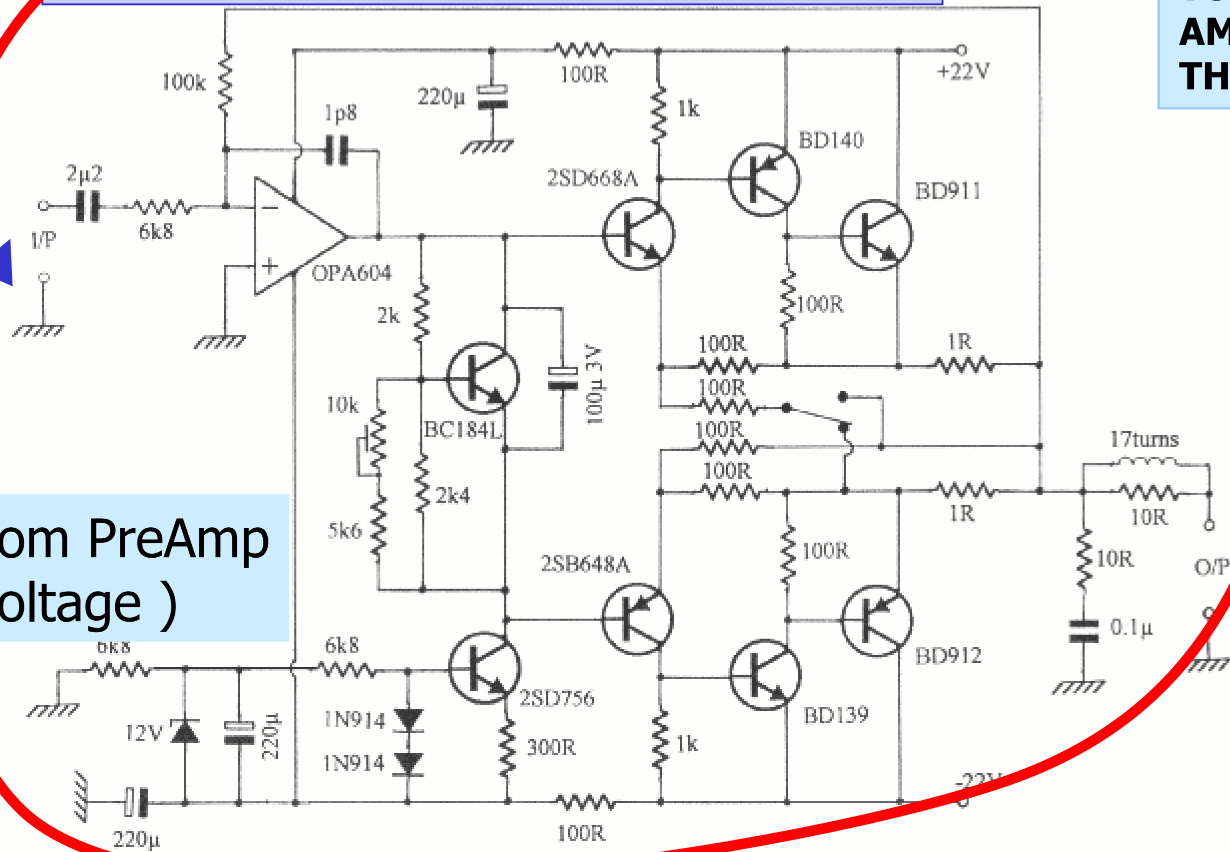
# THEVENIN'S AND NORTON'S THEOREMS

These are some of the most powerful analysis results to be discussed.

They permit to hide information that is not relevant and concentrate in what is important to the analysis

# Low distortion audio power amplifier



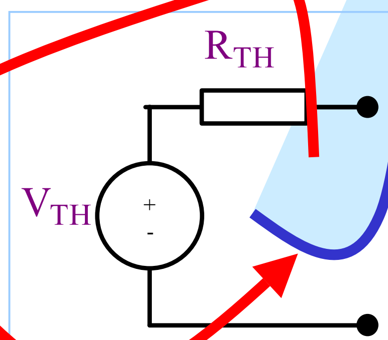

From PreAmp (voltage )

TO MATCH SPEAKERS AND AMPLIFIER ONE SHOULD ANALYZE THIS CIRCUIT

To speakers

Courtesy of M.J. Renardson

http://angelfire.com/ab3/mjramp/index.html




REPLACE AMPLIFIER BY SIMPLER "EQUIVALENT"

TO MATCH SPEAKERS AND AMPLIFIER IT IS MUCH EASIER TO CONSIDER THIS EQUIVALENT CIRCUIT!

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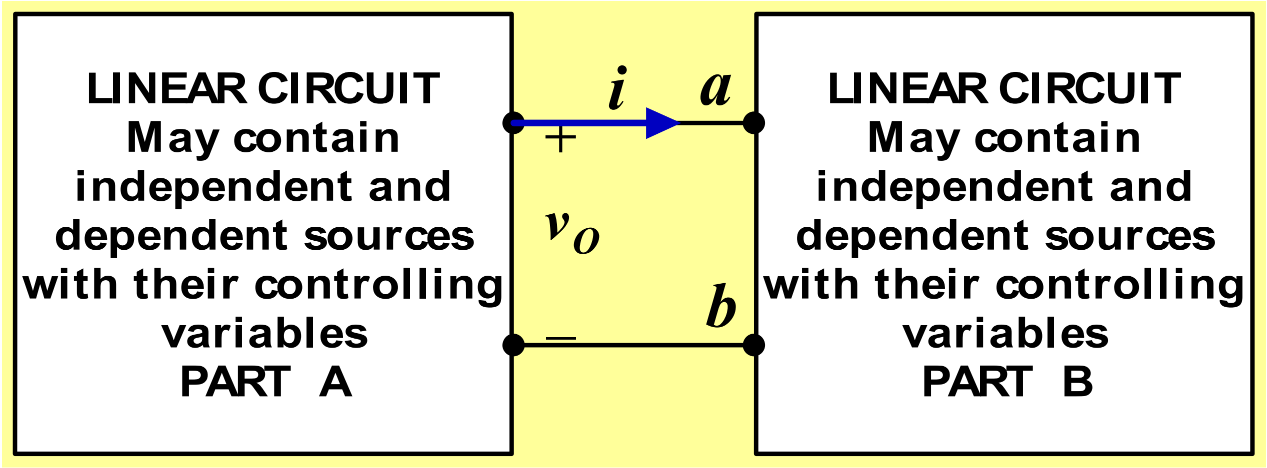
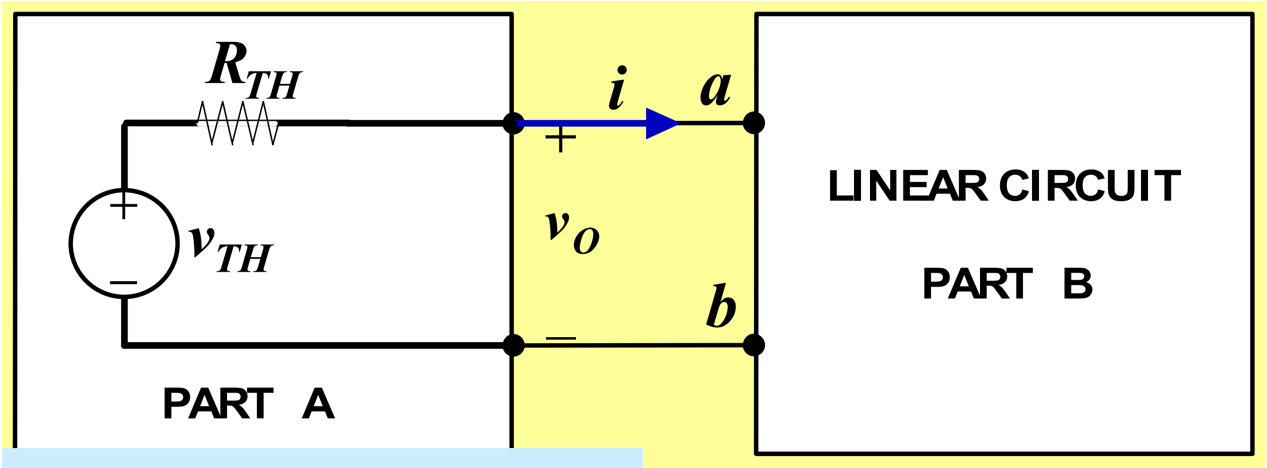
# THEVENIN'S EQUIVALENCE THEOREM



