# ADDITIONAL ANALYSIS TECHNIQUES

## LEARNING GOALS

**REVIEW LINEARITY**
The property has two equivalent definitions.
We show and application of homogeneity

**APPLY SUPERPOSITION**
We discuss some implications of the superposition property in linear circuits

**DEVELOP THEVENIN'S AND NORTON'S THEOREMS**
These are two very powerful analysis tools that allow us to focus on parts of a circuit and hide away unnecessary complexities

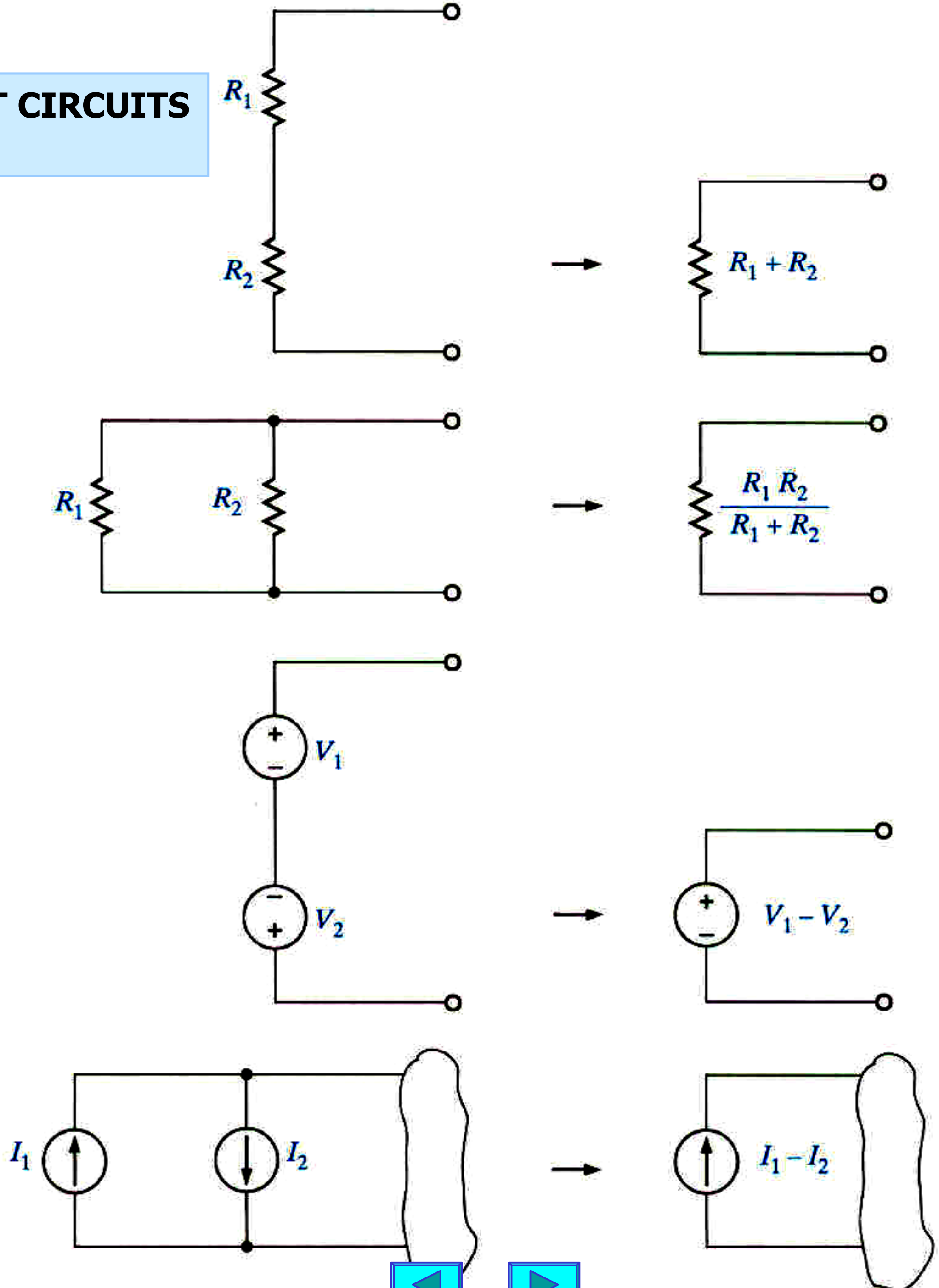
**MAXIMUM POWER TRANSFER**
This is a very useful application of Thevenin's and Norton's theorems

**THE METHODS OF NODE AND LOOP ANALYSIS PROVIDE POWERFUL TOOLS TO DETERMINE THE BEHAVIOR OF EVERY COMPONENT IN A CIRCUIT**

**The techniques developed in chapter 2; i.e., combination series/parallel, voltage divider and current divider are special techniques that are more efficient than the general methods, but have a limited applicability.**
**It is to our advantage to keep them in our repertoire and use them when they are more efficient.**

**In this section we develop additional techniques that simplify the analysis of some circuits.**
**In fact these techniques expand on concepts that we have already introduced: linearity and circuit equivalence**

**SOME EQUIVALENT CIRCUITS ALREADY USED**




GEAUX

# LINEARITY

**THE MODELS USED ARE ALL LINEAR. MATHEMATICALLY THIS IMPLIES THAT THEY SATISFY THE PRINCIPLE OF SUPERPOSITION**

THE MODEL $y = Tu$ IS LINEAR IFF

$$T(\alpha_1 u_1 + \alpha_2 u_2) = \alpha_1 T u_1 + \alpha_2 T u_2$$

for all possible input pairs $u_1, u_2$

and all possible scalars $\alpha_1, \alpha_2$

**AN ALTERNATIVE, AND EQUIVALENT, DEFINITION OF LINEARITY SPLITS THE SUPERPOSITION PRINCIPLE IN TWO.**

THE MODEL $y = Tu$ IS LINEAR IFF

1. $T(u_1 + u_2) = Tu_1 + Tu_2 \ , \forall u_1, u_2$ additivity
2. $T(\alpha u) = \alpha T u, \ \forall \alpha, \forall u$ homogeneity

**NOTICE THAT, TECHNICALLY, LINEARITY CAN NEVER BE VERIFIED EMPIRICALLY ON A SYSTEM. BUT IT COULD BE DISPROVED BY A SINGLE COUNTER EXAMPLE.**
**IT CAN BE VERIFIED MATHEMATICALLY FOR THE MODELS USED.**

