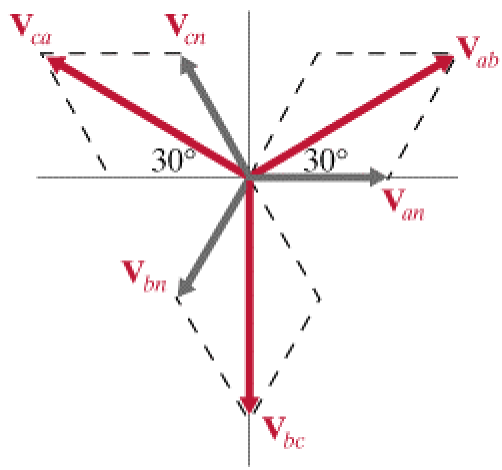
BALANCED Y-Y CONNECTION

Line voltages



$$V_{an} = |V_p| \angle 0°$$
$$V_{bn} = |V_p| \angle -120°$$
$$V_{cn} = |V_p| \angle 120°$$

Positive sequence phase voltages



$$\begin{aligned} V_{ab} &= V_{an} - V_{bn} \\ &= |V_p| \angle 0° - |V_p| \angle -120° \\ &= |V_p| \left(1 - (\cos 120 - j \sin 120)\right) \\ &= |V|_p - |V_p| \left(\frac{1}{2} - j \frac{\sqrt{3}}{2} \right) \\ &= \sqrt{3} |V_p| \angle 30° \end{aligned}$$

$$V_{bc} = \sqrt{3} |V_p| \angle -90°$$
$$V_{ca} = \sqrt{3} |V_p| \angle -210°$$

$V_L = \sqrt{3} |V_p| =$ Line Voltage

$$I_a = \frac{V_{an}}{Z_Y}; I_b = \frac{V_{bn}}{Z_Y}; I_c = \frac{V_{cn}}{Z_Y}$$

$$I_a = |I_L| \angle \theta°; I_b = |I_L| \angle \theta - 120°; I_c = |I_L| \angle \theta + 120°$$

$$I_a + I_b + I_c = I_n = 0$$

For this balanced circuit it is enough to analyze one phase

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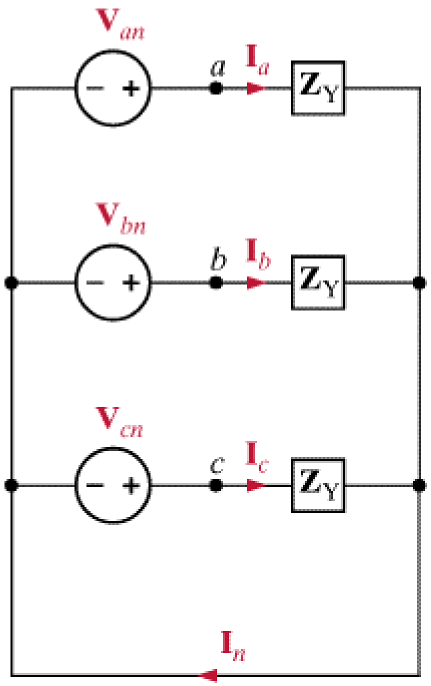
LEARNING EXAMPLE

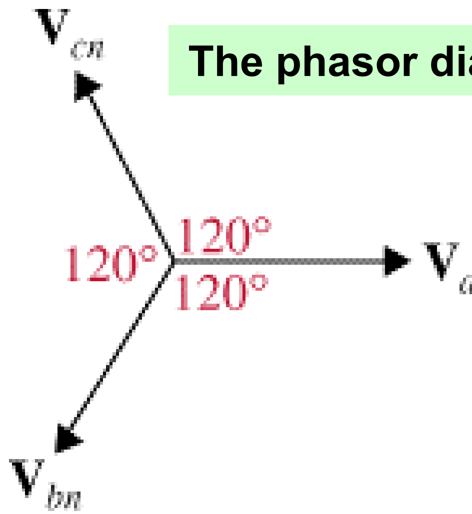
For an abc sequence, balanced Y - Y three phase circuit

$$V_{ab} = 208\angle -30°$$

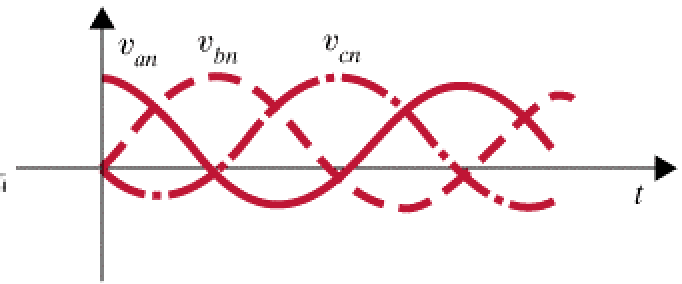
Determine the phase voltages



Balanced Y - Y



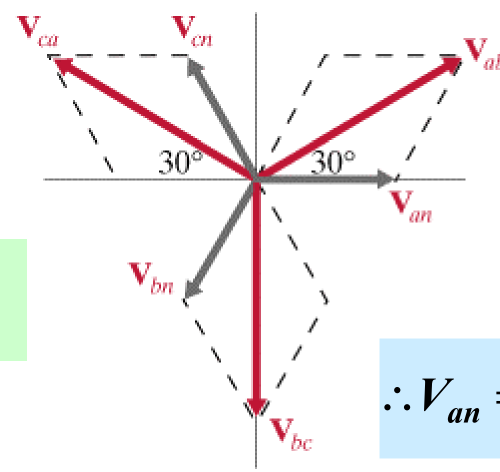
The phasor diagram could be rotated by any angle



Positive sequence a-b-c

$$V_{an} = |V_p|\angle 0°$$
$$V_{bn} = |V_p|\angle -120°$$
$$V_{cn} = |V_p|\angle 120°$$

Positive sequence phase voltages



$$V_{ab} = \sqrt{3}|V_p|\angle 30°$$

V_{an} lags V_{ab} by 30°

$$V_{ab} = 208\angle -30°$$

$$\therefore V_{an} = \frac{|V_{ab}|}{\sqrt{3}}\angle(-30° - 30°)$$

Relationship between phase and line voltages

$$V_{an} = 120\angle -60°$$
$$V_{bn} = 120\angle -180°$$
$$V_{an} = 120\angle 60°$$

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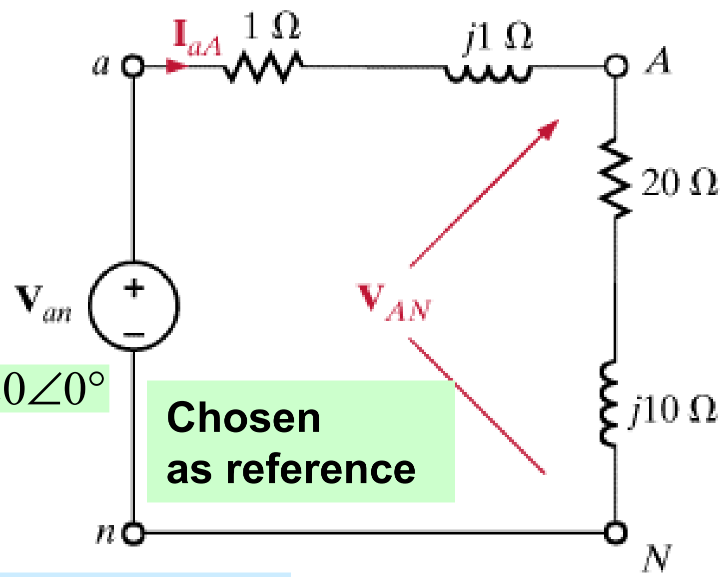
LEARNING EXAMPLE

For an abc sequence, balanced Y - Y three phase circuit

source $|V_{phase}| = 120(V)_{rms}$, $Z_{line} = 1 + j1\Omega$, $Z_{phase} = 20 + j10\Omega$

Determine line currents and load voltages

Because circuit is balanced data on any one phase are sufficient



$120\angle 0°$

Chosen as reference

$$V_{an} = 120\angle 0°$$
$$V_{bn} = 120\angle -120°$$
$$V_{an} = 120\angle 120°$$

Abc sequence

$$I_{aA} = \frac{V_{an}}{21 + j11} = \frac{120\angle 0°}{23.71\angle 27.65°} = 5.06\angle -27.65°(A)rms$$

$$I_{bB} = 5.06\angle -120 - 27.65°(A)rms$$
$$I_{cC} = 5.06\angle 120 - 27.65°(A)rms$$

$$V_{AN} = I_{aA} \times (20 + j10) = I_{aA} \times 22.36\angle 26.57°$$
$$V_{AN} = 113.15\angle -1.08°(V)rms$$

$$V_{BN} = 113.15\angle -121.08°(V)rms$$
$$V_{CN} = 113.15\angle 118.92°(V)rms$$

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LEARNING EXTENSION

For an abc sequence, balanced Y - Y three phase circuit

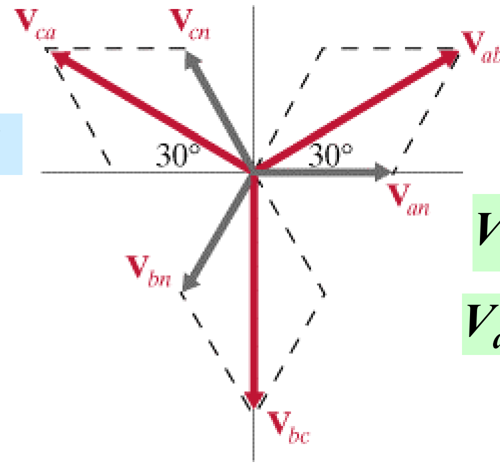
$V_{an} = 120\angle 90°(V)rms$. Find the line voltages

V_{ab} leads V_{an} by 30°

$$V_{ab} = \sqrt{3}\times 120\angle 120°(V)rms$$

$$V_{bc} = \sqrt{3}\times 120\angle 0°(V)rms$$

$$V_{ca} = \sqrt{3}\times 120\angle 240°(V)rms$$



$$V_{ab} = \sqrt{3}\,|V_p|\angle 30°$$

V_{an} lags V_{ab} by 30°

Relationship between phase and line voltages

$V_{ab} = 208\angle 0°(V)rms$. Find the phase voltages

V_{an} lags V_{ab} by 30°

$$V_{an} = \frac{208}{\sqrt{3}}\angle -30°\ (V)rms$$

$$V_{bn} = \frac{208}{\sqrt{3}}\angle -150°(V)rms$$

$$V_{cn} = \frac{208}{\sqrt{3}}\angle 90°(V)rms$$

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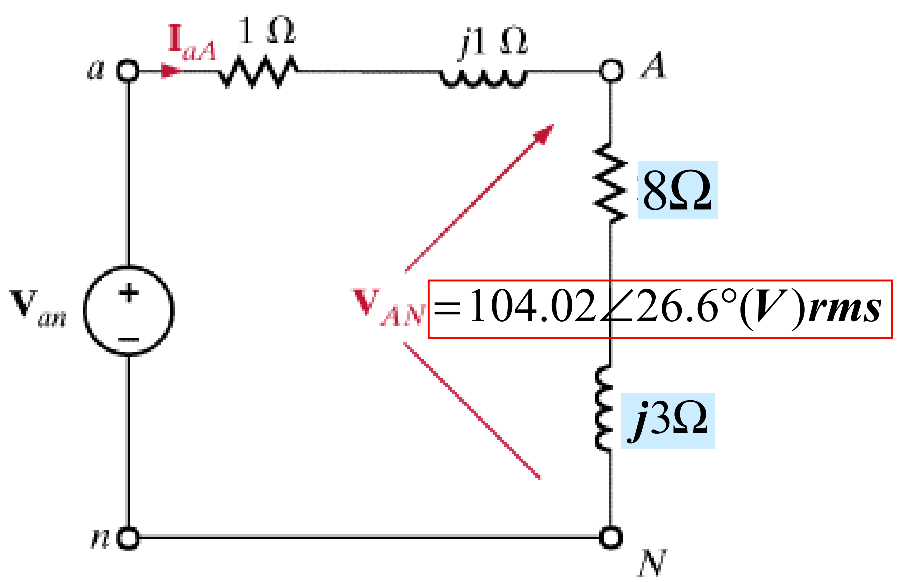
LEARNING EXTENSION