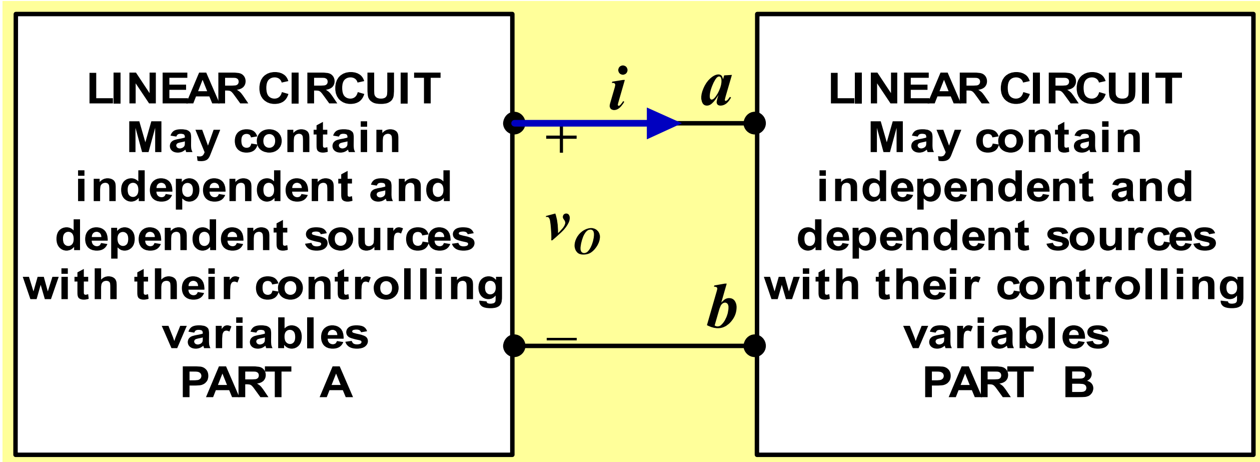



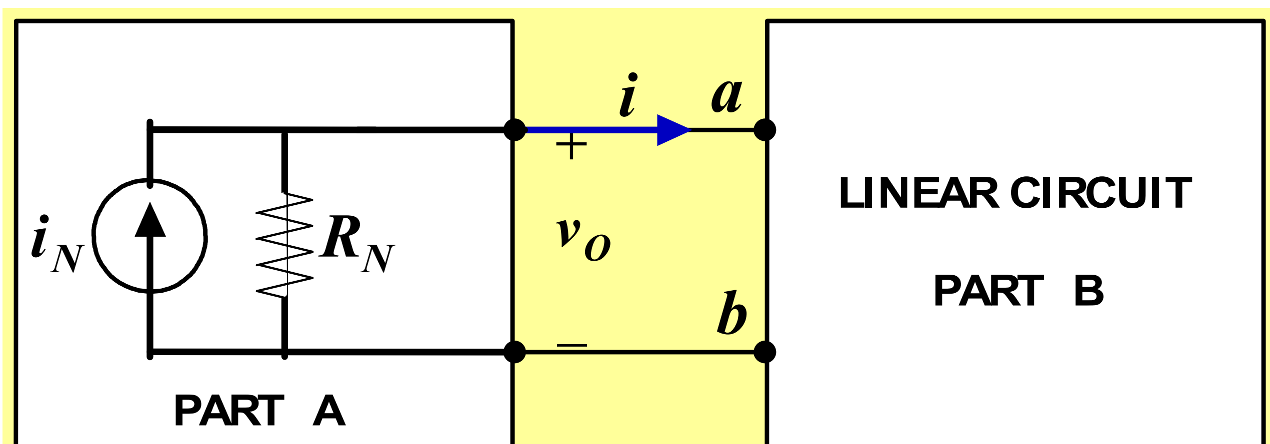

**Thevenin Equivalent Circuit for PART A**

$v_{TH}$ Thevenin Equivalent Source

$R_{TH}$ Thevenin Equivalent Resistance

# NORTON'S EQUIVALENCE THEOREM




**Norton Equivalent Circuit for PART A**

$i_N$ Thevenin Equivalent Source

$R_N$ Thevenin Equivalent Resistance

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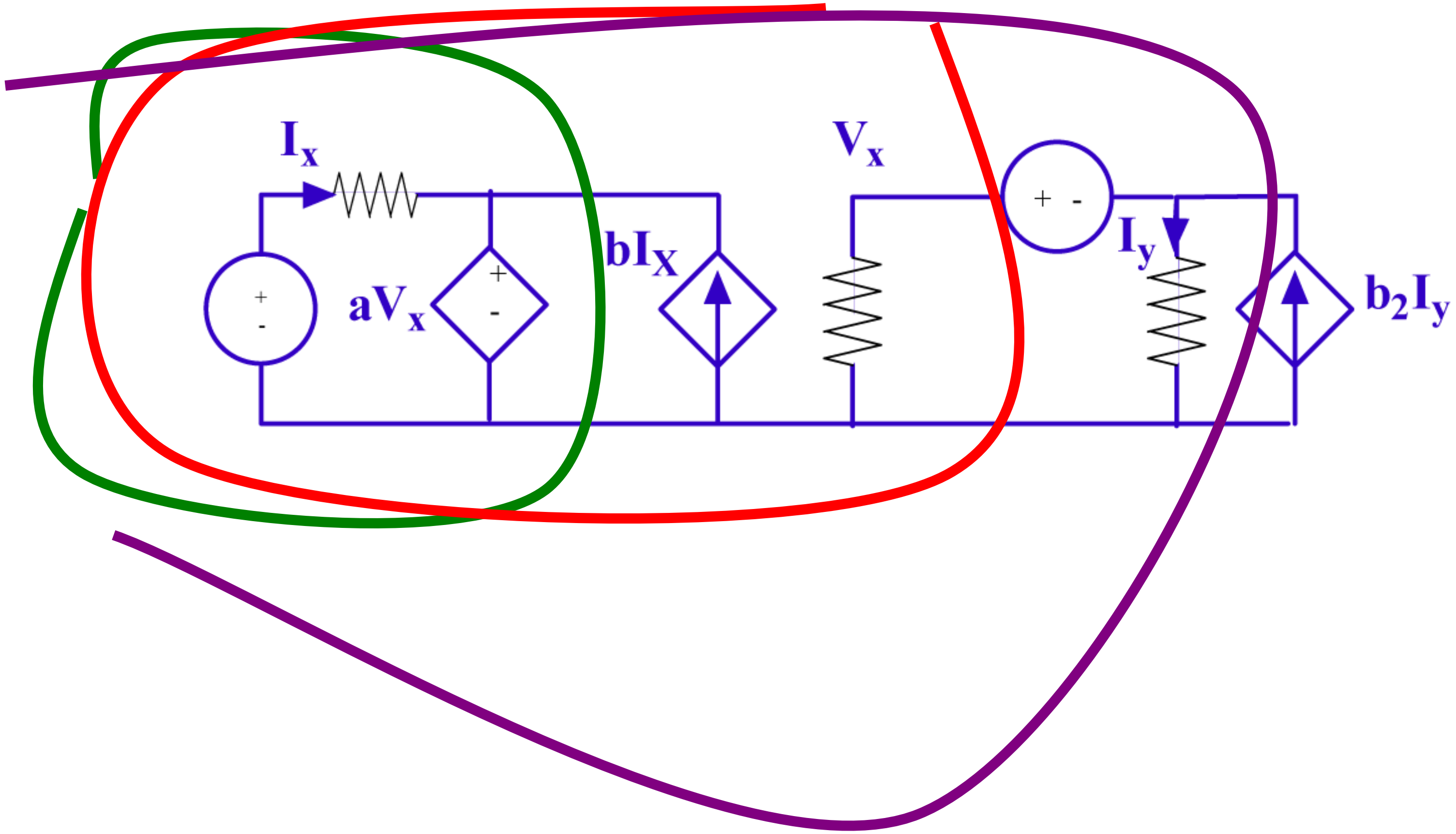
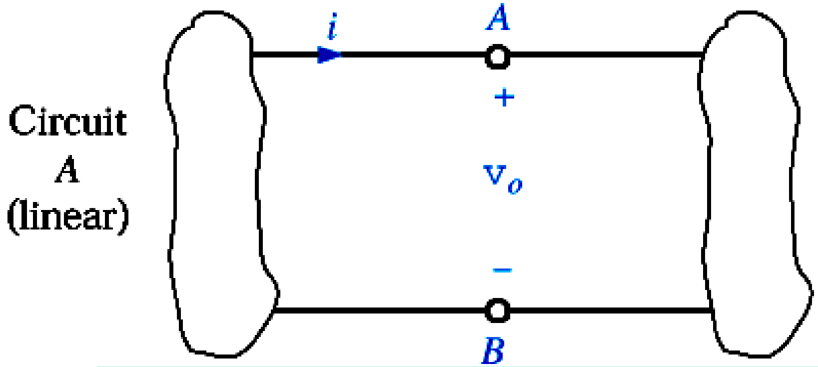
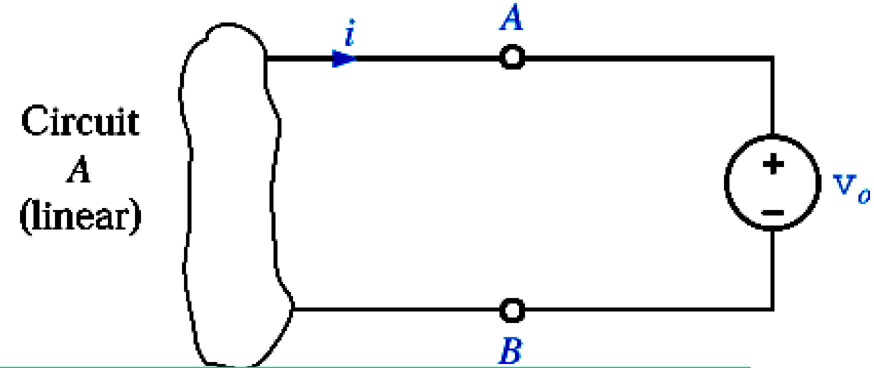
# Examples of Valid and Invalid Partitions