**USING NODE ANALYSIS FOR RESISTIVE CIRCUITS ONE OBTAINS MODELS OF THE FORM** $Av = f$

***v*** **IS A VECTOR CONTAINING ALL THE NODE VOLTAGES AND *f* IS A VECTOR DEPENDING ONLY ON THE INDEPENDENT SOURCES.**
**IN FACT THE MODEL CAN BE MADE MORE DETAILED AS FOLLOWS**

$$Av = Bs$$

**HERE, *A,B,* ARE MATRICES AND *s* IS A VECTOR OF ALL INDEPENDENT SOURCES**

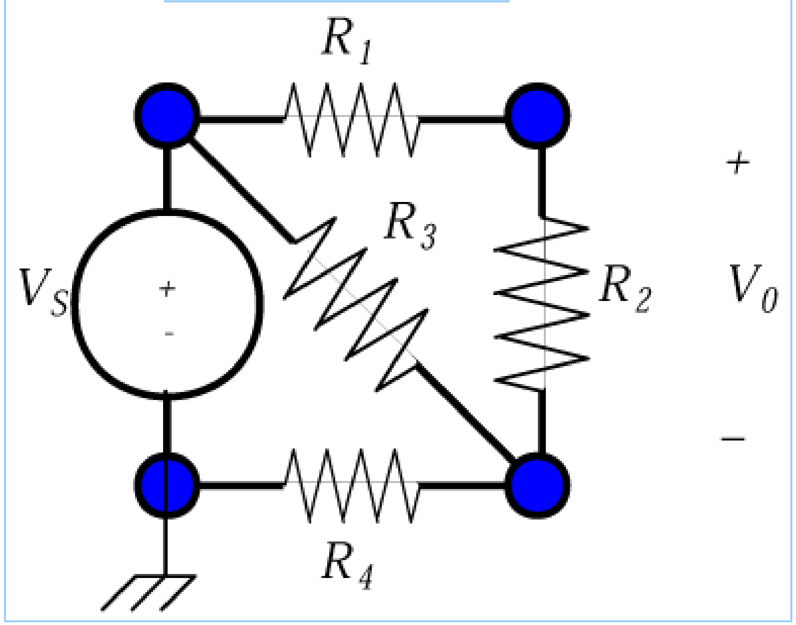
**FOR CIRCUIT ANALYSIS WE CAN USE THE LINEARITY ASSUMPTION TO DEVELOP SPECIAL ANALYSIS TECHNIQUES**

FIRST WE REVIEW THE TECHNIQUES CURRENTLY AVAILABLE

GEAUX

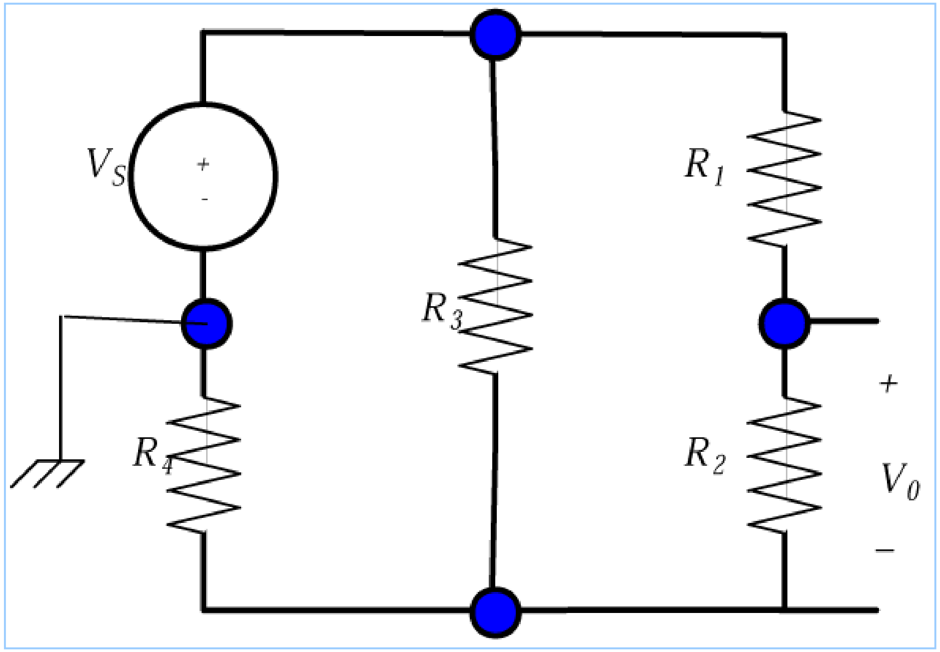
# A CASE STUDY TO REVIEW PAST TECHNIQUES

DETERMINE $V_O$

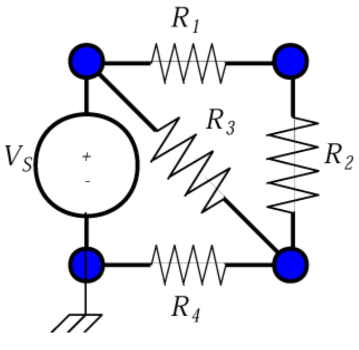



**SOLUTION TECHNIQUES AVAILABLE??**

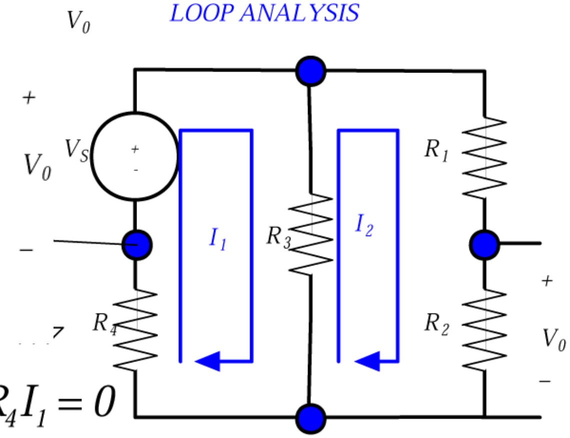
**Redrawing the circuit may help us in recognizing special cases**