For an abc sequence, balanced Y - Y three phase circuit load, $|V_{phase}| = 104.02\angle 26.6°(V)_{rms}$, $Z_{line} = 1 + j1\Omega$, $Z_{phase} = 8 + j3\Omega$

Determine source phase voltages



Currents are not required. Use inverse voltage divider

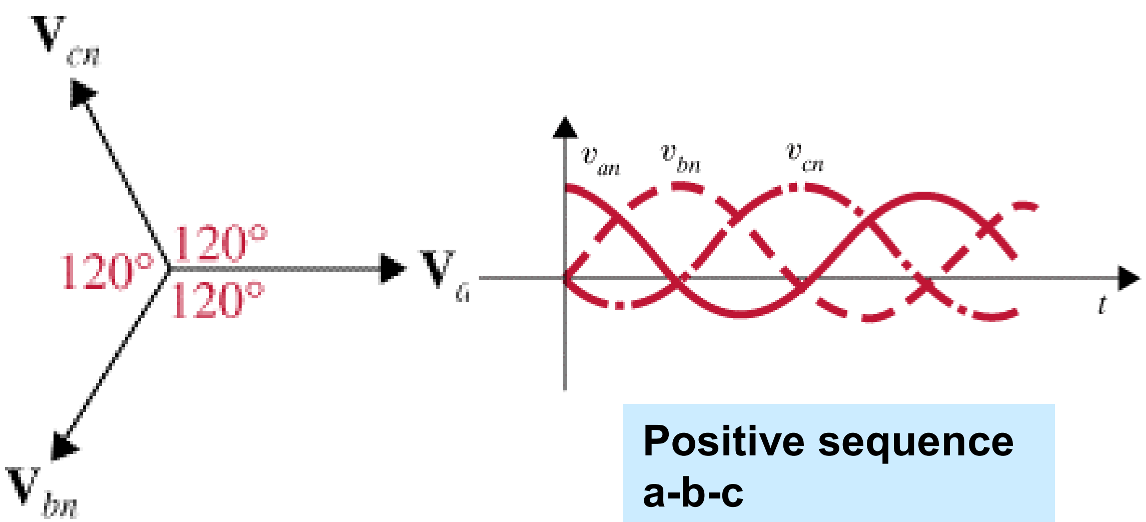
$$V_{an} = \frac{(8 + j3) + (1 + j1)}{8 + j3} V_{AN}$$

$$\frac{9 + j4}{8 + j3} \times \frac{8 - j3}{8 - j3} = \frac{84 + j5}{73} = 1.15\angle 3.41°$$

$$V_{an} = 120\angle 30°$$

$$V_{bn} = 120\angle -90°$$

$$V_{cn} = 120\angle 150°$$



Positive sequence a-b-c

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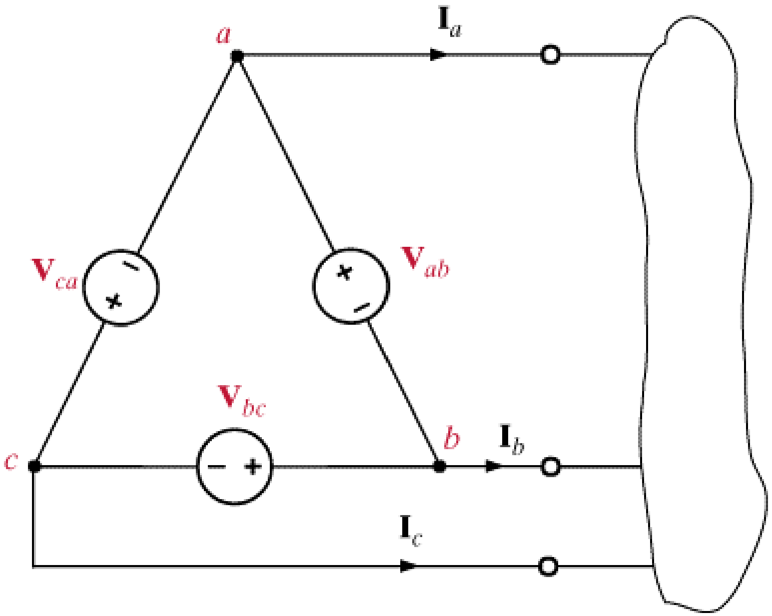
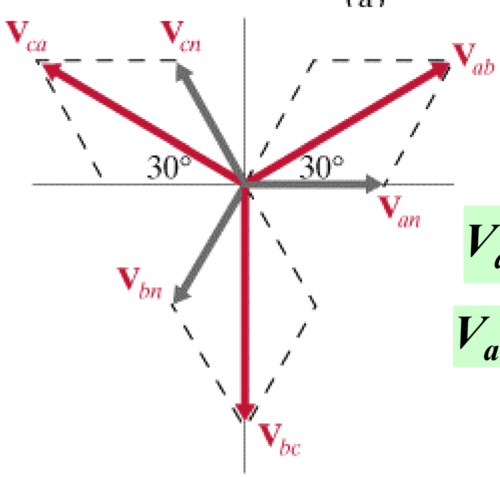
DELTA CONNECTED SOURCES



(a)

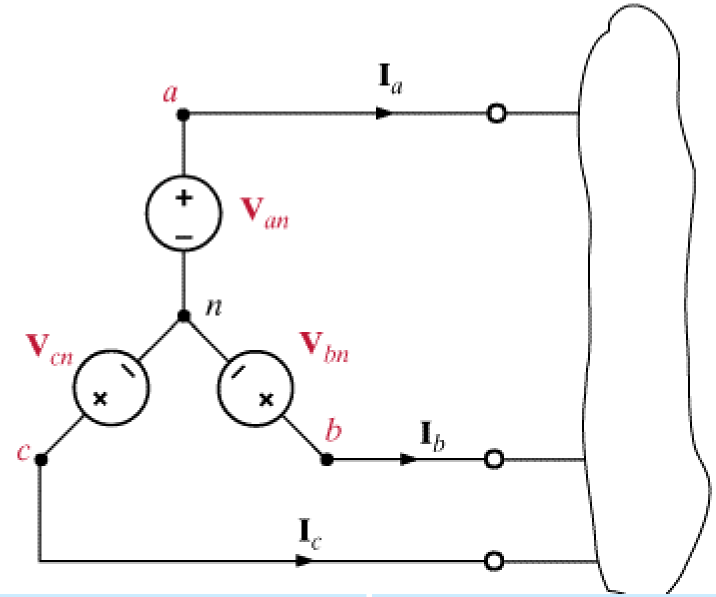


$$V_{ab} = \sqrt{3}\,|V_p|\,\angle 30^\circ$$

V_{an} lags V_{ab} by 30°

Relationship between phase and line voltages

Convert to an equivalent Y connection



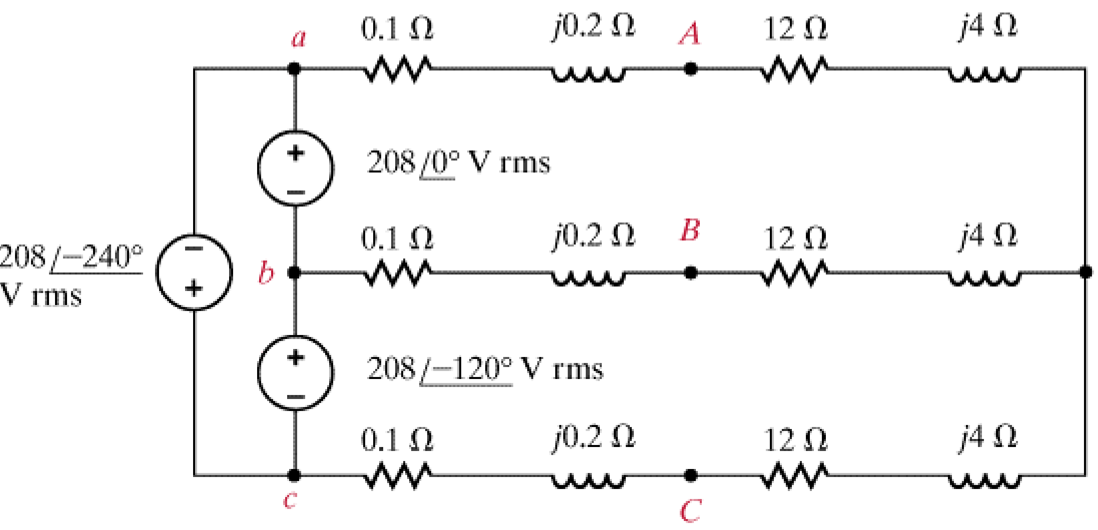
$$\left.\begin{aligned} V_{ab} &= V_L\angle 0^\circ \\ V_{bc} &= V_L\angle -120^\circ \\ V_{ca} &= V_L\angle 120^\circ \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} V_{an} &= \frac{V_L}{\sqrt{3}}\angle -30^\circ \\ V_{bn} &= \frac{V_L}{\sqrt{3}}\angle -150^\circ \\ V_{cn} &= \frac{V_L}{\sqrt{3}}\angle 90^\circ \end{aligned}\right.$$

Example

$$\left.\begin{aligned} V_{ab} &= 208\angle 60^\circ \\ V_{bc} &= 208\angle -60^\circ \\ V_{ca} &= 208\angle 180^\circ \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} V_{an} &= 120\angle 30^\circ \\ V_{bn} &= 120\angle -90^\circ \\ V_{cn} &= 120\angle 150^\circ \end{aligned}\right.$$

LEARNING EXAMPLE

Determine line currents and line voltages at the loads

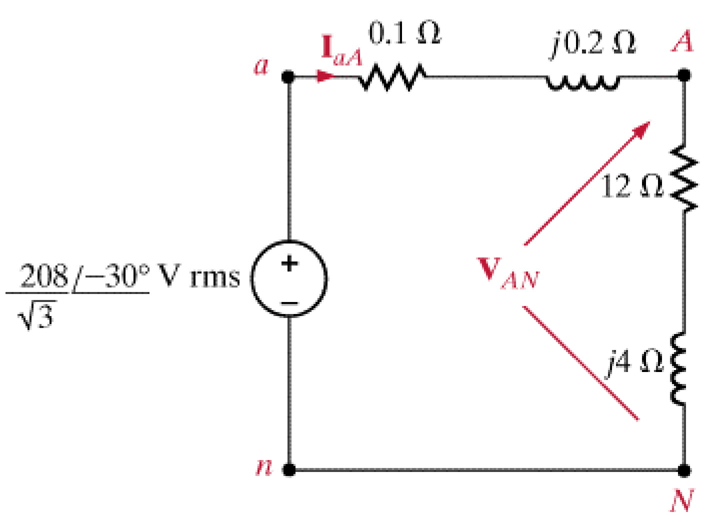


(a)