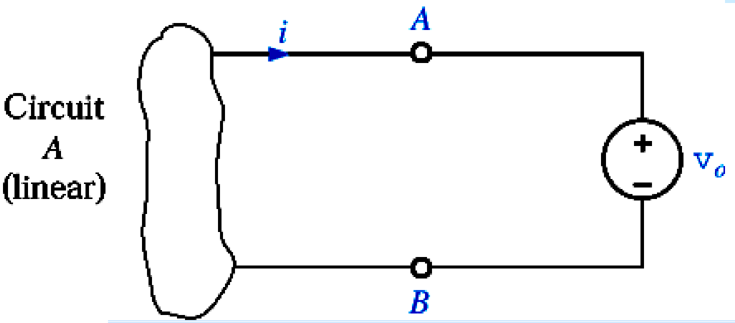
# OUTLINE OF PROOF - version 1




**If Circuit A is unchanged then the current should be the same FOR ANY Vo**

**USE SOURCE SUPERPOSITION**

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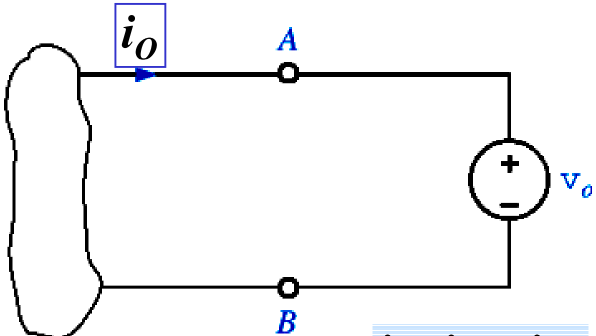
**All independent sources set to zero in A**

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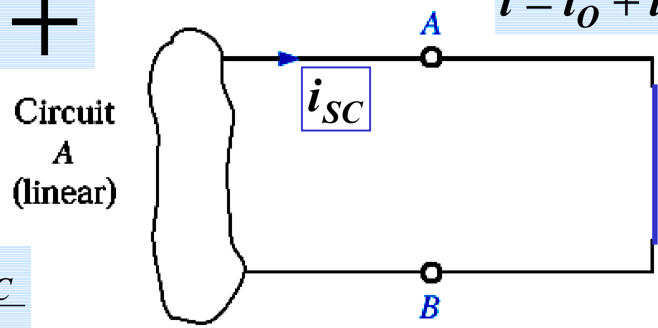


$i = i_O + i_{SC}$

DEFINE $R_{TH} = -\frac{v_O}{i_O} \Rightarrow i = -\frac{v_O}{R_{TH}} + i_{SC}; \quad \forall v_O$

SPECIAL CASE: OPEN CIRCUIT $(i = 0)$

$$v_O = v_{OC} \Rightarrow 0 = -\frac{v_{OC}}{R_{TH}} + i_{SC} \Rightarrow R_{TH} = \frac{v_{OC}}{i_{SC}} \Rightarrow i_{SC} = \frac{v_{OC}}{R_{TH}}$$

$$i = -\frac{v_O}{R_{TH}} + i_{SC} \Rightarrow v_O = v_{OC} - R_{TH} i$$

**HOW DO WE INTERPRET THIS RESULT?**

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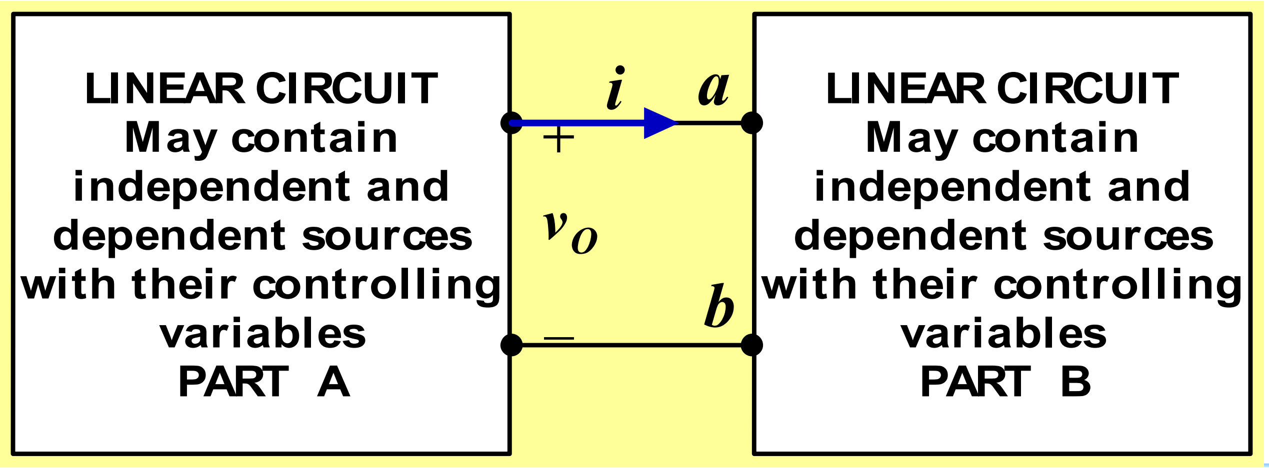
# OUTLINE OF PROOF - version 2




1. Because of the linearity of the models, for any Part B the relationship between *Vo* and the current, *i*, has to be of the form $v_O = m*i + n$

2. Result must hold for "every valid Part B" that we can imagine

3. If part B is an open circuit then i=0 and... $n = v_{OC}$

4. If Part B is a short circuit then Vo is zero. In this case

$$0 = m*i_{SC} + v_{OC} \Rightarrow m = -\frac{v_{OC}}{i_{SC}} = -R_{TH}$$

$$v_O = -R_{TH} i + v_{OC}$$

How do we interpret this?

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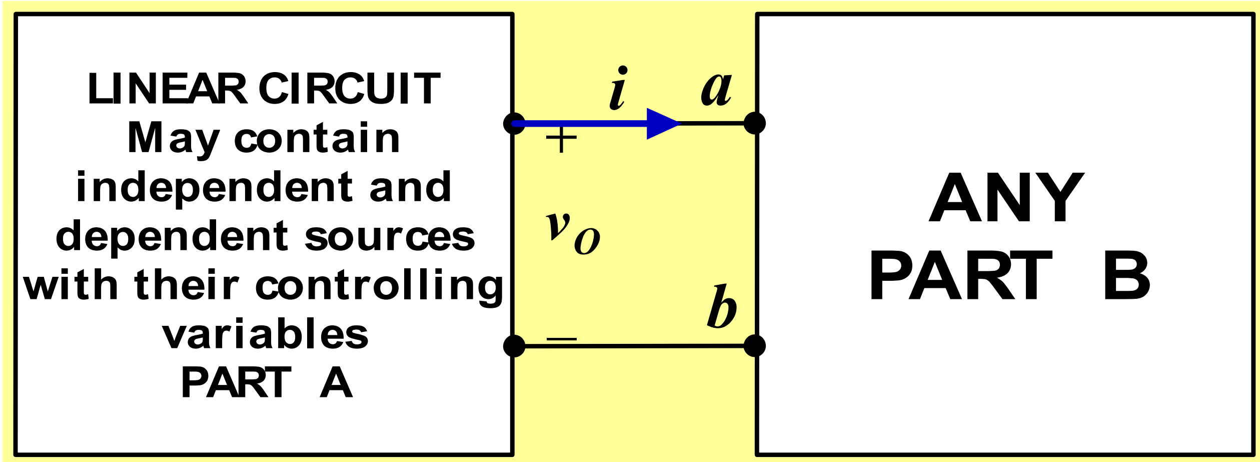
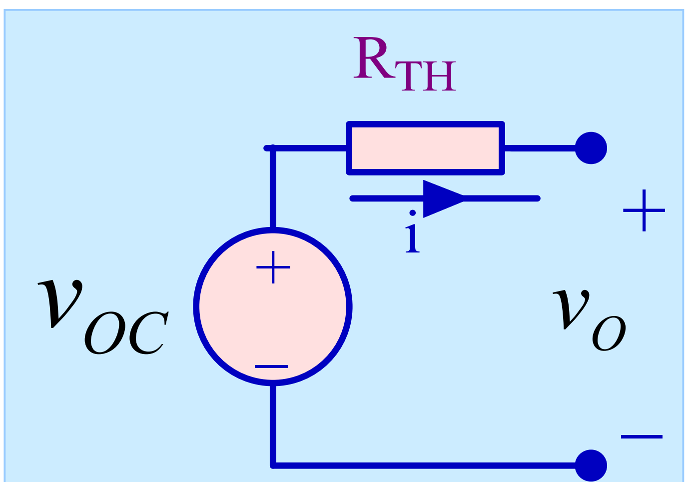
# THEVENIN APPROACH