GEAUX




## LOOP ANALYSIS

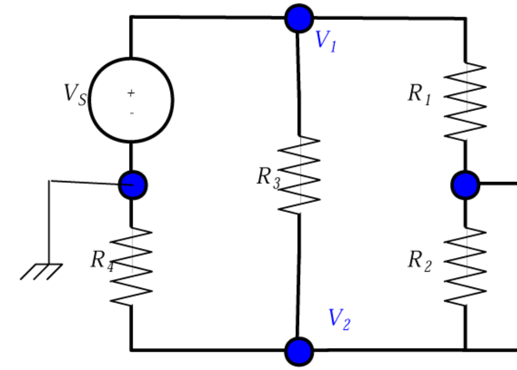



$$V_S + R_3(I_1 \quad I_2) + R_4 I_1 = 0$$

$$R_3(I_2 \quad I_1) + (R_1 + R_2)I_2 = 0$$

$$V_0 = R_2 I_2$$

## NODE ANALYSIS

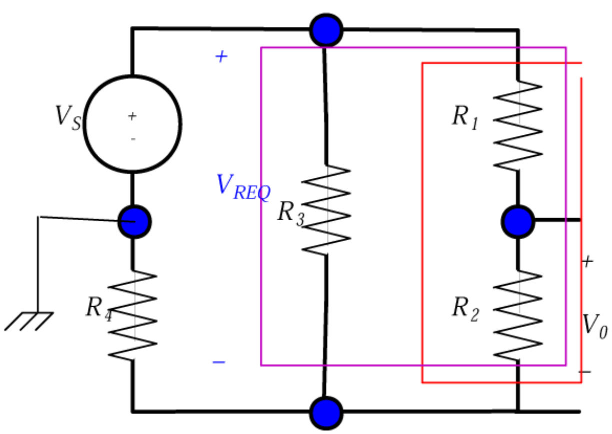



$$V_1 = V_S$$

$$\frac{V_2 \quad V_1}{R_1} + \frac{V_2}{R_4} + \frac{V_2 \quad V_0}{R_2} = 0$$

$$\frac{V_0 \quad V_2}{R_2} + \frac{V_0 \quad V_1}{R_1} = 0$$

## COMBINATION SERIES/PARALLEL




$$R_{EQ} = \frac{R_3(R_1 + R_2)}{R_1 + R_2 + R_3}$$

$R_{EQ} = ||R_3, (R_1 + R_2)||$

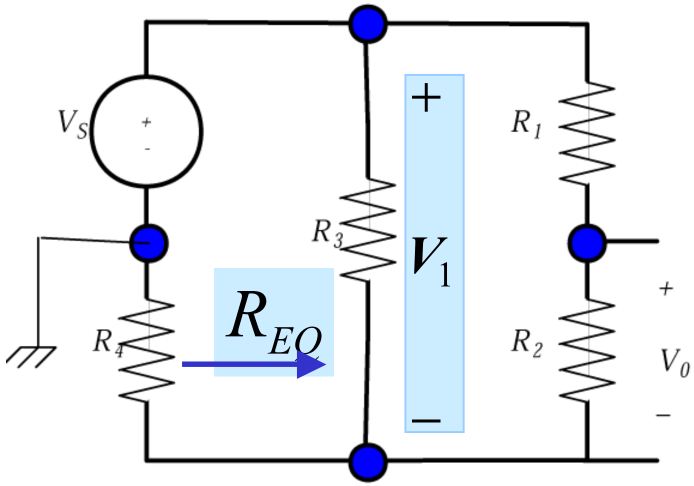
## VOLTAGE DIVIDER

$$V_{REQ} = \frac{R_{EQ}}{R_{EQ} + R_4} V_S$$

$$V_0 = \frac{R_2}{R_2 + R_3} V_{REQ}$$

GEAUX

# USING HOMOGENEITY




**Assume that the answer is known. Can we Compute the input in a very easy way ?!!**

**If Vo is given then V1 can be computed using an inverse voltage divider.**

$$V_1 = \frac{R_1 + R_2}{R_2} V_0$$

**... And Vs using a second voltage divider**

$$V_S = \frac{R_4 + R_{EQ}}{R_{EQ}} V_1 = \frac{R_4 + R_{EQ}}{R_{EQ}} \frac{R_1 + R_2}{R_2} V_0$$

**Solve now for the variable Vo**

**The procedure can be made entirely algorithmic**

1. **Give to Vo any arbitrary value (e.g., V'o =1 )**
2. **Compute the resulting source value and call it V'_s**
3. **Use linearity.** $V_S' \rightarrow V_0' \Rightarrow kV_S' \rightarrow kV_0', \forall k$
4. **The given value of the source (V_s) corresponds to** $k = \frac{V_S}{V_S'}$

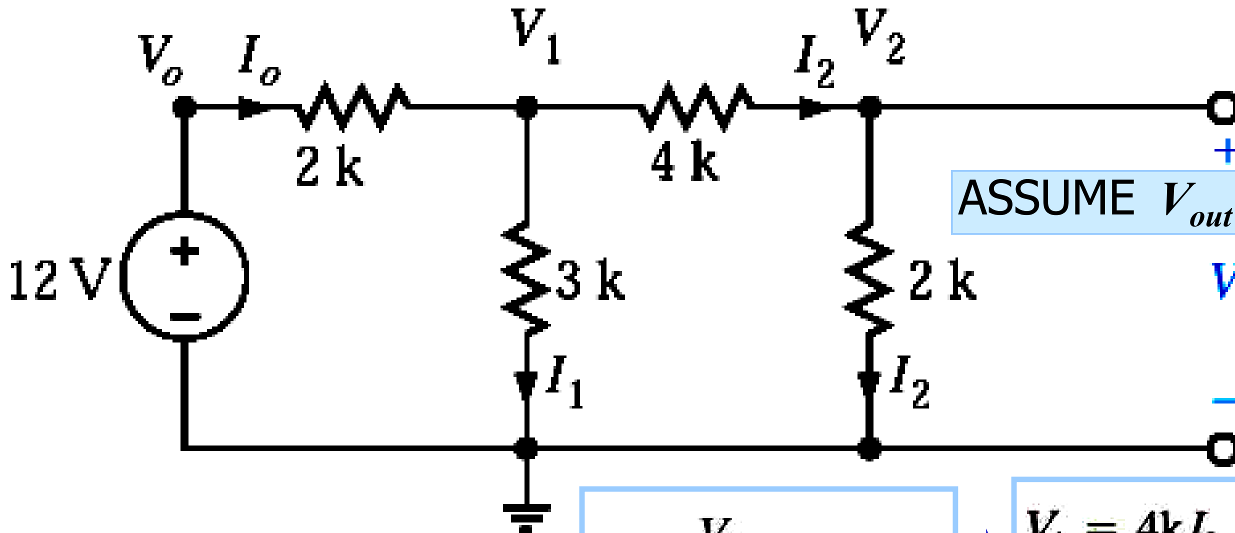
**Hence the desired output value is**

$$V_0 = kV_0' = \frac{V_S}{V_S'} V_0'$$

**This is a nice little tool for special problems. Normally when there is only one source and when in our judgement solving the problem backwards is actually easier**