Source is Delta connected.
Convert to equivalent Y

$$\left.\begin{aligned} V_{ab} &= V_L\angle 0° \\ V_{bc} &= V_L\angle -120° \\ V_{ca} &= V_L\angle 120° \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} V_{an} &= \frac{V_L}{\sqrt{3}}\angle -30° \\ V_{bn} &= \frac{V_L}{\sqrt{3}}\angle -150° \\ V_{cn} &= \frac{V_L}{\sqrt{3}}\angle 90° \end{aligned}\right.$$

Analyze one phase



(b)

$$I_{aA} = \frac{(208/\sqrt{3})\angle -30°}{12.1 + j4.2} = 9.38\angle -49.14°(A)rms$$

$$V_{AN} = (12 + j4)\times 9.38\angle -49.19° = 118.65\angle -30.71°(V)rms$$

$$V_{ab} = \sqrt{3}\,|V_p|\angle 30° \qquad V_{AB} = \sqrt{3}\times 118.65\angle 0.71°$$

Determine the other phases using the balance

$$I_{bB} = 9.38\angle -169.14°(A)rms \qquad V_{BC} = \sqrt{3}\times 118.65\angle -119.29°$$

$$I_{cC} = 9.38\angle -71.86°(A)rms \qquad V_{CA} = \sqrt{3}\times 118.65\angle 120.71°$$

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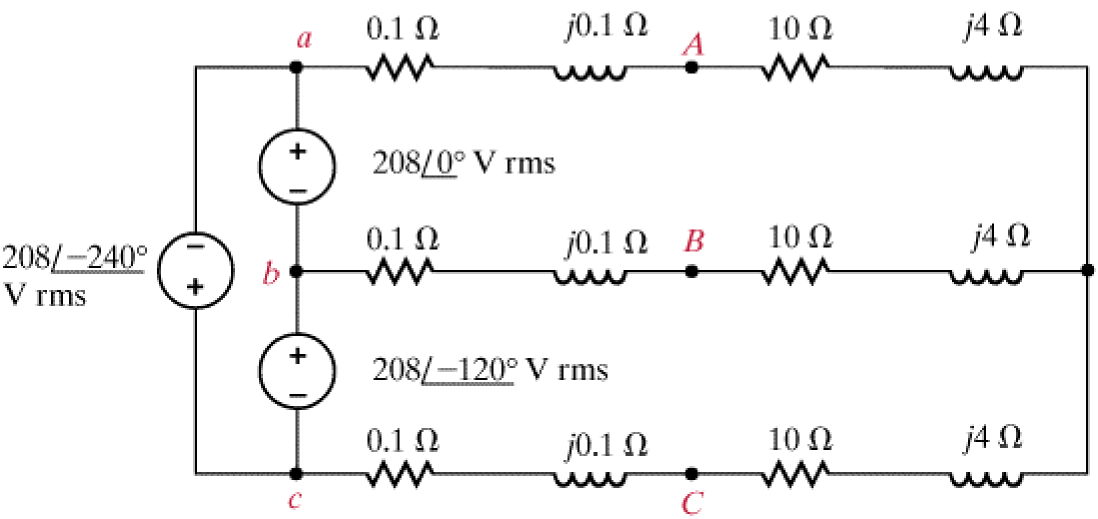
LEARNING EXTENSION

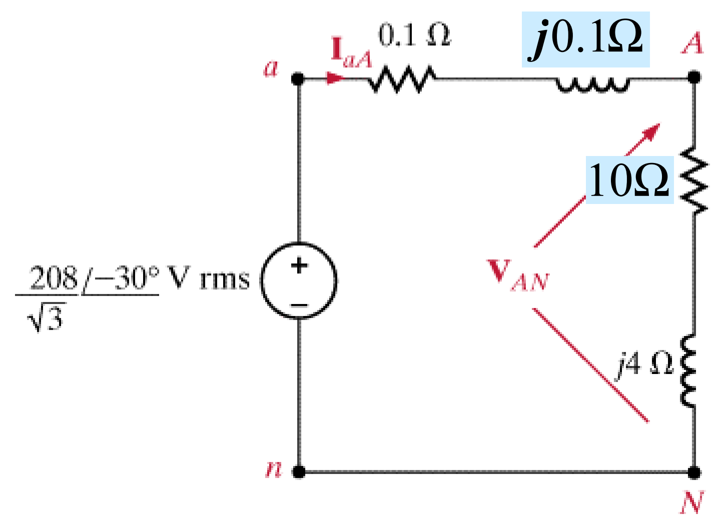
Compute the magnitude of the line voltage at the load



Source is Delta connected. Convert to equivalent Y

$$\left.\begin{aligned} V_{ab} &= V_L\angle 0° \\ V_{bc} &= V_L\angle -120° \\ V_{ca} &= V_L\angle 120° \end{aligned}\right\} \Rightarrow \left\{\begin{aligned} V_{an} &= \frac{V_L}{\sqrt{3}}\angle -30° \\ V_{bn} &= \frac{V_L}{\sqrt{3}}\angle -150° \\ V_{cn} &= \frac{V_L}{\sqrt{3}}\angle 90° \end{aligned}\right.$$

Analyze one phase



(b)

$$V_{AN} = \frac{10 + j4}{10.1 + j4.1} 120\angle -30°$$

Only interested in magnitudes!

$$|V_{AN}| = 120\frac{10.77}{10.90} = 118.57(V)rms$$

$$V_{ab} = \sqrt{3}\,|V_p|\,\angle 30°$$

$$|V_{AB}| = 205.4(V)rms$$

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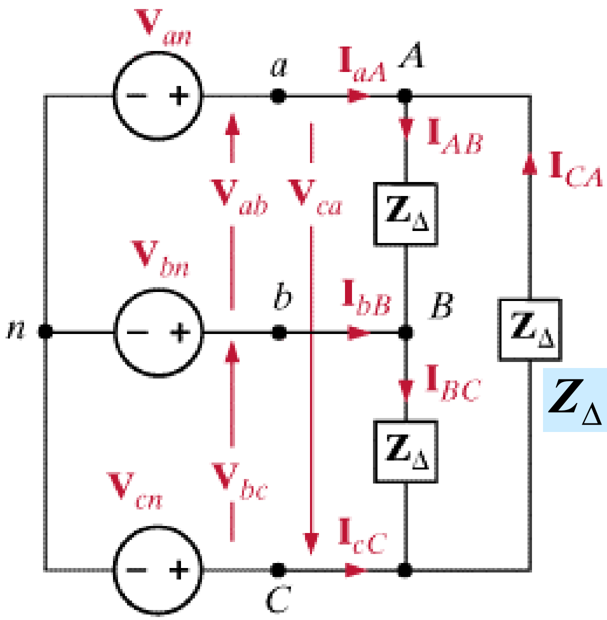
DELTA-CONNECTED LOAD



$$Z_\Delta = |Z_L| \angle \theta_Z$$

Method 1: Solve directly

$$V_{an} = |V_p| \angle 0°$$
$$V_{bn} = |V_p| \angle -120°$$
$$V_{cn} = |V_p| \angle 120°$$

Positive sequence phase voltages

$$V_{ab} = \sqrt{3}|V_p| \angle 30°$$
$$V_{bc} = \sqrt{3}|V_p| \angle -90°$$
$$V_{ca} = \sqrt{3}|V_p| \angle -210°$$

$$|I_{line}| = \sqrt{3}|I_\Delta|$$
$$\theta_{line} = \theta_\Delta - 30°$$

Line-phase current relationship

Load phase currents

$$I_{AB} = \frac{V_{AB}}{Z_\Delta} = |I_\Delta| \angle \theta_\Delta$$

$$I_{BC} = \frac{V_{BC}}{Z_\Delta} = |I_\Delta| \angle \theta_\Delta - 120°$$

$$I_{CA} = \frac{V_{CA}}{Z_\Delta} = |I_\Delta| \angle \theta_\Delta + 120°$$

$$\theta_\Delta = 30° - \theta_Z$$

Line currents

$$I_{aA} = I_{AB} - I_{CA}$$
$$I_{bB} = I_{BC} - I_{AB}$$
$$I_{cC} = I_{CA} - I_{BC}$$

Method 2: We can also convert the delta connected load into a Y connected one. The same formulas derived for resistive circuits are applicable to impedances

Balanced case $Z_Y = \frac{Z_\Delta}{3}$

$$I_{aA} = \frac{V_{an}}{Z_Y} = |I_{aA}| \angle \theta_L \Rightarrow \begin{cases} |I_{aA}| = \dfrac{|V_{AB}|/\sqrt{3}}{|Z_\Delta|/3} \\ \theta_L = -\theta_Z \end{cases}$$

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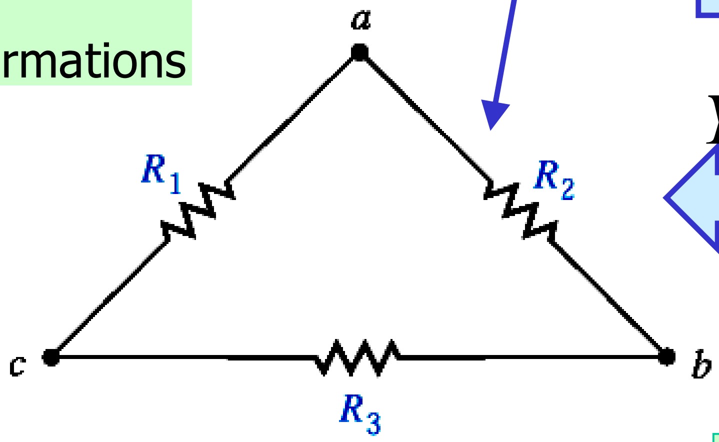
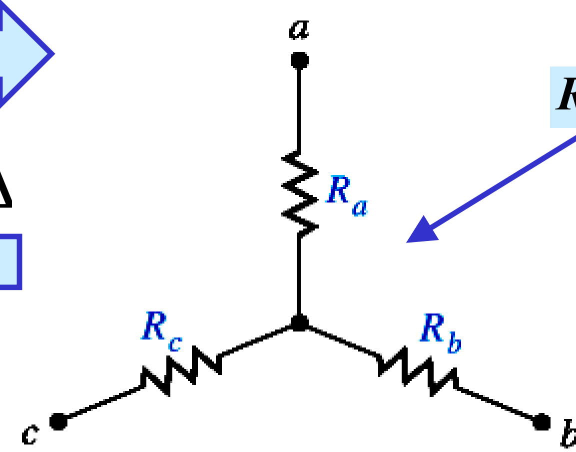
REVIEW OF $\Delta \leftrightarrow Y$ Transformations