$$v_O = -R_{TH} i + v_{OC}$$

For ANY circuit in Part B




PART A MUST BEHAVE LIKE THIS CIRCUIT

This is the Thevenin equivalent circuit for the circuit in Part A

The voltage source is called the THEVENIN EQUIVALENT SOURCE

The resistance is called the THEVENIN EQUIVALENT RESISTANCE

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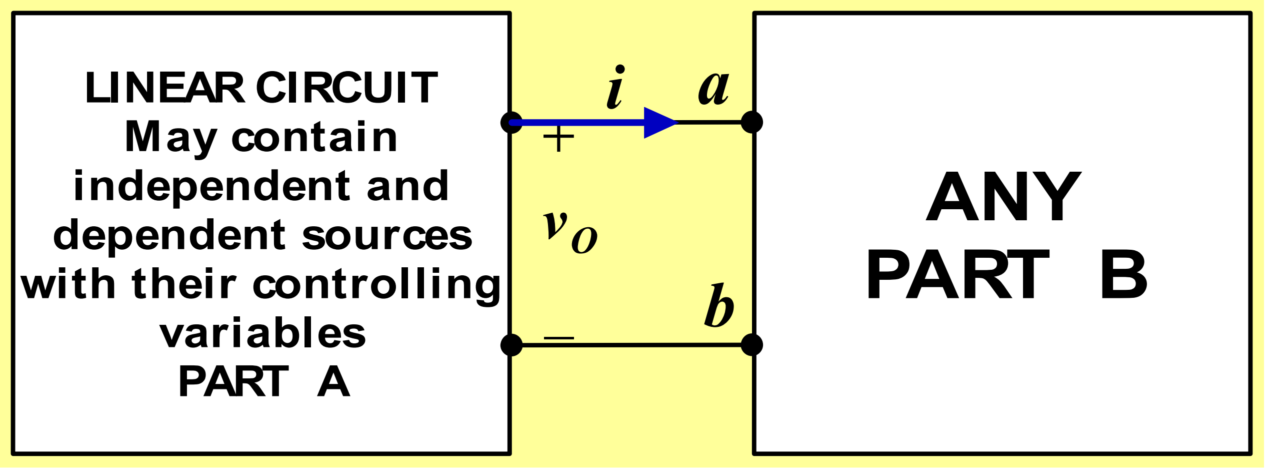
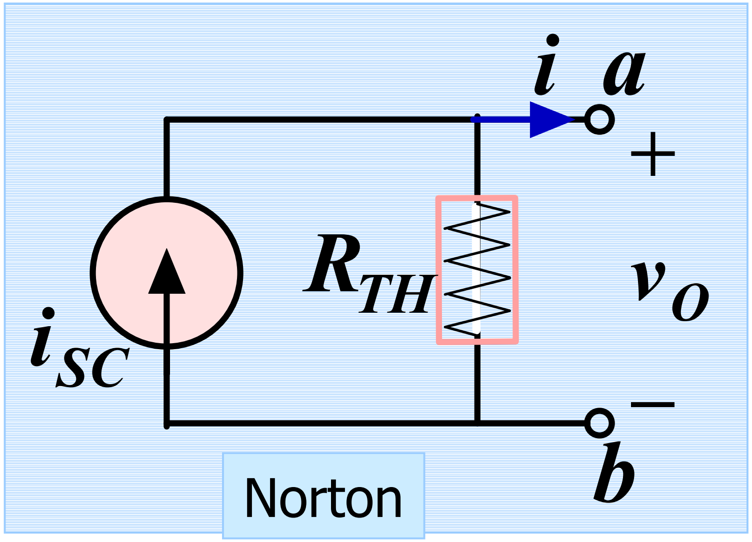
# Norton Approach

$$v_O = v_{OC} - R_{TH} i \Rightarrow i = \frac{v_{OC}}{R_{TH}} - \frac{v_O}{R_{TH}}$$

$$\frac{v_{OC}}{R_{TH}} = i_{SC}$$

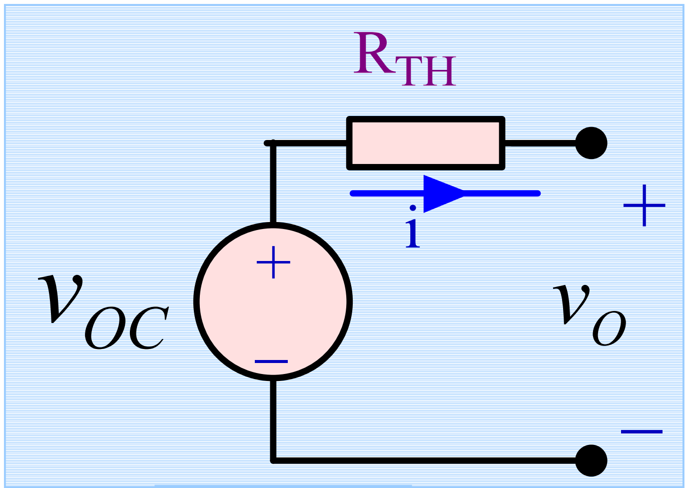


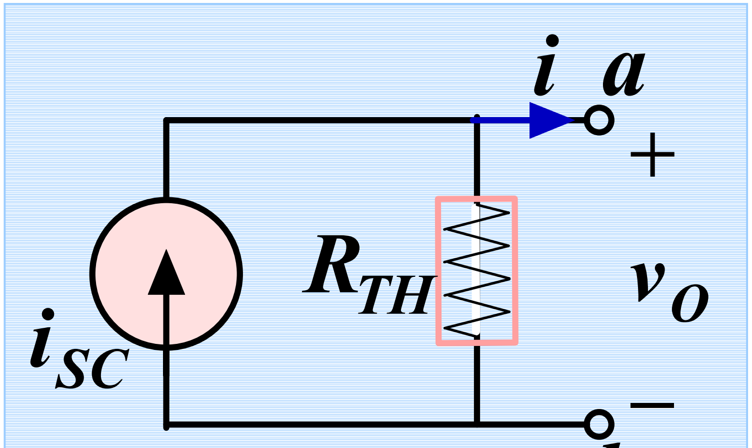

Norton Equivalent
Representation for Part A

$i_{SC}$ Norton Equivalent Source

# ANOTHER VIEW OF THEVENIN'S AND NORTON'S THEOREMS




Thevenin




Norton

$$i_{SC} = \frac{v_{OC}}{R_{TH}}$$

This equivalence can be viewed as a source transformation problem It shows how to convert a voltage source in series with a resistor into an equivalent current source in parallel with the resistor

**SOURCE TRANSFORMATION CAN BE A GOOD TOOL TO REDUCE THE COMPLEXITY OF A CIRCUIT**

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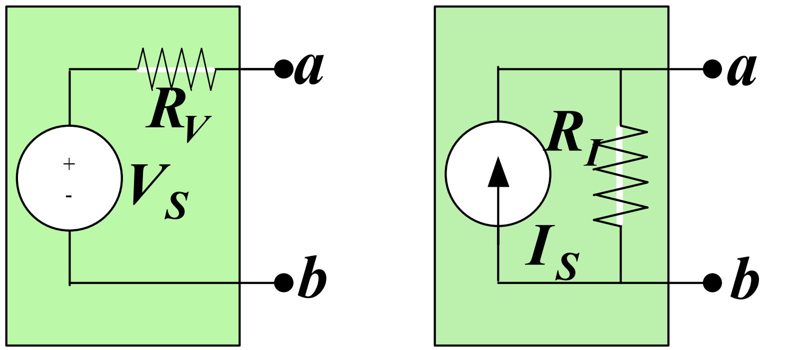
Source transformation is a good tool to reduce complexity in a circuit ...

**WHEN IT CAN BE APPLIED!!**

"ideal sources" are not good models for real behavior of sources

A real battery does not produce infinite current when short-circuited




Improved model for voltage source

Improved model for current source

THE MODELS ARE EQUIVALENTS WHEN

$$R_V = R_I = R$$

$$V_S = RI_S$$

Source Transformationcan be used to determine the Thevenin or Norton Equivalent...

**BUT THERE MAY BE MORE EFFICIENT TECHNIQUES**

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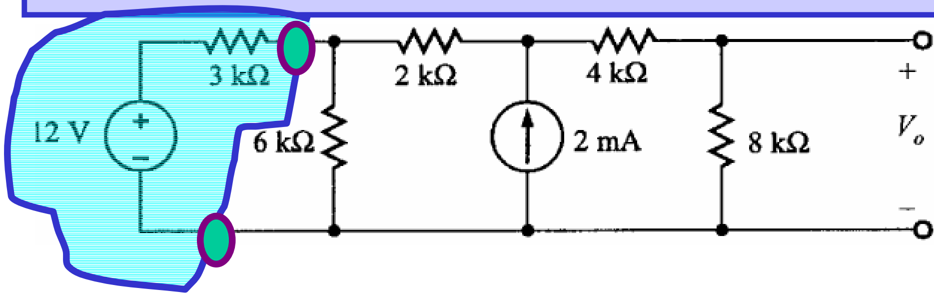
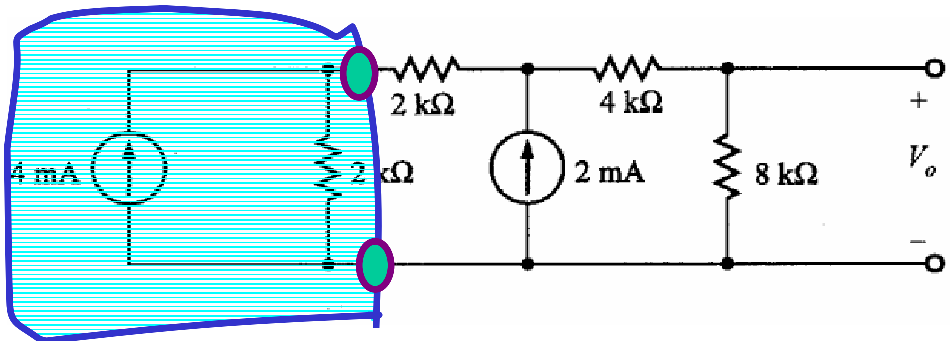
# EXAMPLE: SOLVE BY SOURCE TRANSFORMATION