GEAUX

# SOLVE USING HOMOGENEITY




ASSUME $V_{out} = V_2 = 1[V]$

$$I_2 = \frac{V_2}{2k} = 0.5\ \text{mA} \Rightarrow V_1 = 4kI_2 + V_2 = 3\ \text{V} \Rightarrow I_1$$

$$I_1 = \frac{V_1}{3k} = 1\ \text{mA} \Rightarrow I_o = I_1 + I_2 = 1.5\ \text{mA} \Rightarrow V_O$$

$$V_o = 2kI_o + V_1 = 6\ \text{V}$$

**NOW USE HOMOGENEITY**

$$V_O = 6[V] \rightarrow V_{out} = 1[V]$$

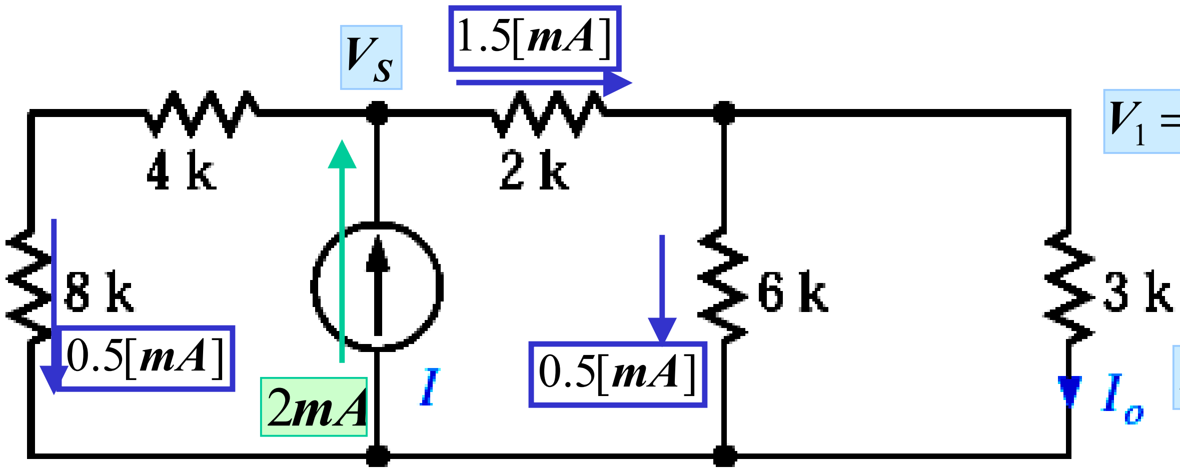
$$V_O = 12[V] \rightarrow V_{out} = 2[V]$$

GEAUX

# LEARNING EXTENSION

COMPUTE $I_O$ USING HOMOGENEITY. USE $I = 6mA$

$$V_S = 1.5[mA] \times 2k\Omega + V_1 = 6[V]$$




ASSUME $I_O = 1mA$

USE HOMOGENEITY

$I = 2mA \rightarrow I_O = 1mA$

$I = 6mA \rightarrow I_O =$ _____

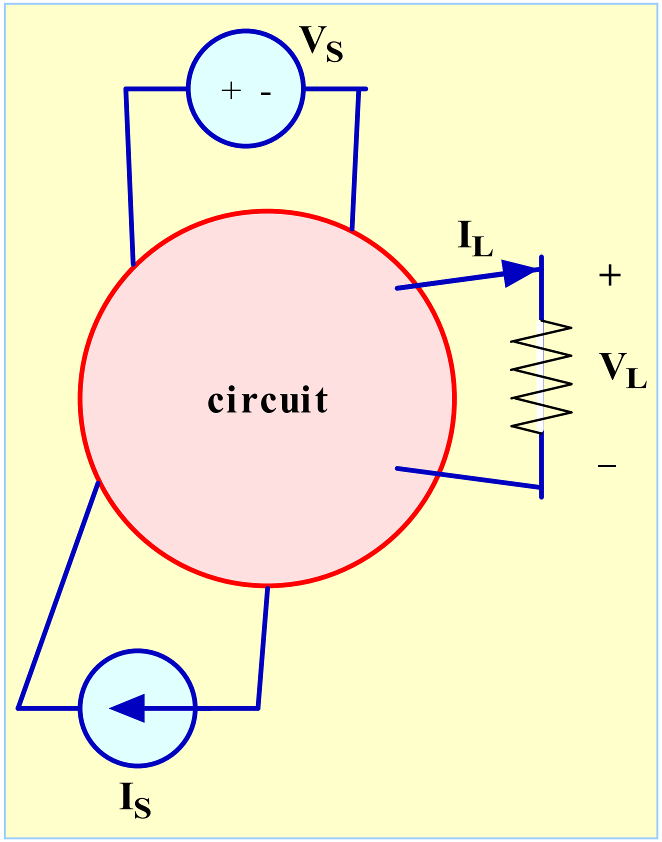
GEAUX

# Source Superposition

This technique is a direct application of linearity.

**It is normally useful when the circuit has only a few sources.**

FOR CLARITY WE SHOW A CIRCUIT WITH ONLY TWO SOURCES




## Due to Linearity

$$V_L = a_1 V_S + a_2 I_S$$

CONTRIBUTION BY $V_S$: $V_L^1$

CONTRIBUTION BY $I_S$: $V_L^2$

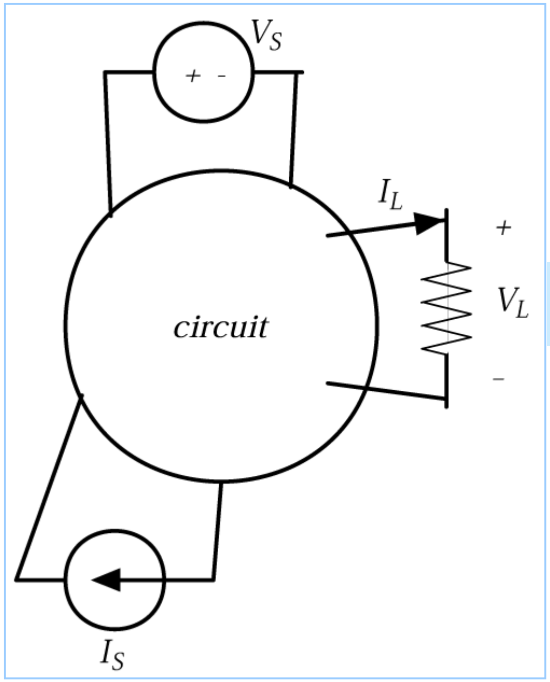
$V^1{}_L$ — Can be computed by setting the current source to zero and solving the circuit