$$R_{ab} = R_2 \,||\, (R_1 + R_3)$$



$\Delta \rightarrow Y$

$Y \rightarrow \Delta$



$$R_{ab} = R_a + R_b$$

$$R_a + R_b = \frac{R_2(R_1 + R_3)}{R_1 + R_2 + R_3}$$

$$R_b + R_c = \frac{R_3(R_1 + R_2)}{R_1 + R_2 + R_3}$$

$$R_c + R_a = \frac{R_1(R_2 + R_3)}{R_1 + R_2 + R_3}$$

SUBTRACT THE FIRST TWO THEN ADD TO THE THIRD TO GET Ra

$$R_a = \frac{R_1 R_2}{R_1 + R_2 + R_3}$$

$$R_b = \frac{R_2 R_3}{R_1 + R_2 + R_3}$$

$$R_c = \frac{R_3 R_1}{R_1 + R_2 + R_3}$$

$\Delta \rightarrow Y$

$$\frac{R_a}{R_b} = \frac{R_1}{R_3} \Rightarrow R_3 = \frac{R_b R_1}{R_a}$$

$$\frac{R_b}{R_c} = \frac{R_2}{R_1} \Rightarrow R_2 = \frac{R_b R_1}{R_c}$$

REPLACE IN THE THIRD AND SOLVE FOR R1

$$R_1 = \frac{R_a R_b + R_b R_c + R_c R_a}{R_b}$$

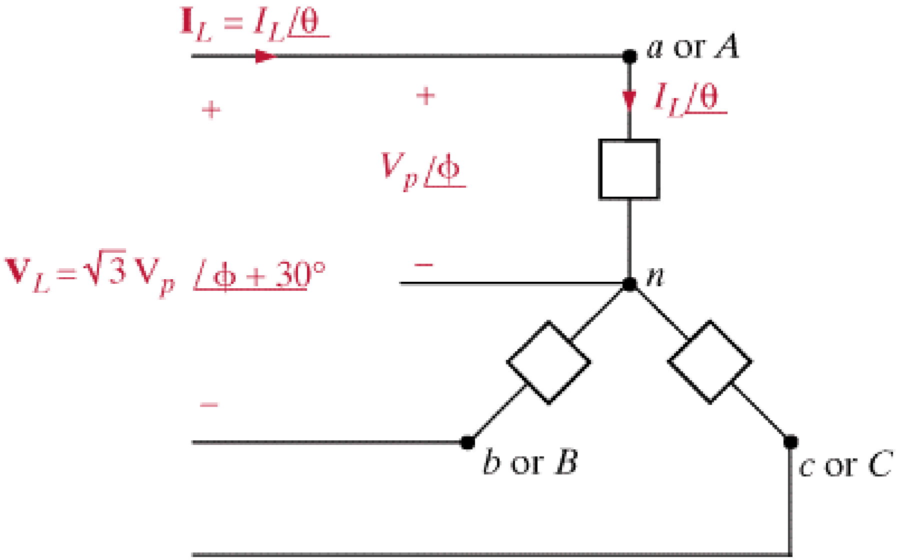
$$R_2 = \frac{R_a R_b + R_b R_c + R_c R_a}{R_c}$$

$$R_3 = \frac{R_a R_b + R_b R_c + R_c R_a}{R_a}$$

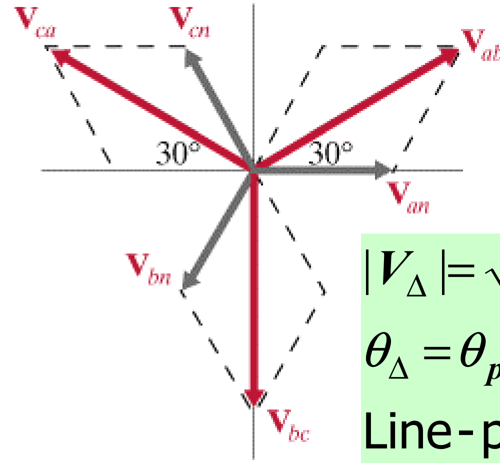
$Y - \Delta$

$$R_\Delta = R_1 = R_2 = R_3 \Rightarrow R_Y = \frac{R_\Delta}{3}$$

GEAUX



(a)

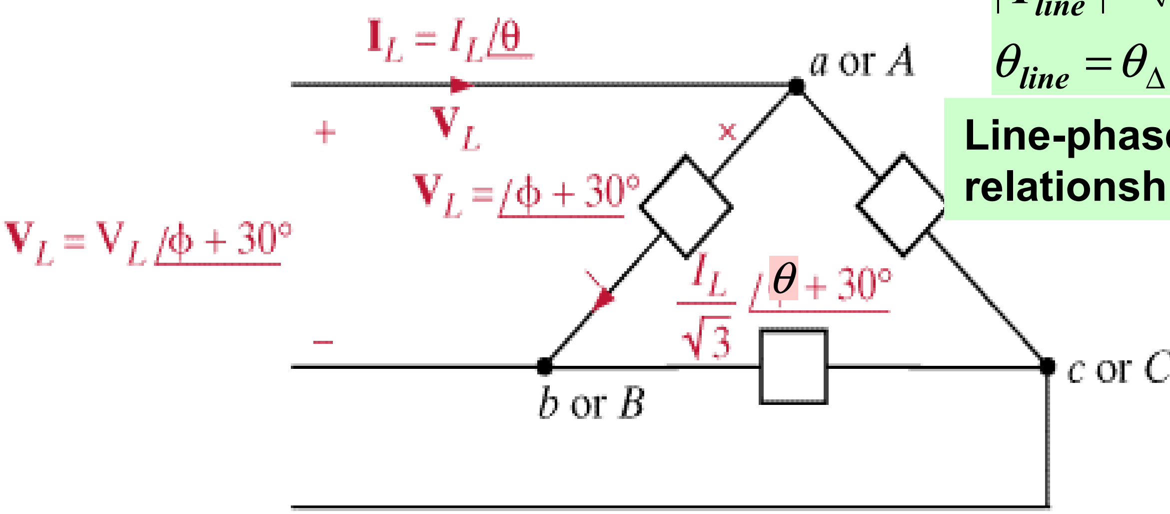


(b)

$$|V_\Delta| = \sqrt{3}\,|V_{phase}|$$

$$\theta_\Delta = \theta_{phase} + 30°$$

Line-phase voltage relationship



(b)

$$|I_{line}| = \sqrt{3}\,|I_\Delta|$$

$$\theta_{line} = \theta_\Delta - 30°$$

Line-phase current relationship

LEARNING EXTENSION

$I_{aA} = 12\angle 40°.$

Find the phase currents

$$I_{AB} = 6.93\angle 70°$$

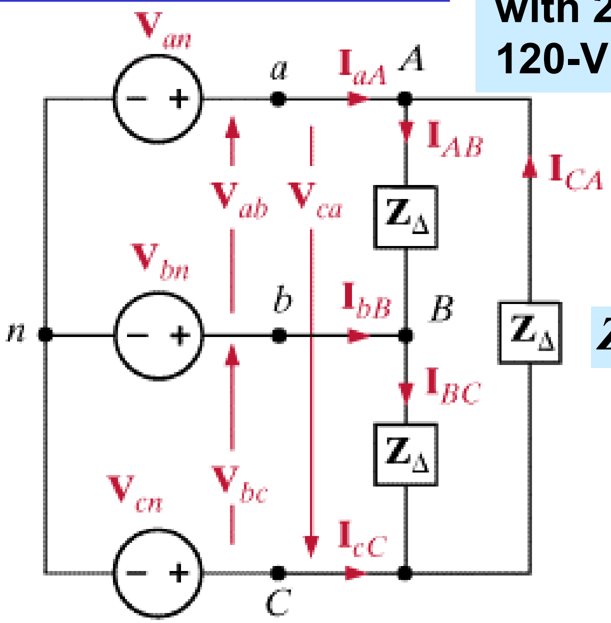
$$I_{BC} = 6.93\angle -50°$$

$$I_{CA} = 6.93\angle 190°$$

GEAUX

LEARNING EXAMPLE

Delta-connected load consists of 10-Ohm resistance in series with 20-mH inductance. Source is Y-connected, abc sequence, 120-V rms, 60Hz. Determine all line and phase currents



$$V_{an} = 120\angle 30°(V)rms$$

$$Z_{\text{inductance}} = 2\pi \times 60 \times 0.020 = 7.54\Omega$$

$$Z_\Delta = 10 + j7.54\Omega = 12.52\angle 37.02° \quad \Rightarrow Z_Y = 4.17\angle 37.02°$$

$$I_{AB} = \frac{V_{AB}}{Z_\Delta} = \frac{120\sqrt{3}\angle 60°}{10 + j7.54} = 16.60\angle 22.98°(A)rms$$

$$I_{BC} = 16.60\angle -97.02°(A)rms$$

$$I_{CA} = 16.60\angle 142.98°(A)rms$$

$$|V_\Delta| = \sqrt{3}\,|V_{phase}|$$

$$\theta_\Delta = \theta_{phase} + 30°$$

Line-phase voltage relationship

$$I_{aA} = 28.75\angle -7.02°(A)rms$$

$$I_{bB} = 28.75\angle -127.02°(A)rms$$

$$I_{cC} = 28.75\angle 112.98°(A)rms$$

$$|I_{line}| = \sqrt{3}\,|I_\Delta|$$

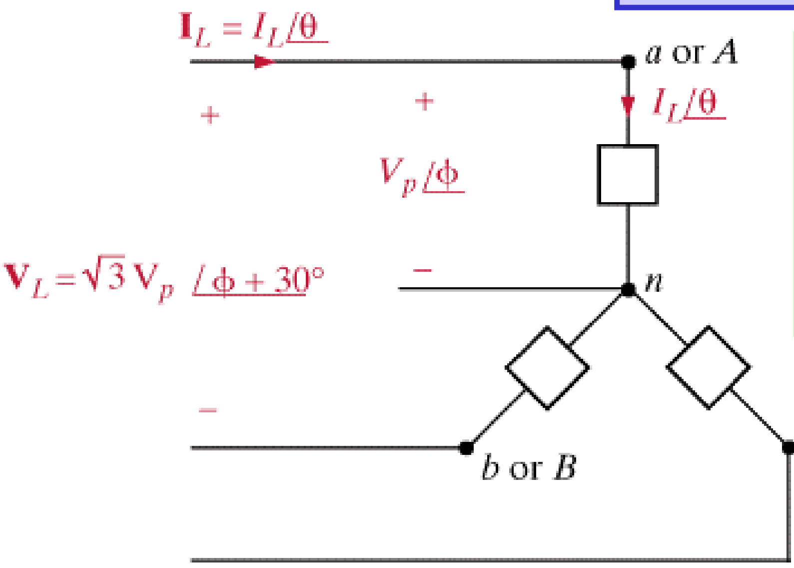
$$\theta_{line} = \theta_\Delta - 30°$$

Line-phase current relationship

Alternatively, determine first the line currents and then the delta currents

GEAUX

POWER RELATIONSHIPS



(a)

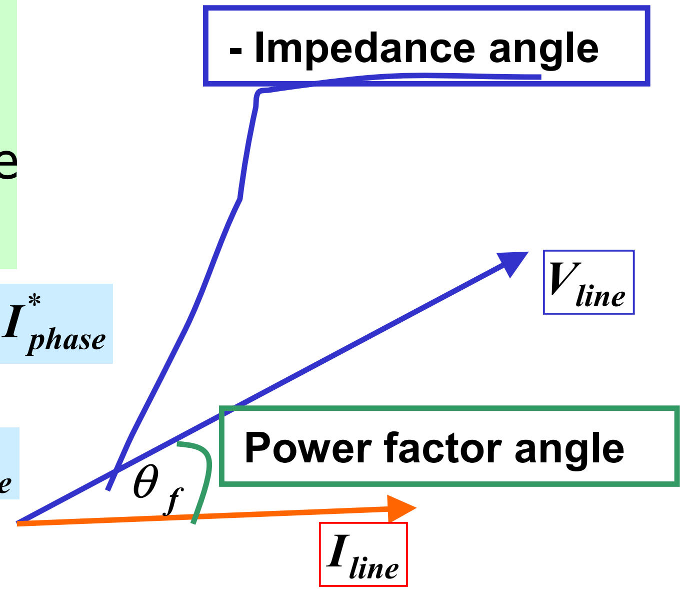
$$|V_\Delta| = \sqrt{3}\,|V_{phase}|$$

$$\theta_\Delta = \theta_{phase} + 30°$$

Line-phase voltage relationship

$$S_{Total} = 3 \times V_{phase} \times I^*_{phase}$$

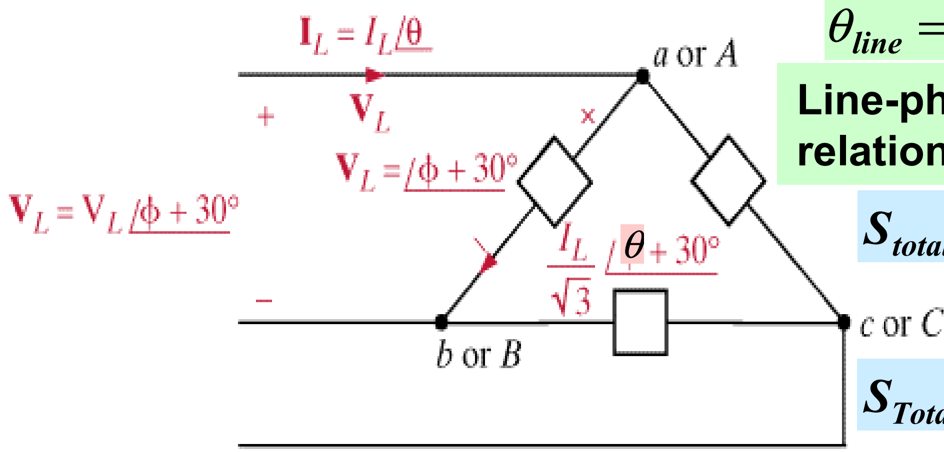
$$S_{Total} = \sqrt{3} V_{line} I^*_{line}$$