In between the terminals we connect a current source and a resistance in parallel

The equivalent current source will have the value 12V/3k

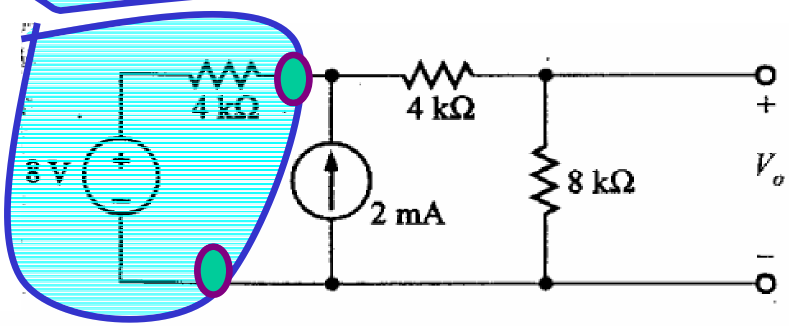
The 3k and the 6k resistors now are in parallel and can be combined




In between the terminals we connect a voltage source in series with the resistor

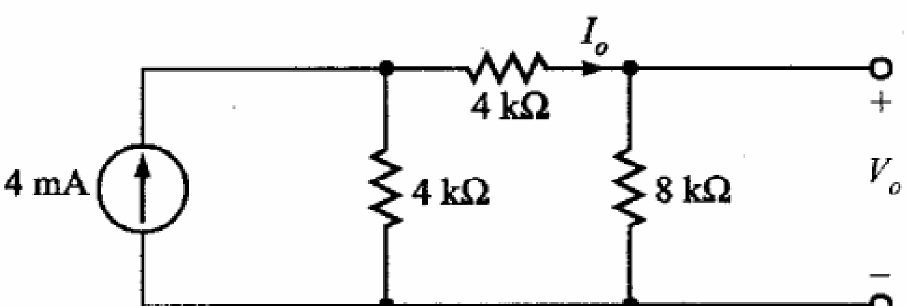
The equivalent source has value 4mA*2k

The 2k and the 2k resistor become connected in series and can be combined




After the transformation the sources can be combined

The equivalent current source has value 8V/4k and the combined current source has value 4mA




Options at this point

1. Do another source transformation and get a single loop circuit

2. Use current divider to compute I_0 and then compute V_0 using Ohm's law

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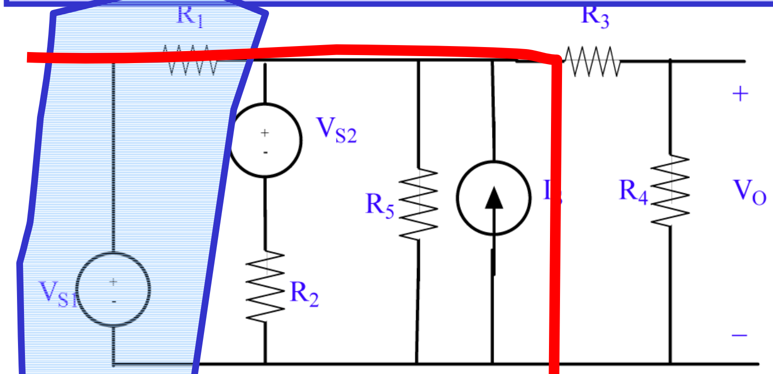
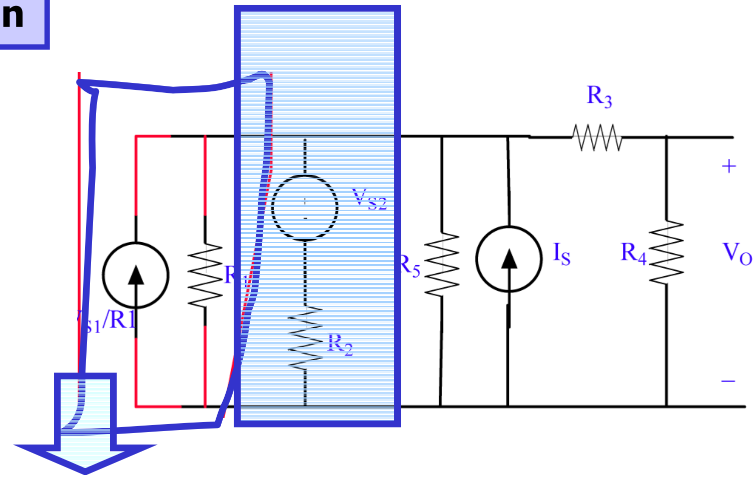
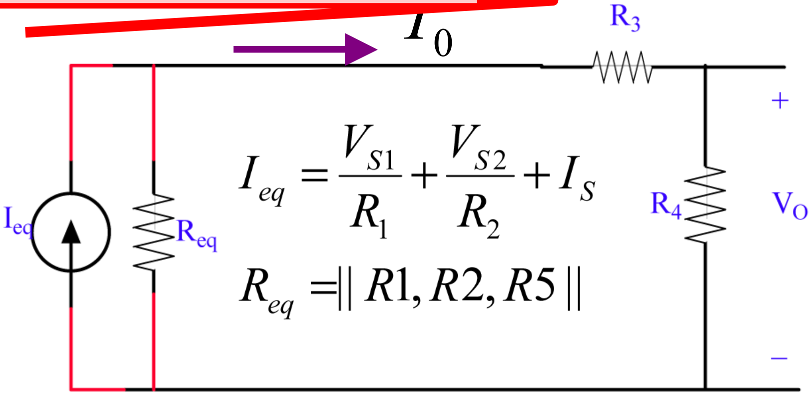
## PROBLEM Compute V_0 using source transformation



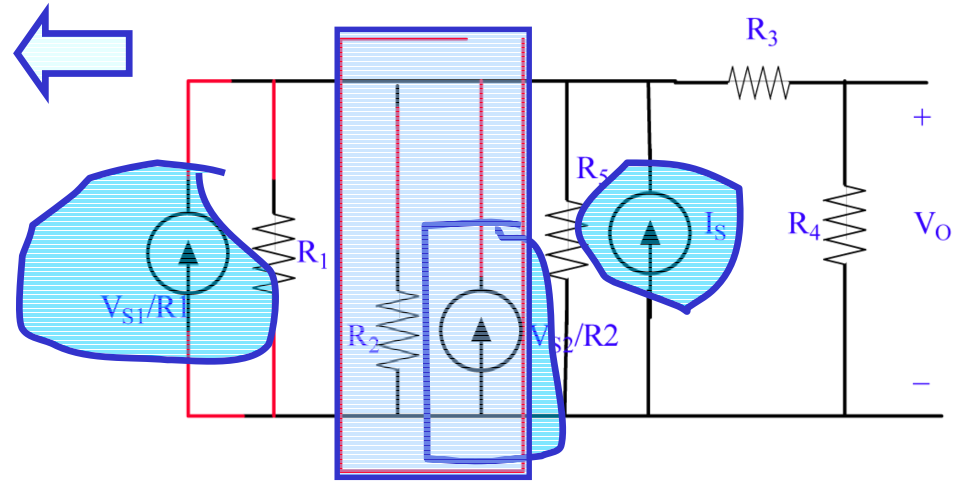




**EQUIVALENT CIRCUITS**

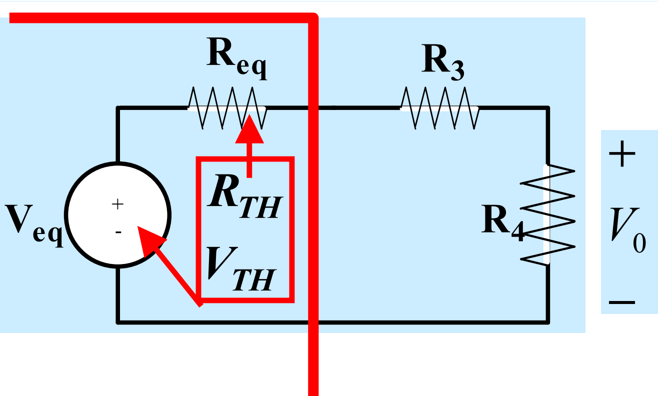



$$I_{eq} = \frac{V_{S1}}{R_1} + \frac{V_{S2}}{R_2} + I_S$$

$$R_{eq} = || R1, R2, R5 ||$$




**Or one more source transformation**




**3 current sources in parallel and three resistors in parallel**

$$V_{eq} = R_{eq} I_{eq}$$

$$V_0 = \frac{R_4}{R_4 + R_3 + R_{eq}} V_{eq}$$

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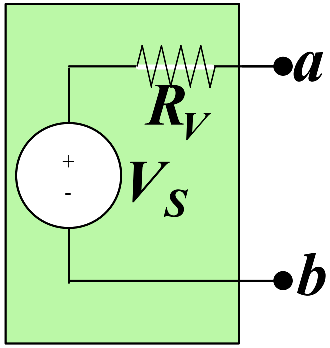
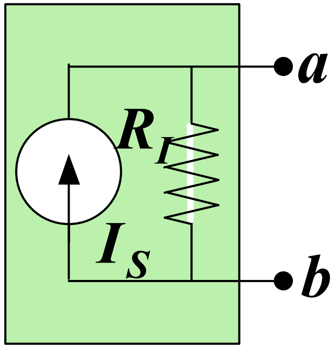
# RECAP OF SOURCE TRANSFORMATION




Improved model
for voltage source




Improved model
for current source

THE MODELS ARE EQUIVALENTS WHEN

$$R_V = R_I = R$$

$$V_S = RI_S$$

Source Transformationcan be used to determine the Thevenin or Norton Equivalent...