$V^2{}_L$ — Can be computed by setting the voltage source to zero and solving the circuit

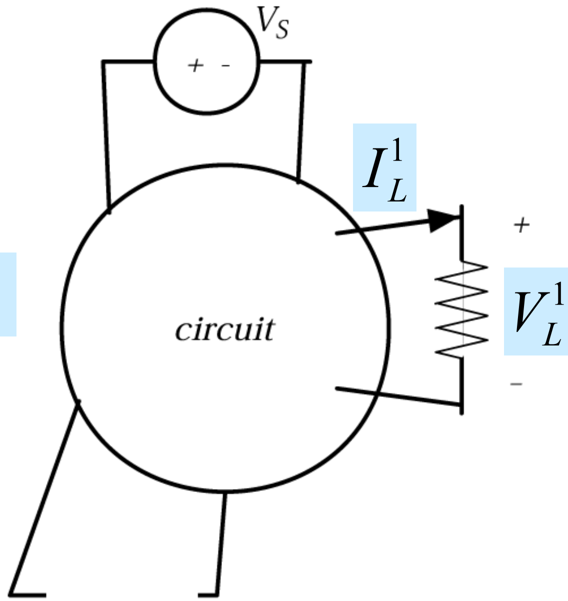
GEAUX

**SOURCE SUPERPOSITION**

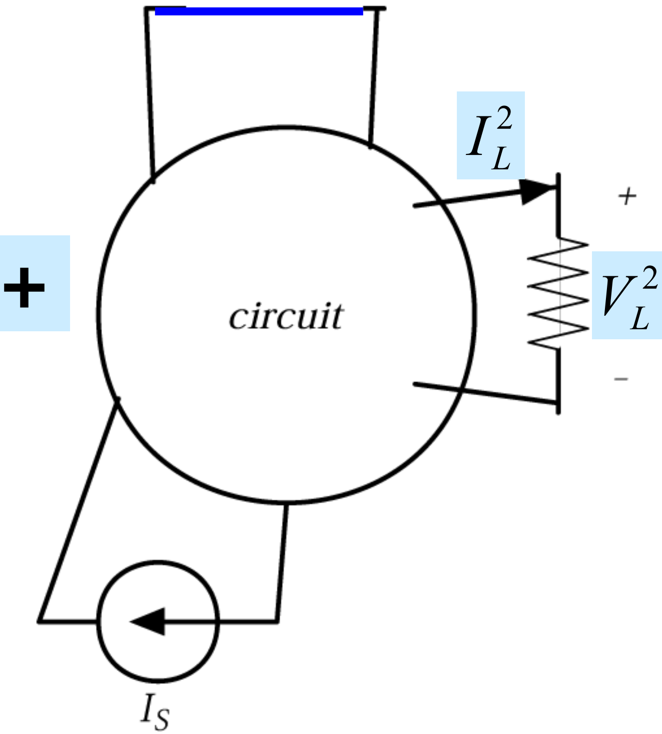
**Circuit with voltage source set to zero (SHORT CIRCUITED)**




=




+




**Circuit with current source set to zero(OPEN)**

**Due to the linearity of the models we must have**

$$I_L = I_L^1 + I_L^2 \qquad V_L = V_L^1 + V_L^2$$

**Principle of Source Superposition**

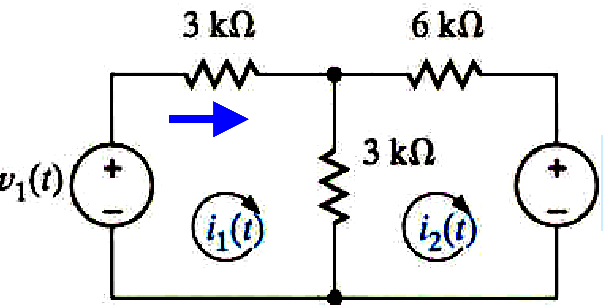
**The approach will be useful if solving the two circuits is simpler, or more convenient, than solving a circuit with two sources**

**We can have any combination of sources. And we can partition any way we find convenient**

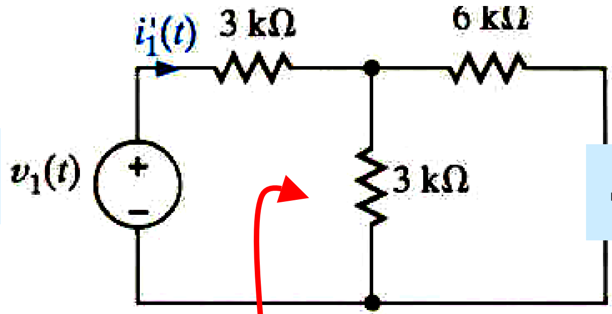
GEAUX

**LEARNING EXAMPLE**

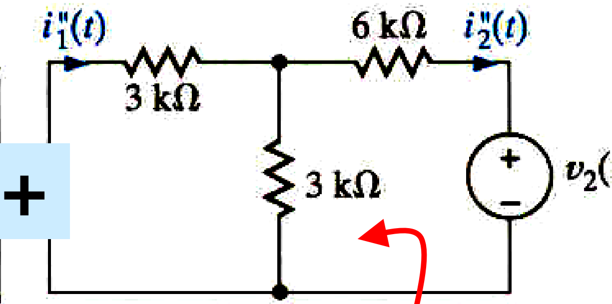
**WE WISH TO COMPUTE THE CURRENT $i_1$**