$$|I_{line}| = \sqrt{3}\,|I_\Delta|$$

$$\theta_{line} = \theta_\Delta - 30°$$

Line-phase current relationship



(b)

$$S_{total} = 3V_{line} \times I^*_\Delta$$

$$S_{Total} = \sqrt{3} V_{line} I^*_{line}$$

$$P_{total} = \sqrt{3}\,|V_{line}|\,|I_{line}|\cos\theta_f$$

$$Q_{total} = \sqrt{3}\,|V_{line}|\,|I_{line}|\sin\theta_f$$

GEAUX

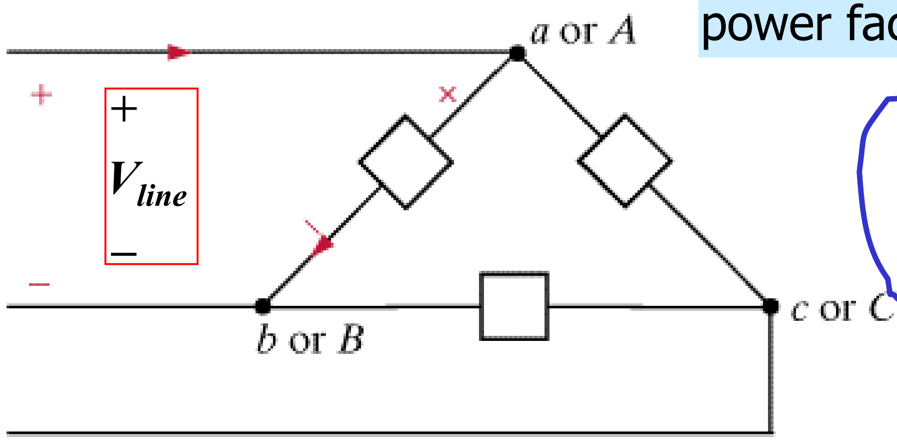
LEARNING EXAMPLE

$|V_{line}| = 208(V)rms$

$P_{total} = 1200W$

power factor angle = 20° lagging

Determine the magnitude of the line currents and the value of load impedance per phase in the delta



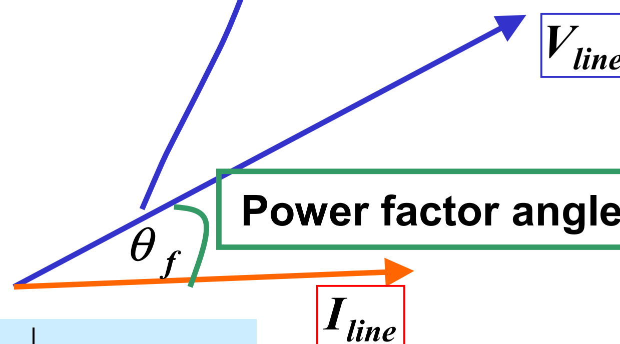
- Impedance angle

$$P_{total} = \sqrt{3}|V_{line}||I_{line}|\cos\theta_f$$

$$Q_{total} = \sqrt{3}|V_{line}||I_{line}|\sin\theta_f$$

$$Z_\Delta = 101.46\angle 20°$$

$$\frac{P_{total}}{3} = \frac{|V_{line}||I_{line}|}{\sqrt{3}}\cos\theta_f \Rightarrow |I_{line}| = 3.54(A)rms$$



Power factor angle

$$|I_{line}| = \sqrt{3}|I_\Delta|$$

$$\theta_{line} = \theta_\Delta - 30°$$

Line-phase current relationship

$$\Rightarrow |I_\Delta| = 2.05(A)rms$$

$$\Rightarrow |Z_\Delta| = \frac{|V_{line}|}{|I_\Delta|} = 101.46\Omega$$

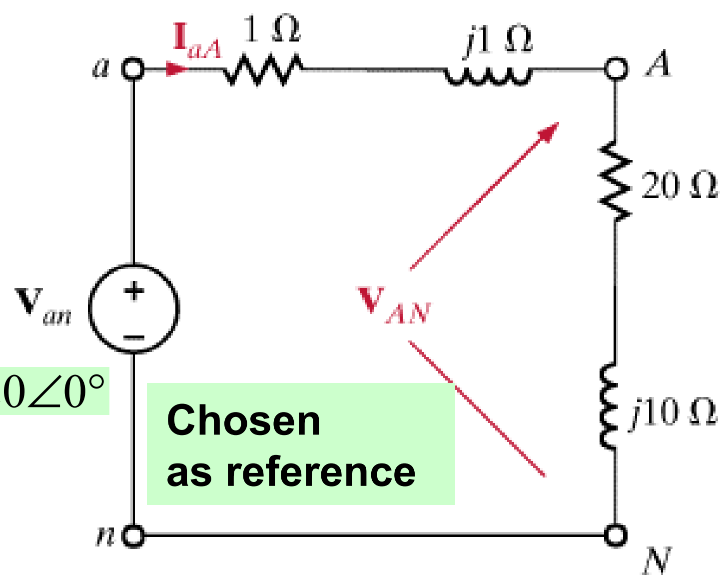
GEAUX

LEARNING EXAMPLE

For an abc sequence, balanced Y - Y three phase circuit

source $|V_{phase}| = 120(V)_{rms}$, $Z_{line} = 1 + j1\Omega$, $Z_{phase} = 20 + j10\Omega$

Determine real and reactive power per phase at the load and total real, reactive and complex power at the source



$120\angle 0°$

Chosen as reference

$$V_{AN} = I_{aA} \times (20 + j10) = I_{aA} \times 22.36\angle 26.57°$$

$$V_{AN} = 113.15\angle -1.08°(V)rms$$

$$S_{phase} = V_{AN} I_{aA}^* = 113.15\angle -1.08° \times 5.06\angle 27.65°$$

$$S_{phase} = 572.54\angle 26.57° = 512 + j256.09(VA)rms$$

$P_{\text{per phase}}$ $Q_{\text{per phase}}$

$$S_{\text{source phase}} = V_{an} \times I_{aA}^* = 120\angle 0° \times 5.06\angle 27.65°$$

$$V_{an} = 120\angle 0°$$
$$V_{bn} = 120\angle -120°$$
$$V_{an} = 120\angle 120°$$

Because circuit is balanced data on any one phase are sufficient

Abc sequence

$$I_{aA} = \frac{V_{an}}{21 + j11} = \frac{120\angle 0°}{23.71\angle 27.65°}$$
$$= 5.06\angle -27.65°(A)rms$$

$$S_{\text{source phase}} = 607.2\angle 27.65°$$
$$= 537.86 + j281.78VA$$

$$P_{\text{total source}} = 3 \times 537.86(W)$$
$$Q_{\text{total source}} = 3 \times 281.78(VA)$$

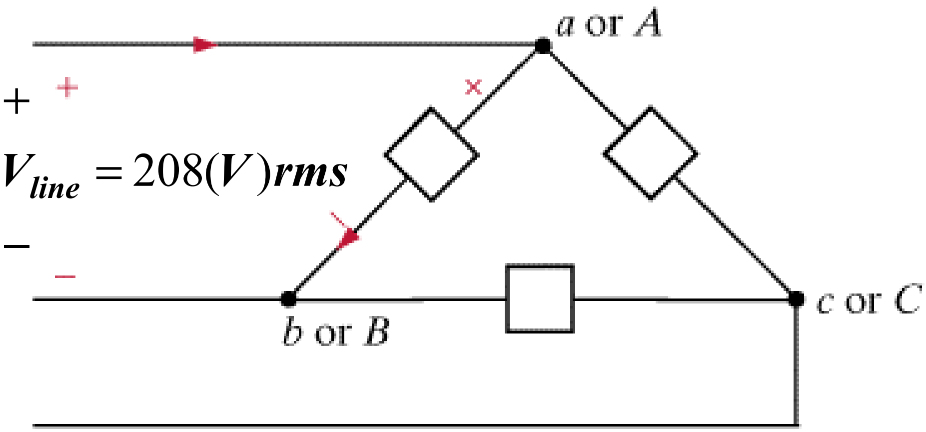
$$S_{\text{total source}} = P_{\text{total source}} + Q_{\text{total source}}$$
$$= 1613.6 + j845.2(VA)$$

$$|S_{\text{total source}}| = 1821.6(VA)$$

GEAUX

LEARNING EXAMPLE

Determine the line currents and the combined power factor



$V_{line} = 208(V)rms$

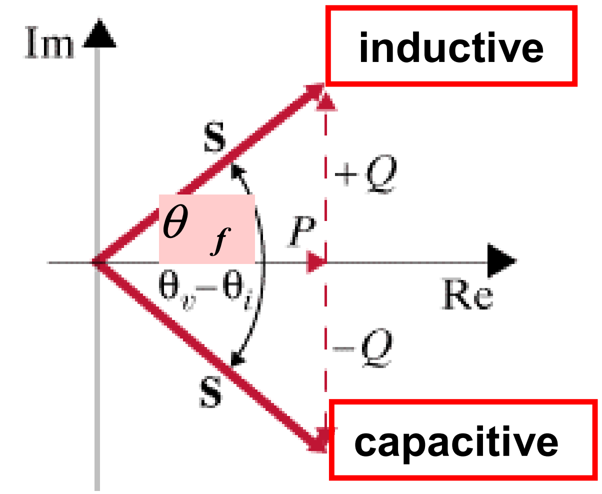
Circuit is balanced

Load 1: 24kW at pf = 0.6 lagging

Load 2: 10kW at pf = 1

Load 3: 12kVA at pf = 0.8 leading

$$S = P + jQ$$
$$P = |S|\cos\theta_f$$
$$Q = |S|\sin\theta_f$$
$$pf = \cos\theta_f$$
$$S_{total} = S_1 + S_2 + S_3$$



$$\left.\begin{matrix} P_1 = 24kW \\ pf = 0.6\,lagging \end{matrix}\right\} \Rightarrow |S_1| = 40kVA$$

$$|Q_1| = \sqrt{|S_1|^2 - |P_1|^2} = 32kVA$$

$$lagging \Rightarrow inductive \therefore S_1 = 24 + j32\,kVA$$

Load 2

$$\left.\begin{matrix} P_2 = 10kW \\ pf = 1 \end{matrix}\right\} \Rightarrow S_2 = 10 + j0\,kVA$$