**WE NOW REVIEW SEVERAL EFFICIENT APPROACHES TO DETERMINE THEVENIN OR NORTON EQUIVALENT CIRCUITS**

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# A General Procedure to Determine the Thevenin Equivalent

$v_{TH}$ Open Circuit voltage

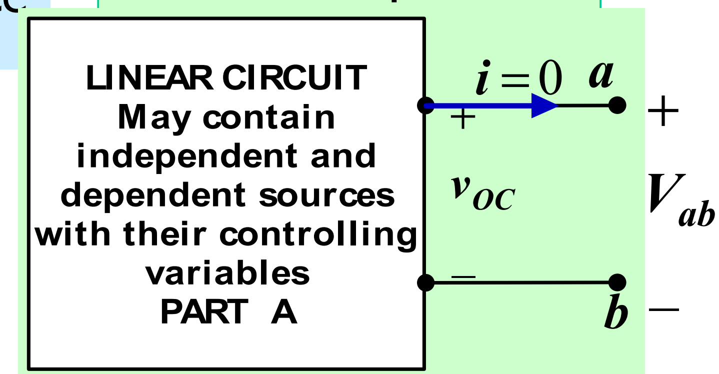
voltage at a - b if Part B is removed

$i_{SC}$ Short Circuit Current

current through a - b if Part B is replaced by a short circuit

$R_{TH} = \frac{v_{TH}}{i_{SC}}$ Thevenin Equivalent Resistance

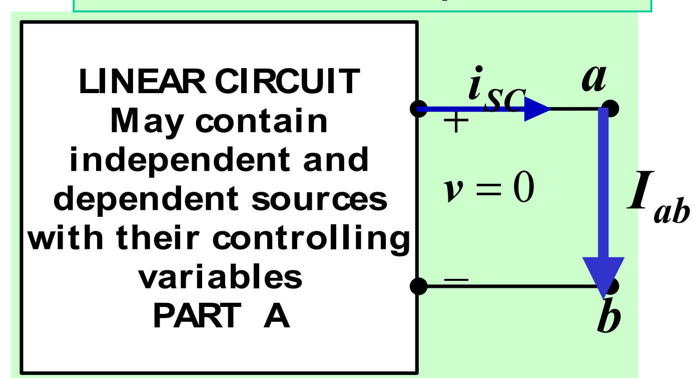
One circuit problem




1. Determine the Thevenin equivalent source

Remove part B and compute the OPEN CIRCUIT voltage $V_{ab}$

Second circuit problem




2. Determine the SHORT CIRCUIT current

Remove part B and compute the SHORT CIRCUIT current $I_{ab}$

$$v_{TH} = v_{OC}, R_{TH} = \frac{v_{OC}}{i_{SC}}$$

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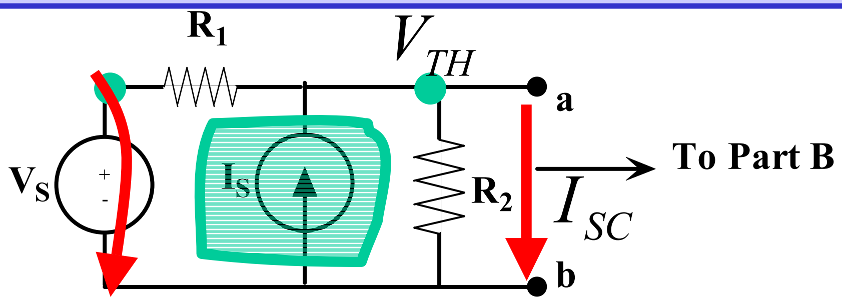
# AN EXAMPLE OF DETERMINING THE THEVENIN EQUIVALENT




**Part B is irrelevant.**
**The voltage V_ab will be the value of the Thevenin equivalent source.**

**What is an efficient technique to compute the open circuit voltage?**

**NODE ANALYSIS**

$$\frac{V_{TH}}{R_2} + \frac{V_{TH} - V_S}{R_1} - I_S = 0$$

$$\left(\frac{1}{R_1} + \frac{1}{R_2}\right)V_{TH} = \frac{V_S}{R_1} + I_S$$

$$V_{TH} = \frac{R_2}{R_1 + R_2}V_S + \frac{R_1 R_2}{R_1 + R_2}I_S$$

$$V_{TH} = \frac{R_1 R_2}{R_1 + R_2}\left(\frac{V_S}{R_1} + I_S\right)$$

**Now for the short circuit current**
**Lets try source superposition**

**When the current source is open the current through the short circuit is** $I_{SC}^1 = \frac{V_S}{R_1}$

**When the voltage source is set to zero, the current through the short circuit is** $I_{SC}^2 = I_S$

$$I_{SC} = I_S + \frac{V_S}{R_1}$$

**To compute the Thevenin resistance we use**

$$R_{TH} = \frac{V_{TH}}{I_{SC}}$$

$$R_{TH} = \frac{R_1 R_2}{R_1 + R_2}$$

**For this case the Thevenin resistance can be computed as the resistance from a - b when all independent sources have been set to zero**

**Is this a general result?**

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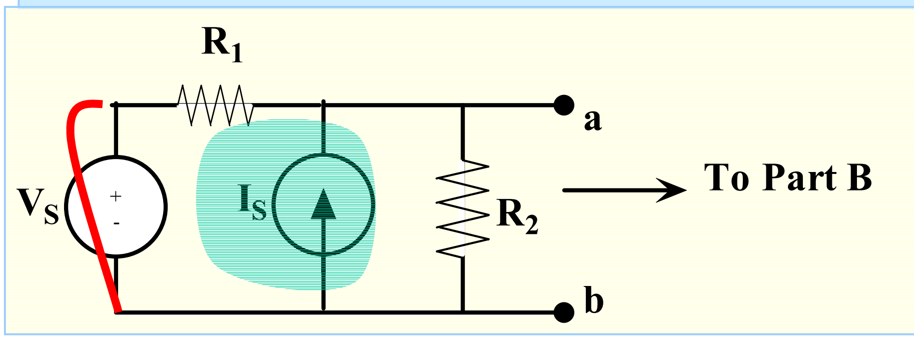
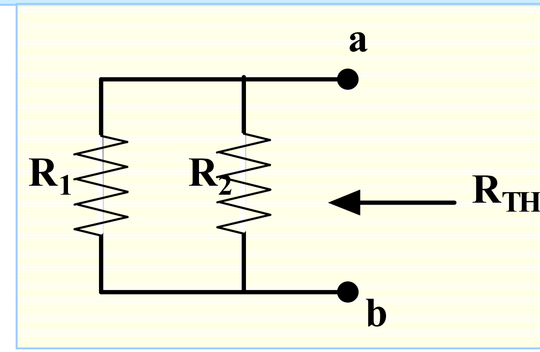
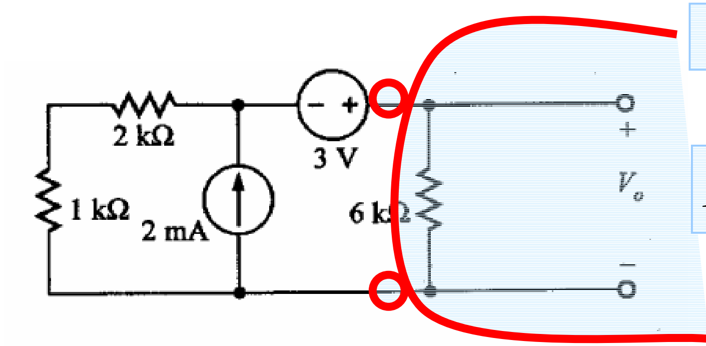
# Determining the Thevenin Equivalent in Circuits with Only INDEPENDENT SOURCES

**The Thevenin Equivalent Source is computed as the open loop voltage**

**The Thevenin Equivalent Resistance CAN BE COMPUTED by setting to zero all the sources and then determining the resistance seen from the terminals where the equivalent will be placed**