=




+




$$R_{eq} = 3 + 3 \| 6 \; [k]$$

$$R_{eq} = 6 + (3 \| 3) \; [k]$$

$$i_2'' = \frac{v_2}{R_{eq}}$$

$$6ki_1(t) - 3ki_2(t) = v_1(t)$$

$$-3ki_1(t) + 9ki_2(t) = -v_2(t)$$

**Loop equations**

$$i_1'(t) = \frac{v_1(t)}{3k + \dfrac{(3k)(6k)}{3k + 6k}} = \frac{v_1(t)}{5k}$$

**Contribution of v1**

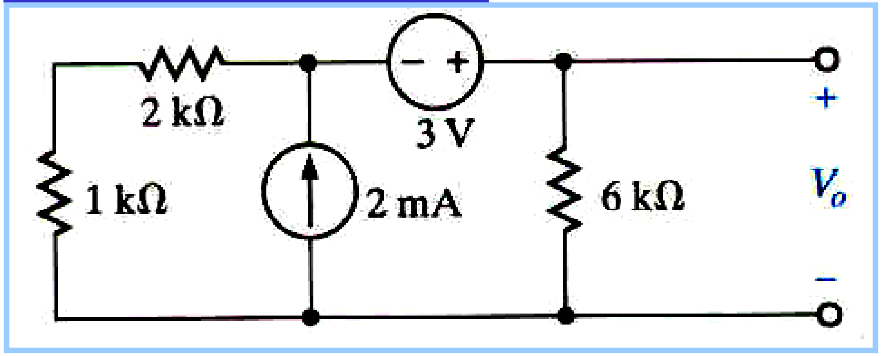
$$i_1''(t) = \frac{-2v_2(t)}{15k}\left(\frac{3k}{3k + 3k}\right) = \frac{-v_2(t)}{15k}$$

**Contribution of v2**

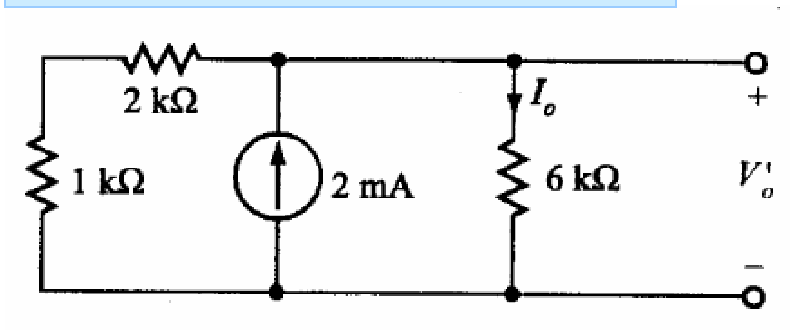
**Once we know the "partial circuits" we need to be able to solve them in an efficient manner**