Load 3

$$\left.\begin{matrix} |S_3| = 12kVA \\ pf = 0.8 \end{matrix}\right\} \Rightarrow \left\{\begin{matrix} P_3 = 9.6kW \\ |Q_3| = 7.2kVA \end{matrix}\right.$$

$$leading\ \ pf \Rightarrow capacitive \therefore S_3 = 9.6 - j7.2kVA$$

$$S_{TOTAL} = S_1 + S_2 + S_3 = 43.6 + j24.8kVA = 50.160\angle 29.63° kVA$$

$$P_{total} = \sqrt{3}|V_{line}||I_{line}|\cos\theta_f$$
$$Q_{total} = \sqrt{3}|V_{line}||I_{line}|\sin\theta_f$$

$$\Rightarrow \left\{\begin{matrix} |S_{total}| = \sqrt{3}|V_{line}| \times |I_{line}| \\ \theta_f = 29.63° \end{matrix}\right.$$

$$pf = 0.869\ lagging$$

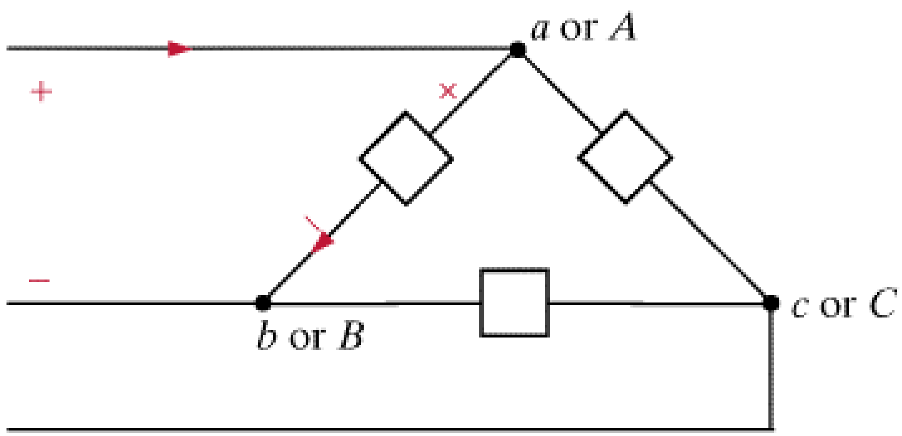
$$|I_{line}| = 139.23(A)rms$$

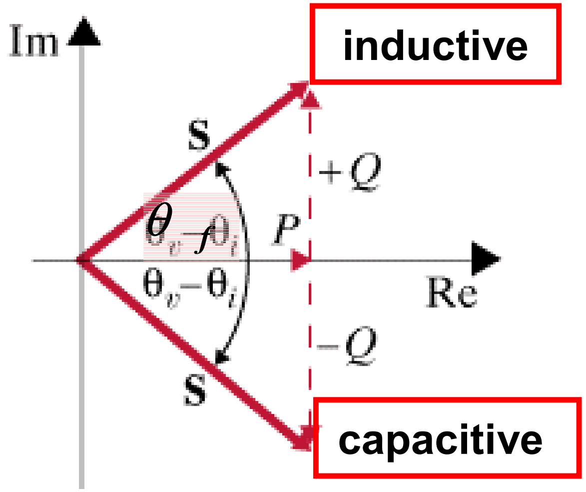
Continued ...

GEAUX

LEARNING EXAMPLE continued ….

If the line impedances are $Z_{line} = 0.05 + j0.02\Omega$ determine line voltages and power factor at the source





$|I_{line}| = 139.23(A)rms$

$$S_{line} = 3\times(Z_{line}I_{line})I^{*}_{line} = 3\times Z_{line}|I_{line}|^2$$

$$S_{line} = 2908 + j1163(VA)$$

$$S_{\text{load total}} = 43.6 + j24.8kVA = 50.160\angle 29.63° kVA$$

$$S_{\text{source total}} = 46.508 + j25.963 = 53.264\angle 29.17° kVA$$

$$\begin{cases} |S_{total}| = \sqrt{3}\,|V_{line}|\times|I_{line}| \\ \theta_f = 29.17° \end{cases}$$

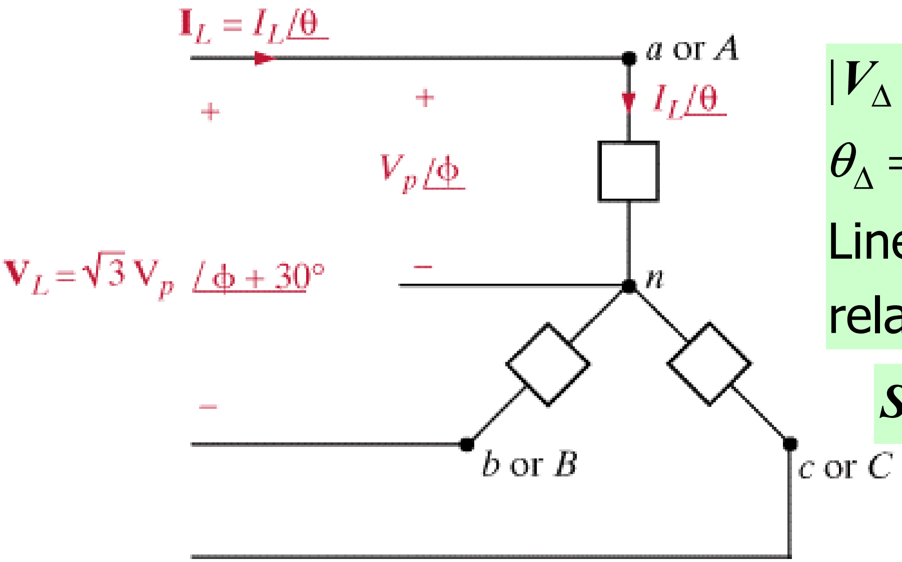
$$V_{line} = \frac{53{,}264}{\sqrt{3}\times 139.13} = 220.87(V)rms$$

$$pf = \cos\theta_f = \cos(29.17°) = 0.873\,lagging$$

GEAUX

LEARNING EXTENSION

A Y -Y balanced three-phase circuit has a line voltage of 208-Vrms. The total real power absorbed by the load is 12kW at pf=0.8 lagging. Determine the per-phase impedance of the load



$$|V_\Delta| = \sqrt{3}\,|V_{phase}|$$

$$\theta_\Delta = \theta_{phase} + 30°$$

Line-phase voltage relationship

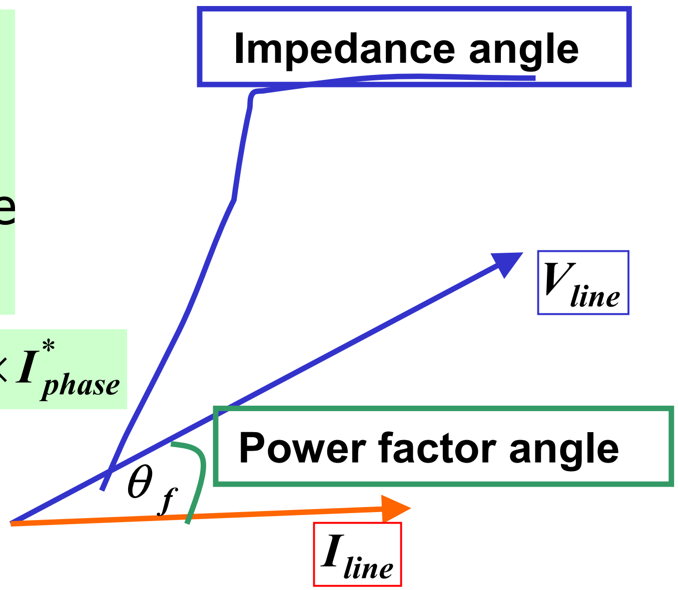
$$S_{Total} = 3 \times V_{phase} \times I^*_{phase}$$

$$S = P + jQ$$

$$P = |S|\cos\theta_f$$

$$Q = |S|\sin\theta_f$$

$$pf = \cos\theta_f$$



$$\Rightarrow |V_{phase}| = \frac{208}{\sqrt{3}} = 120(V)rms$$

$$S_{total} = 3V_{phase} \times \left(\frac{V_{phase}}{Z_{phase}}\right)^* = 3 \times \frac{|V_{phase}|^2}{Z^*_{phase}}$$

$$|Z_{phase}| = \frac{3 \times |V_{phase}|^2}{|S_{total}|} = 2.88\Omega$$

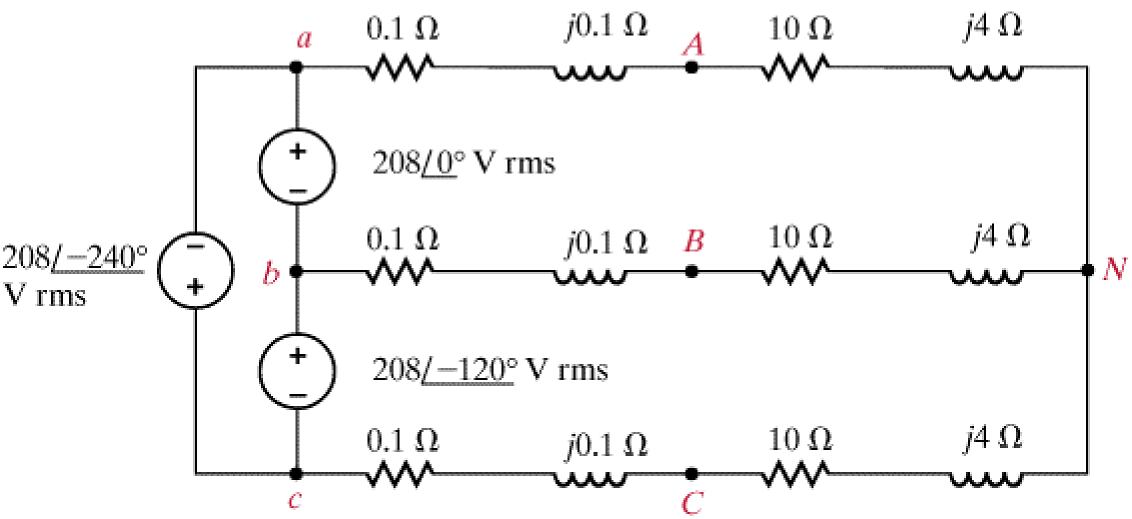
$$pf = 0.8 = \cos\theta_f \Rightarrow \theta_f = 36.87°$$

$$|S_{total}| = \frac{P_{total}}{pf} = 15kVA$$

$$Z_{pahse} = 2.88\angle 36.87°\Omega$$

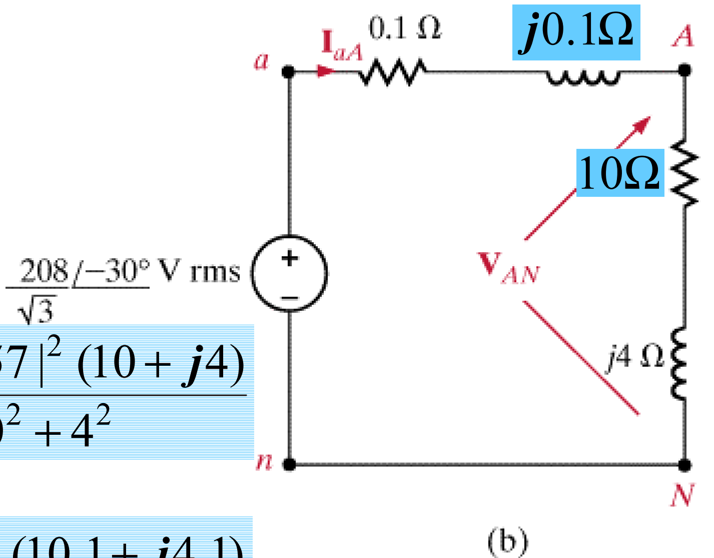
GEAUX

LEARNING EXTENSION Determine real, reactive and complex power at both load and source



Source is Delta connected. Convert to equivalent Y

Analyze one phase



$$S_{load} = 3 \times V_{AN} I_{aA}^{*} = 3 \times \frac{|V_{AN}|^2}{Z_{phase}^{*}} = \frac{3 \times |118.57|^2}{10 - j4} = \frac{3 \times |118.57|^2 (10 + j4)}{10^2 + 4^2}$$

$$S_{source} = 3 \times V_{an} I_{aA}^{*} = 3 \times \frac{|V_{an}|^2}{Z_{total\ phase}^{*}} = \frac{3 \times |120|^2}{10.1 - j4.1} = \frac{3 \times |120|^2 (10.1 + j4.1)}{(10.1)^2 + (4.1)^2}$$

$$V_{AN} = \frac{10 + j4}{10.1 + j4.1} 120\angle -30^\circ$$

$$|V_{AN}| = 120 \frac{10.77}{10.90} = 118.57 (V) rms$$

$$S_{load} = 3 \times (1{,}212.0 + j484.8)$$

$$S_{source} = 3 \times (1224.1 + j496)$$

GEAUX

LEARNING EXTENSION

A 480-V rms line feeds two balanced 3-phase loads. The loads are rated
Load 1: 5kVA at 0.8 pf lagging
Load 2: 10kVA at 0.9 pf lagging.