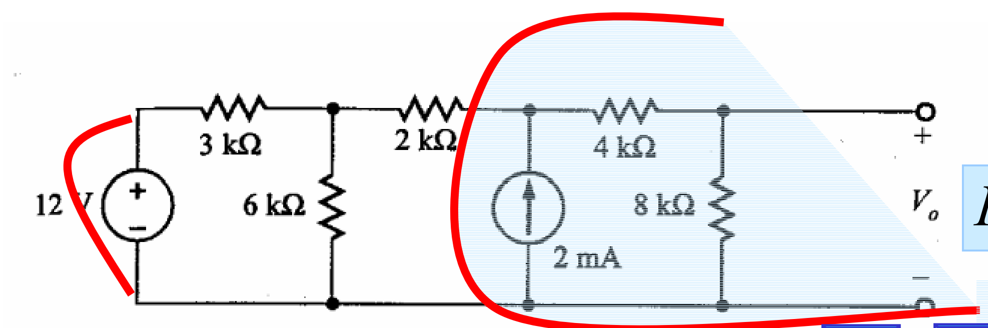




"Part B"

$$R_{TH} = 3k\Omega$$

**Since the evaluation of the Thevenin equivalent can be very simple, we can add it to our toolkit for the solution of circuits!!**




$$R_{TH} = 4k\Omega$$

"Part B"

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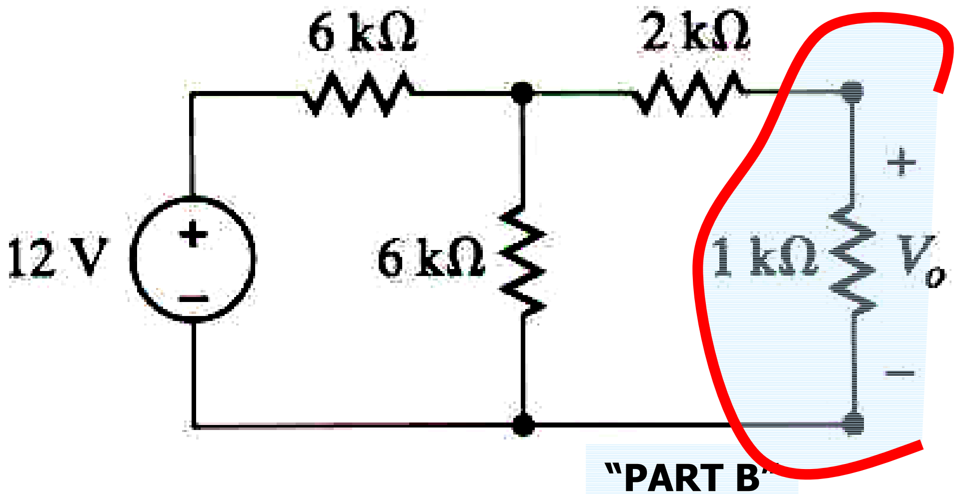
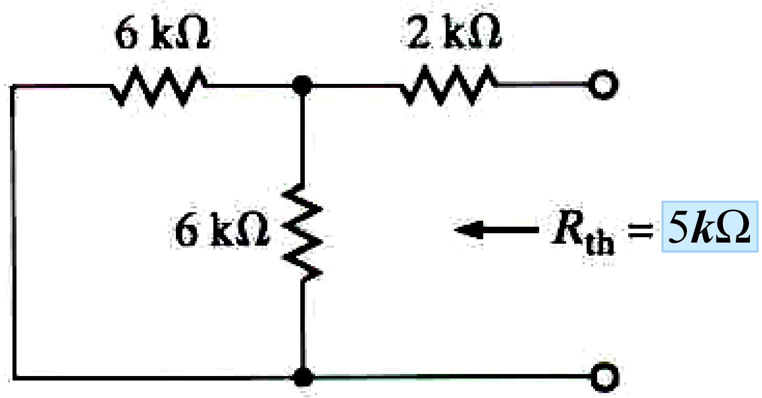
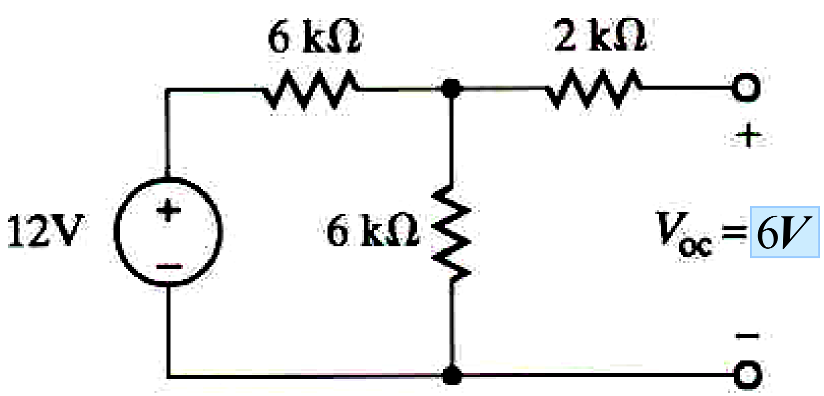
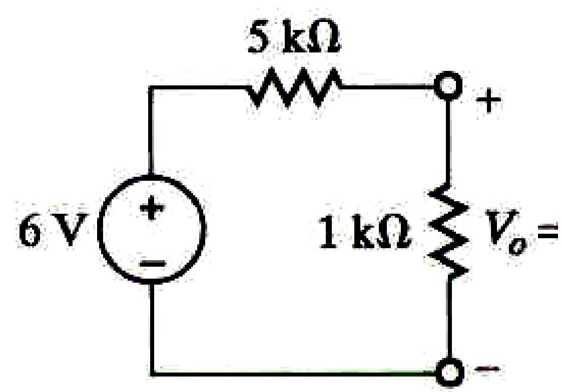
## LEARNING BY DOING

Find $V_o$ in the following network using Thévenin's theorem.




$$V_O = \frac{1k}{1k + 5k}(6V) = 1[V]$$

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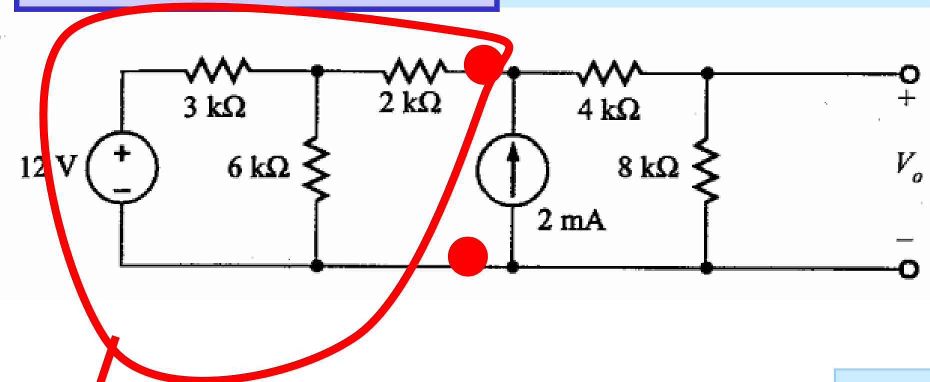
# LEARNING EXAMPLE — COMPUTE Vo USING THEVENIN




In the region shown, one could use source transformation twice and reduce that part to a single source with a resistor.

... Or we can apply Thevenin Equivalence to that part (viewed as "Part A")

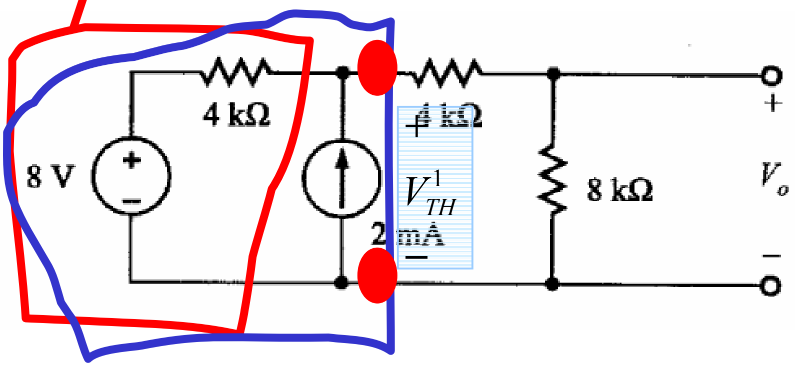
$$R_{TH} = 4k\Omega$$

For the open loop voltage the part outside the region is eliminated

$$V_{TH} = \frac{6}{3+6} 12[V] = 8[V]$$

The original circuit becomes...

And one can apply Thevenin one more time!