**LEARNING EXAMPLE**

Compute $V_0$ using source superposition




**We set to zero the voltage source**



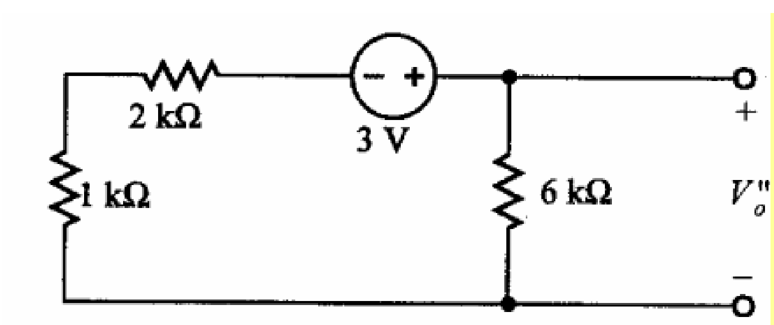

$$I_o = (2 \times 10^{-3}) \left( \frac{1k + 2k}{1k + 2k + 6k} \right)$$

**Current division**

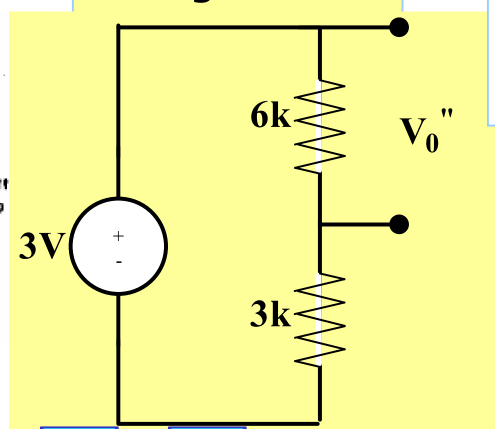
$$V'_0 = I_0(6k) = 4\text{ V}$$

**Ohm's law**

**Now we set to zero the current source**




**Voltage Divider**



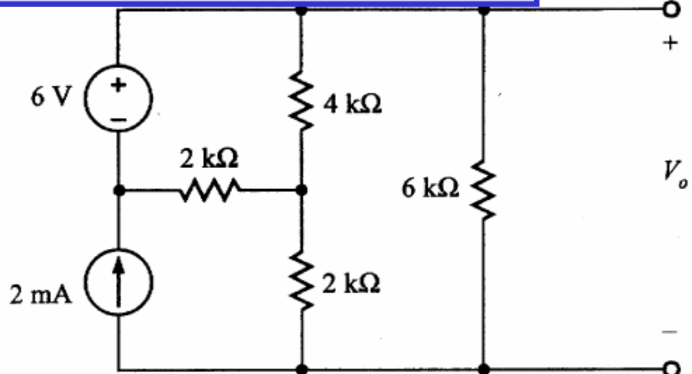

$$V''_o = 3 \left( \frac{6k}{1k + 2k + 6k} \right) = 2[V]$$

$$V_0 = V_0' + V_0'' = 6[V]$$

GEAUX

# LEARNING EXAMPLE

Compute $V_0$ using source superposition

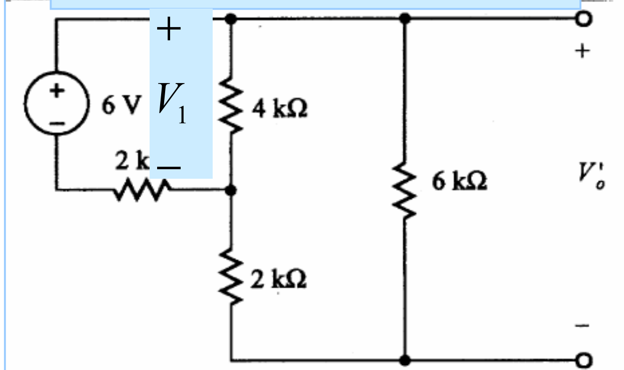


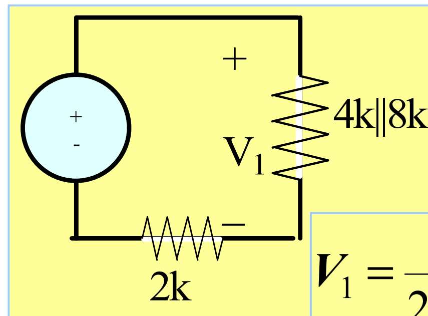

**We must be able to solve each circuit in a very efficient manner!!!**

**If V1 is known then V'o is obtained using a voltage divider**

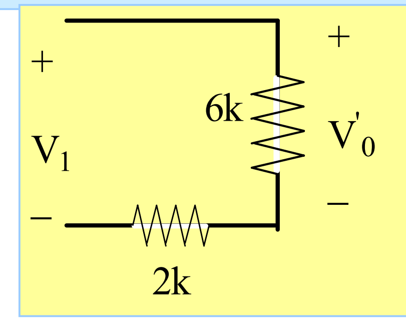
**V1 can be obtained by series parallel reduction and divider**