Determine the magnitude of the line current from the 408-V rms source

$$|S_1| = 5kVA = \frac{P}{0.8} \Rightarrow P_1 = 4kW$$

$$Q_1 = \sqrt{|S_1|^2 - P_1^2} = 3.0kVA$$

$$pf\ lagging \Rightarrow S_1 = 4 + j3kVA$$

$$|S_2| = 10kVA = \frac{P}{0.9} \Rightarrow P = 9kW$$

$$Q_2 = \sqrt{|S_2|^2 - P_2^2} = 4.36kVA$$

$$S_2 = 9 + j4.36kVA$$

$$S_{total} = 13 + j7.36kVA$$

$$S = P + jQ$$

$$P = |S|\cos\theta_f$$

$$Q = |S|\sin\theta_f$$

$$pf = \cos\theta_f$$

$$S_{total} = S_1 + S_2$$

$$P_{total} = \sqrt{3}|V_{line}||I_{line}|\cos\theta_f$$

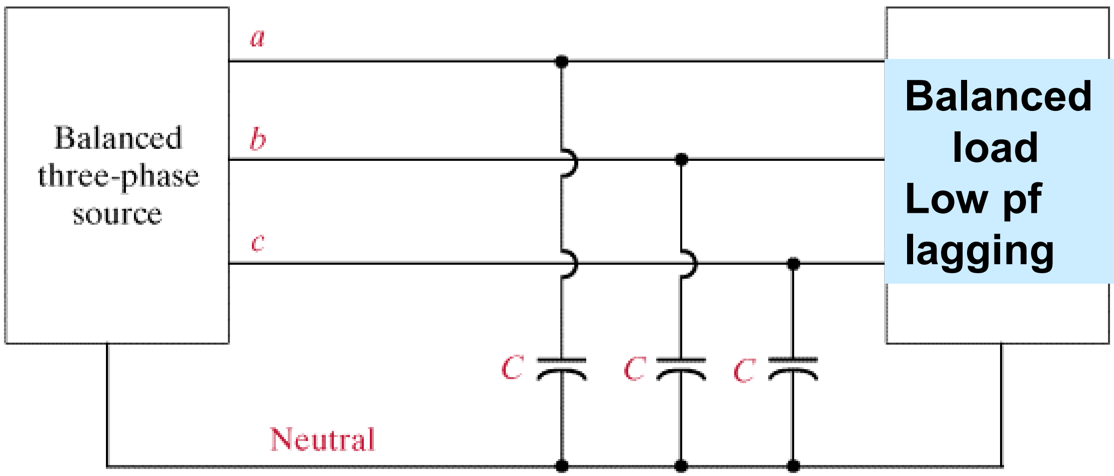
$$Q_{total} = \sqrt{3}|V_{line}||I_{line}|\sin\theta_f$$

$$|S_{total}| = \sqrt{3}|V_{line}||I_{line}|$$

$$|I_{lineq}| = \frac{|S_{total}|}{\sqrt{3} \times |V_{line}|} = \frac{14{,}939}{706.68} = 21.14(A)rms$$

GEAUX

POWER FACTOR CORRECTION



Similar to single phase case. Use capacitors to increase the power factor

Keep clear about total/phase power, line/phase voltages

$$\left.\begin{matrix} S_{old} \\ pf_{old} \end{matrix}\right\} \rightarrow Q_{old}$$

$$\Delta Q = Q_{new} - Q_{old}$$

Reactive Power to be added

$$\left.\begin{matrix} P_{old} \\ pf_{new} \end{matrix}\right\} \rightarrow Q_{new}$$

To use capacitors this value should be negative

$$Q_{\text{per capacitor}} = -\omega C V^2$$

The voltage depends on how the capacitors are connected

$$S = P + jQ$$

$$P = |S| \cos\theta_f$$

$$Q = |S| \sin\theta_f$$

$$pf = \cos\theta_f$$

$$Q = P \tan\theta_f$$

$$lagging \Rightarrow Q > 0$$

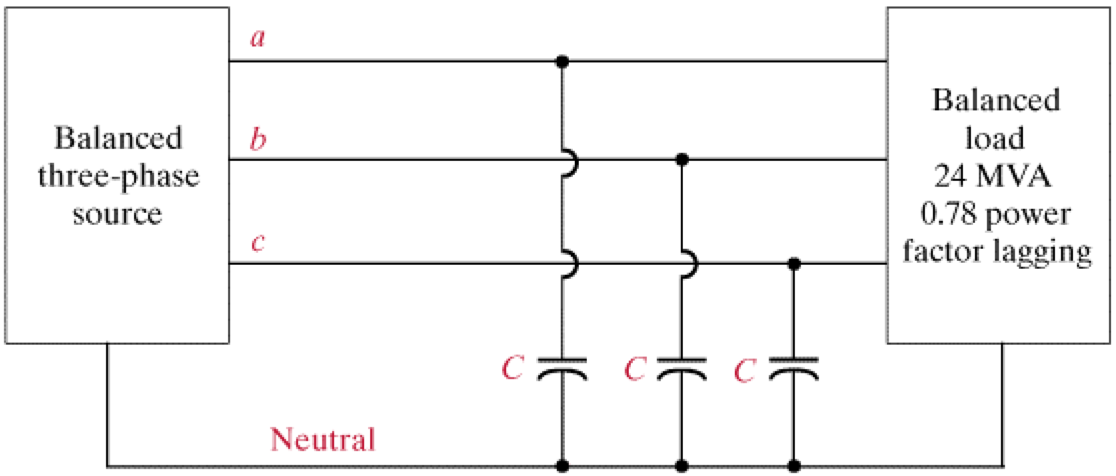
$$pf = \cos\theta_f \Rightarrow \sin\theta_f = \sqrt{1 - pf^2}$$

$$\tan\theta_f = \frac{pf}{\sqrt{1 - pf^2}}$$

GEAUX

LEARNING EXAMPLE

$f = 60Hz, |V_{line}| = 34.5kV\ rms.$ Required: $pf = 0.94\ leading$



$S = P + jQ$

$P = |S|\cos\theta_f$

$Q = |S|\sin\theta_f$

$pf = \cos\theta_f$

$Q = P\tan\theta_f$

$\tan\theta_f = \frac{pf}{\sqrt{1-pf^2}}$

$lagging \Rightarrow Q_{old} > 0$

$pf = \cos\theta_f \Rightarrow \sin\theta_f = \sqrt{1-pf^2} = 0.626$

$|Q_{old}| = 15.02MVA$

$P_{old} = 18.72MW$

$$\left.\begin{matrix} P_{old} = 18.72MW \\ pf_{new} = 0.94\ leading \end{matrix}\right\} \Rightarrow Q_{new} = -6.8MVA$$

$\Delta Q = -6.8 - 15.02 = -21.82MVA$

$Q_{\text{per capacitor}} = -7.273MVA$

$Y-\text{connection} \Rightarrow V_{\text{capacitor}} = \frac{34.5}{\sqrt{3}}kV\ rms$

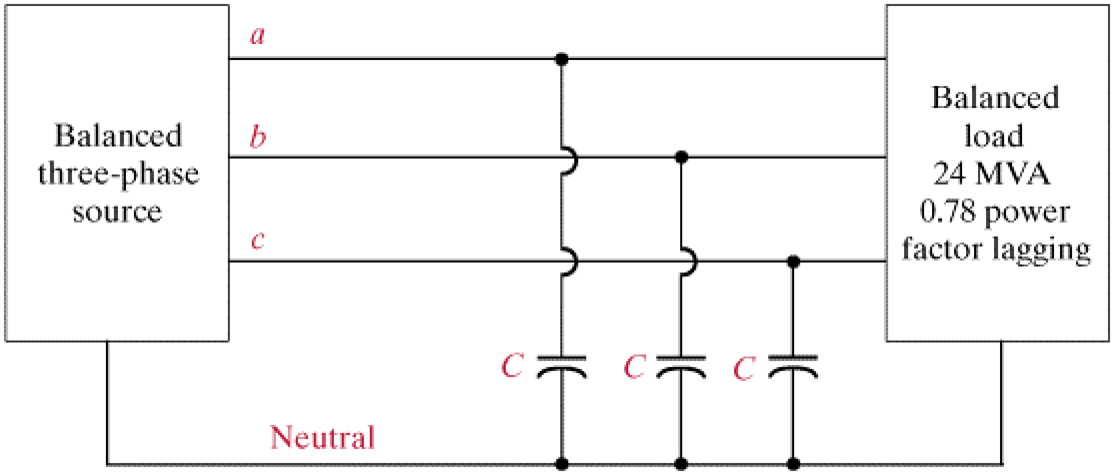
$$-7.273\times10^6 = -2\pi\times60\times C\times\left(\frac{34.5\times10^3}{\sqrt{3}}\right)^2$$

$C = 48.6\mu F$

GEAUX

LEARNING EXAMPLE

$f = 60Hz, |V_{line}| = 34.5kV\ rms$. Required $pf = 0.90\ lagging$



$$S = P + jQ$$
$$P = |S|\cos\theta_f$$
$$Q = |S|\sin\theta_f$$
$$pf = \cos\theta_f$$

$$Q = P\tan\theta_f$$

$$\tan\theta_f = \frac{pf}{\sqrt{1-pf^2}}$$

$$lagging \Rightarrow Q_{old} > 0$$

$$pf = \cos\theta_f \Rightarrow \sin\theta_f = \sqrt{1-pf^2} = 0.626$$

$$|Q_{old}| = 15.02MVA$$
$$P_{old} = 18.72MW$$

$$\left.\begin{array}{l} P_{old} = 18.72MW \\ pf_{new} = 0.90\ lagging \end{array}\right\} \Rightarrow Q_{new} = -9.067MVA$$

$$\Delta Q = 9.067 - 15.02 = -5.953MVA$$
$$Q_{\text{per capacitor}} = -1.984MVA$$

$$Y - \text{connection} \Rightarrow V_{\text{capacitor}} = \frac{34.5}{\sqrt{3}}kV\ rms$$

$$-1.984\times10^6 = -2\pi\times60\times C\times\left(\frac{34.5\times10^3}{\sqrt{3}}\right)^2$$