**Set to zero current source**



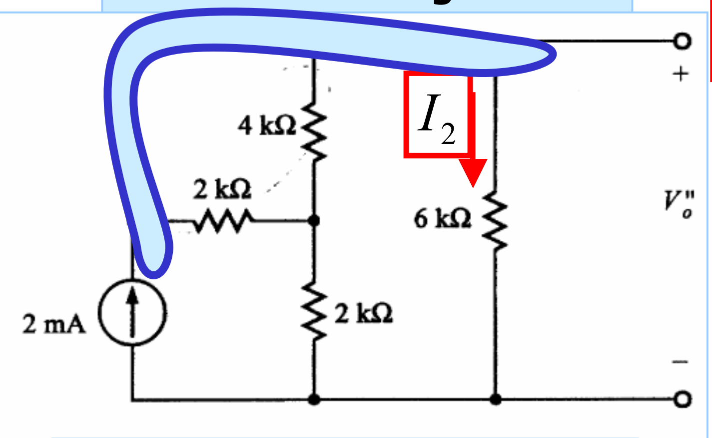




$$V_1 = \frac{8/3}{2+8/3}(6)$$

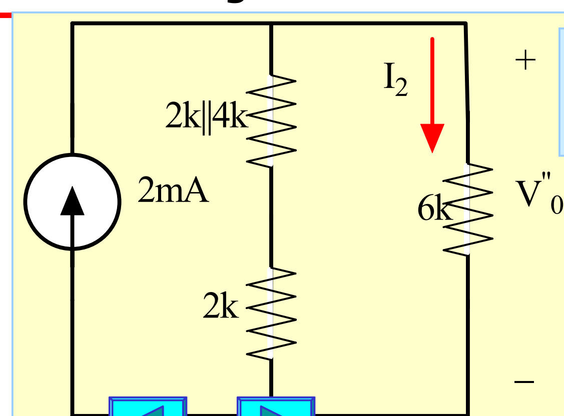



$$V_o' = \frac{6k}{6k+2k}V_1 = \frac{18}{7}[V]$$

**Set to zero voltage source**




**The current I2 can be obtained using a current divider and V"o using Ohm's law**




$$I_2 = \frac{2k+(2k\,||\,4k)}{2k+6k+(2k\,||\,4k)}(2)mA$$

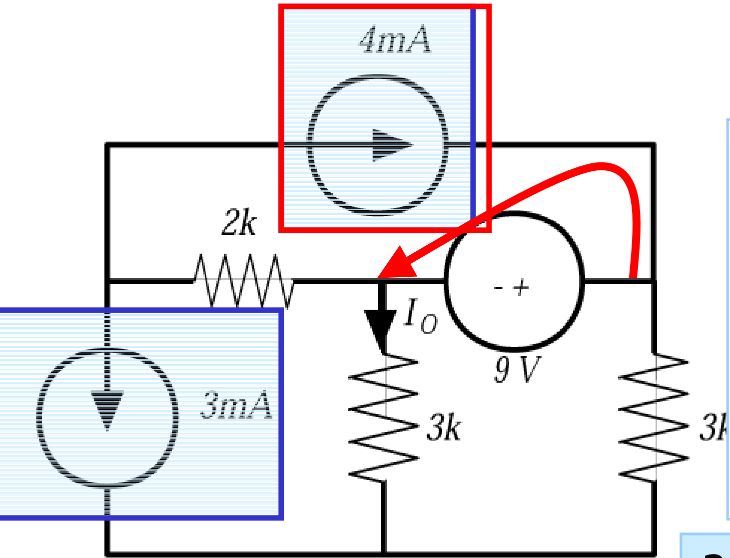
$$V_o'' = 6kI_2$$

$$V_o = V_o' + V_o''$$

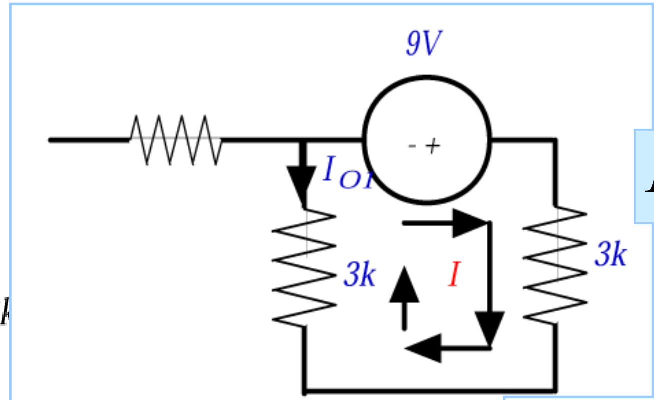
**WHEN IN DOUBT… REDRAW!**

GEAUX

**Sample Problem** COMPUTE $I_0$ USING SOURCE SUPERPOSITION




**1. Consider only the voltage source**

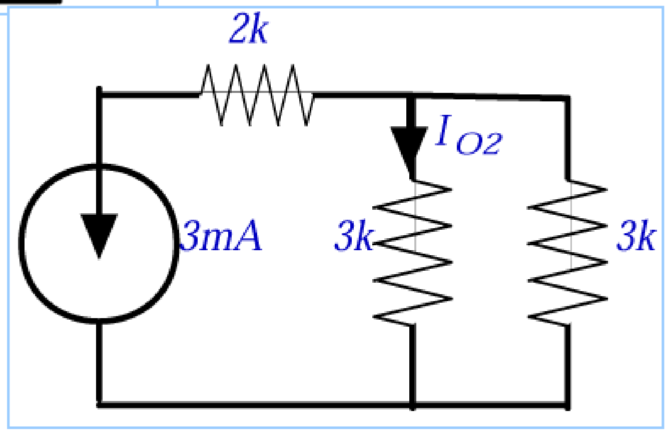



$$I_{01} = -1.5mA$$

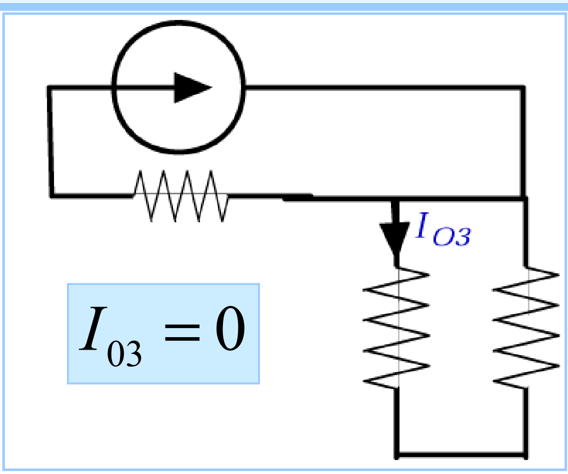
**2. Consider only the 3mA source**

**Current divider**

$$I_{02} = -1.5mA$$




**3. Consider only the 4mA source**



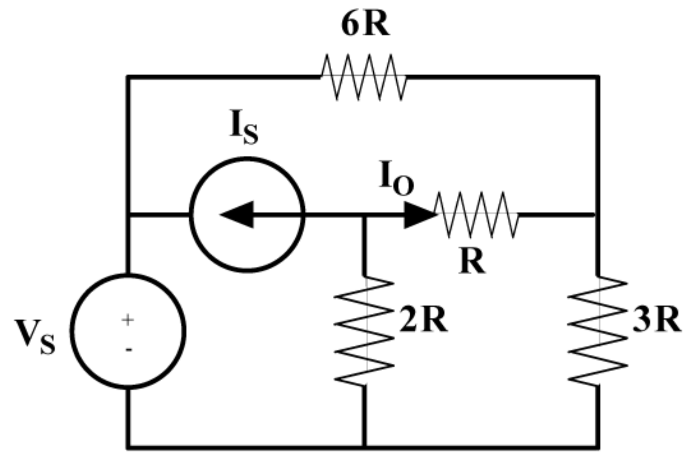

$$I_{03} = 0$$

**Using source superposition**

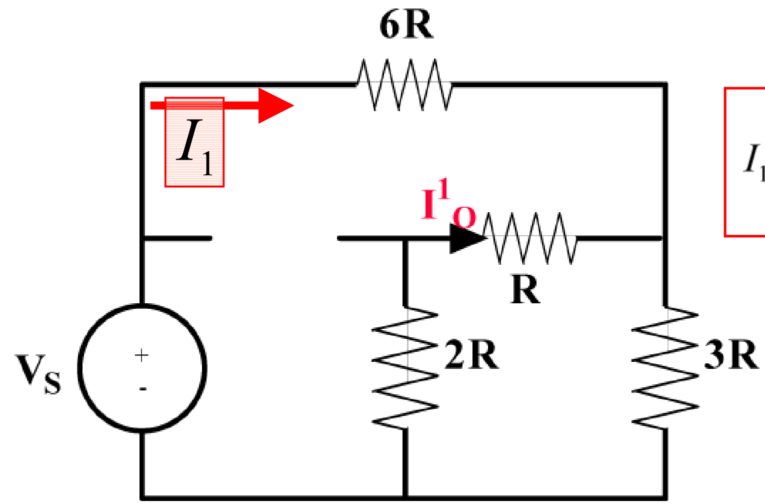
$$I_0 = I_{01} + I_{02} + I_{03} = -3mA$$

GEAUX

# USE SOURCE SUPERPOSITION TO COMPUTE $I_O$



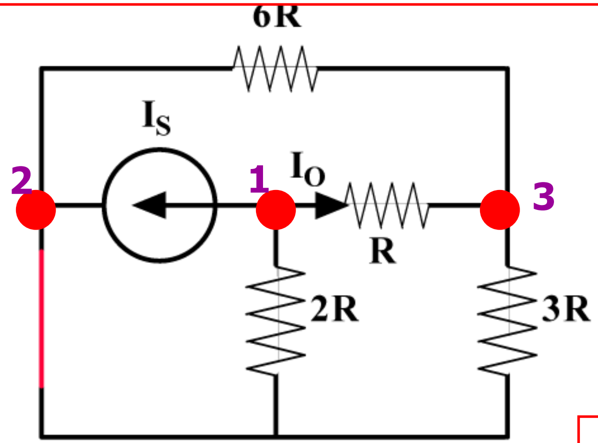

open current source



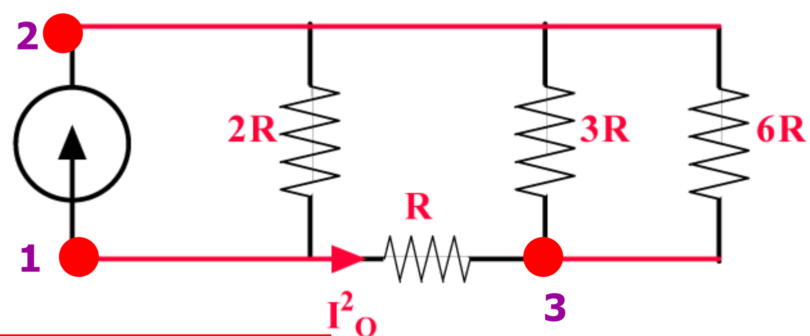

$$I_1 = \frac{V_S}{6R + \|3R, 3R\|}$$

$$I_O^1 = -\frac{I_1}{2}$$

short circuit voltage source




in case of doubt: REDRAW CIRCUIT!




NOW USE CURRENT DIVIDER

$$I^2{}_O = -\frac{2R}{2R + R + \|3R, 6R\|} I_S$$

$$I^2{}_O = -\frac{2}{5} I_S$$

$$I_O = I^1{}_O + I^2{}_O$$

$$I_O = -\frac{V_S}{15R} - \frac{2}{5} I_S$$

Linearity

STOP