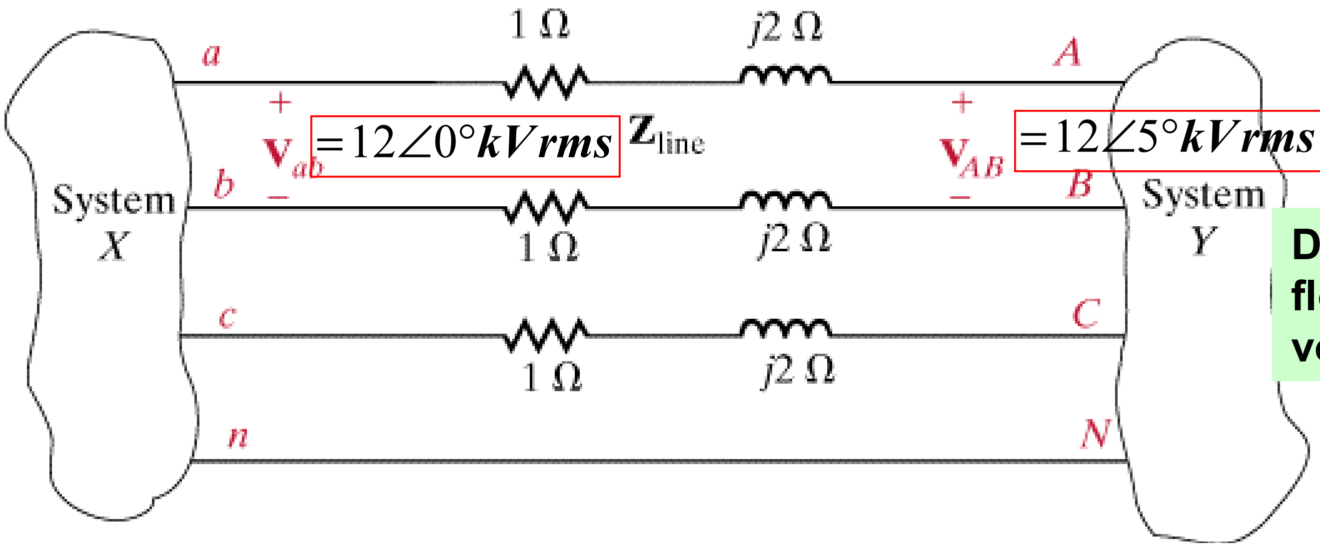
$$C = 13.26\mu F$$

GEAUX

LEARNING EXAMPLE

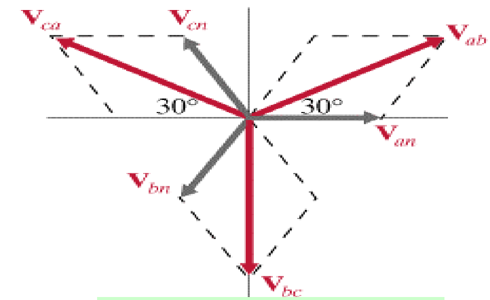
MEASURING POWER FLOW

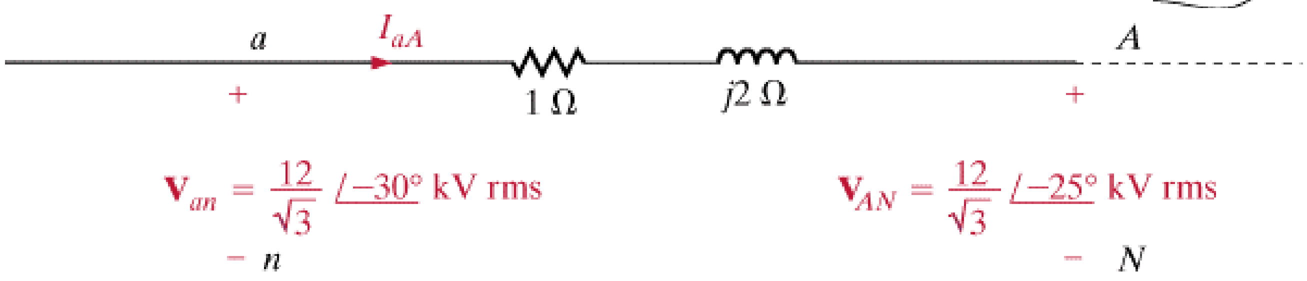
Which circuit is the source and what is the average power supplied?



Phase differences determine direction of power flow!

Determine the current flowing. Convert line voltages to phase voltages





$$S_Y = 3V_{AN} \times I_{aA}^*$$

$$S_X = 3V_{an}(-I_{aA})^*$$

Equivalent 1-phase circuit

$$I_{aA} = \frac{V_{an} - V_{AN}}{1 + j2} = \frac{\frac{12000}{\sqrt{3}}\angle -30° - \frac{12000}{\sqrt{3}}\angle -25°}{1 + j2}$$

$$I_{aA} = 270.30\angle -180.93 (A) rms$$

$$S_Y = \sqrt{3} \times 12 \times 0.2703\angle(-25 + 180.93)° MVA$$

$$S_X = -\sqrt{3} \times 12 \times 0.2703\angle(-30 + 180.93)° MVA$$

$$P_Y = -5.13 MW$$

$$P_X = 4.91 MW$$

$$S = P + jQ$$

$$P = |S| \cos\theta_f$$

System Y is the source

$$P_{loss} = -(P_X + P_Y)$$

GEAUX

CAPACITOR SPECIFICATIONS

Capacitors for power factor correction are normally specified in VARs

$$|Q_{\text{per capacitor}}| = \omega C V^2$$

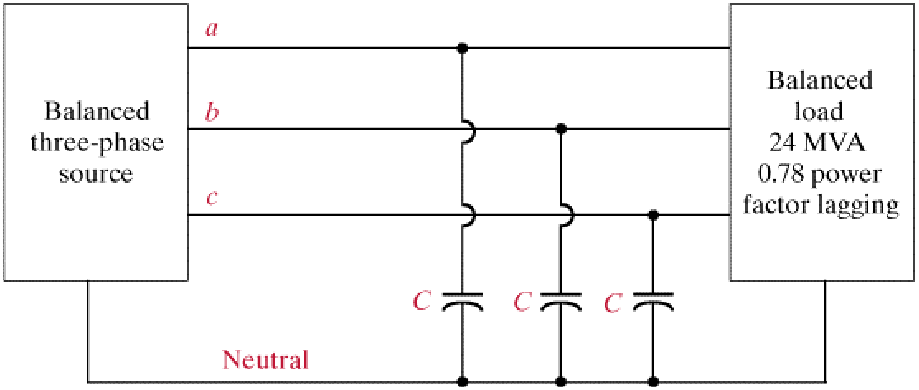
The voltage and frequency must be given in order to know the capacitance

Assume $60\,Hz$ unless other value is given.

LEARNING EXAMPLE

For pf $= 0.94$ leading one needs

$$C = 48.6\mu F$$



$$V_{line} = 34.5kV \Rightarrow V_{phase} = 19.9kV$$

Capacitor 1 is not rated at high enough voltage!

Choices available

Capacity	Rated Voltage (kV)	Rated Q (Mvar)
1	10	4
2	50	25
3	20	7.5

$$C_1 = \frac{4\times 10^6}{2\pi \times 60 \times (10\times 10^3)^2} = 106.1\mu F$$

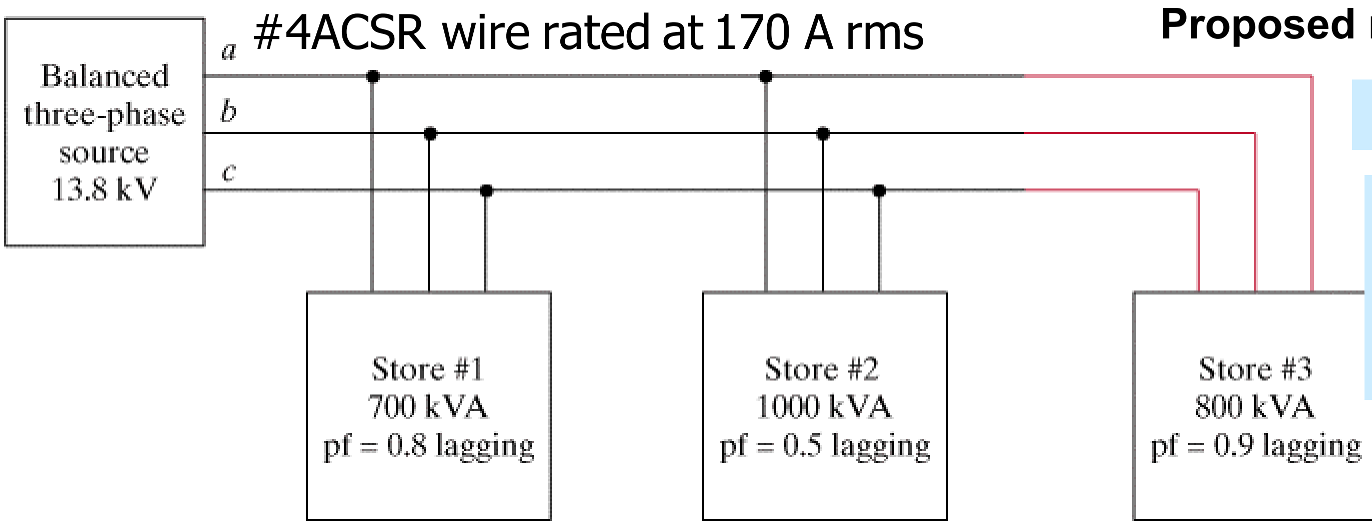
$$C_2 = \frac{25\times 10^6}{2\pi \times 60 \times (50\times 10^3)^2} = 26.53\mu F$$

$$C_3 = \frac{7.5\times 10^6}{2\pi \times 60 \times (20\times 10^3)^2} = 49.7\mu F$$

Capacitor 3 is the best alternative

GEAUX

LEARNING BY DESIGN



1. Is the wire suitable?

2. What capacitance would be required to have a composite pf =0.92 lagging

Capacitors are to be Y - connected

$$S_1 = 700\angle 36.9° = 560 + j420\,kVA$$

$$S_2 = 1000\angle 60°\,kVA = 500 + j866\,kVA$$

$$S_3 = 800\angle 25.8°\,kVA = 720 + j349\,kVA$$

$$S_{total} = S_1 + S_2 + S_3$$

$$S = P + jQ$$

$$P = |S|\cos\theta_f$$

$$Q = |S|\sin\theta_f$$

$$pf = \cos\theta_f$$

$$S_{total} = 1780 + j1635\,kVA = 2417\angle 42.57°\,kVA$$

$$|I_{line}| = \frac{|S_{total}|}{\sqrt{3}\times V_{line}} = \frac{2.417\times 10^6}{\sqrt{3}\times 13.8\times 10^3} = 101.1\,A\,rms$$

Wire is OK

$$S_{Total} = 3\times V_{phase}\times I^*_{phase} = \sqrt{3}\times V_{line}\times I^*_{line}$$

$$\left.\begin{array}{l} P_{old} \\ pf_{new} \end{array}\right\} \rightarrow Q_{new} = P\tan\theta_{f(new)} = 758.28\,kVA$$

$$\Delta Q = Q_{new} - Q_{old} = -876.72\,kVA$$

$$|Q_{\text{per capacitor}}| = \omega C V^2$$

$$V = |V_{phase}| = \frac{13.8\,kV}{\sqrt{3}}$$

$$C = \frac{\left(876.72\times 10^3 / 3\right)}{2\pi\times 60\times\left(13.8\times 10^3\right)^2 / 3} = 12.2\,\mu F$$

Polyphase

STOP