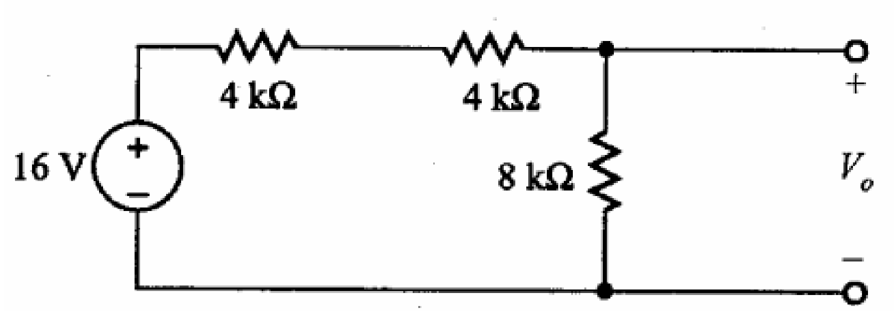



$$R^1_{TH} = 4k\Omega$$

For open loop voltage use KVL

$$V^1_{TH} = 4k * 2mA + 8V = 16V$$

...and we have a simple voltage divider!!




$$V_0 = \frac{8}{8+8} 16[V] = 8V$$

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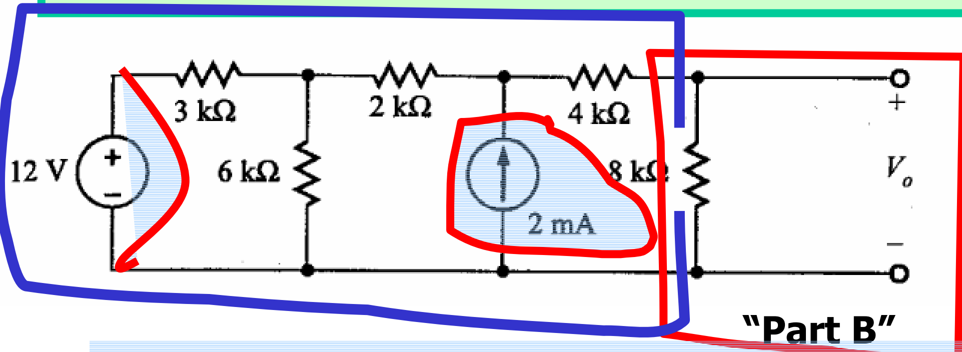
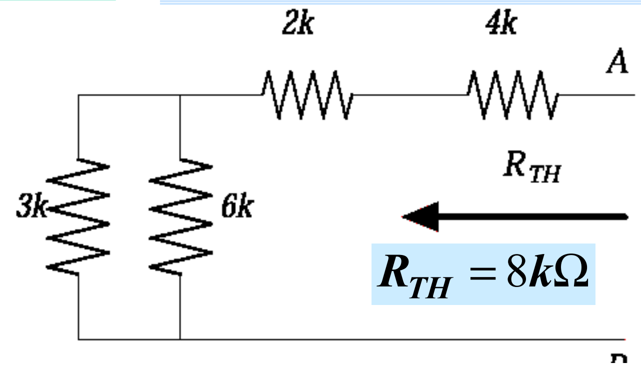
## Or we can use Thevenin only once to get a voltage divider

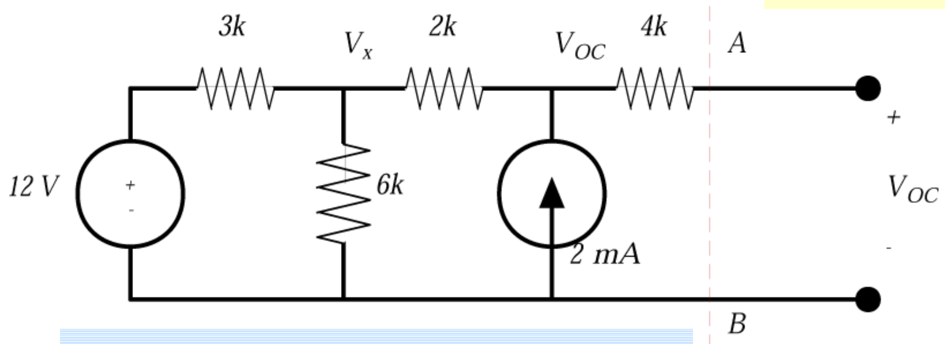



**For the Thevenin resistance**




**For the Thevenin voltage we have to analyze the following circuit**

**METHOD??**




**Source superposition, for example**

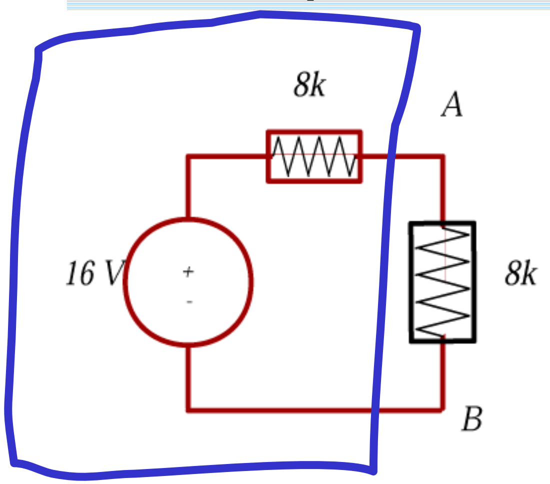
**Contribution of the voltage source**

$$V_{OC}^{1} = \frac{6}{3+6} 12V = 8V$$

**Contribution of the current source**

$$V_{OC}^{2} = (2k + 2k) * (2mA) = 8V$$

**Thevenin Equivalent of "Part A"**




**Simple Voltage Divider**

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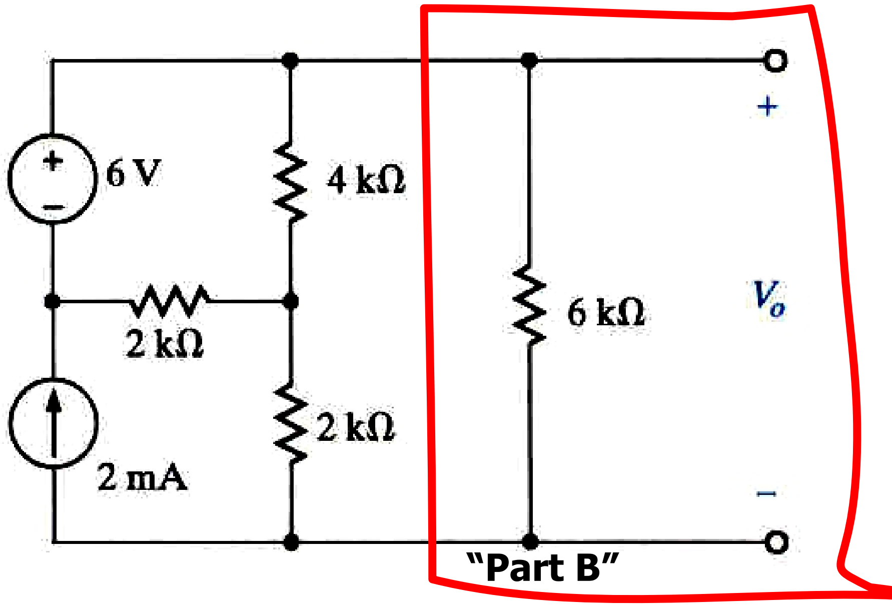
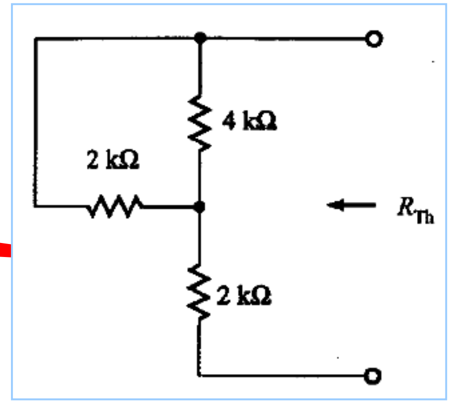
**LEARNING EXAMPLE**

## USE THEVENIN TO COMPUTE Vo




**You have the choice on the way to partition the circuit.**
**Make "Part A" as simple as possible**

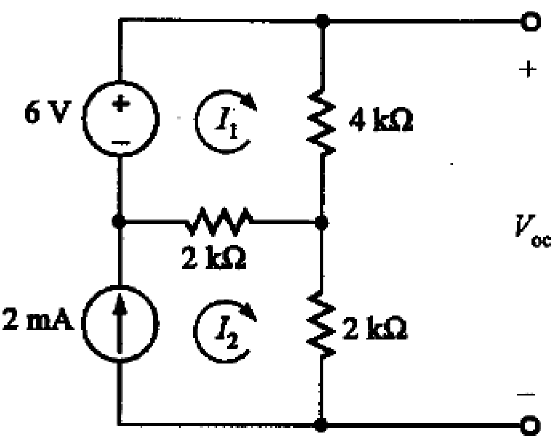
**Since there are only independent sources, for the Thevenin resistance we set to zero all sources and determine the equivalent resistance**




$$R_{TH} = 2 + (2 \| 4)[k\Omega]$$

$$R_{TH} = \frac{10}{3} k\Omega$$

**For the open circuit voltage we analyze the following circuit ("Part A") ...**



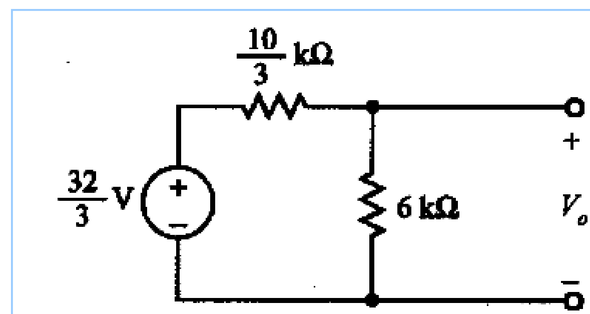

*Loop Analysis*

$$I_2 = 2mA$$

$$-6V + 4kI_1 + 2k(I_1 - I_2) = 0$$

$$I_1 = \frac{6 + 2I_2}{6} mA = \frac{5}{3} mA$$

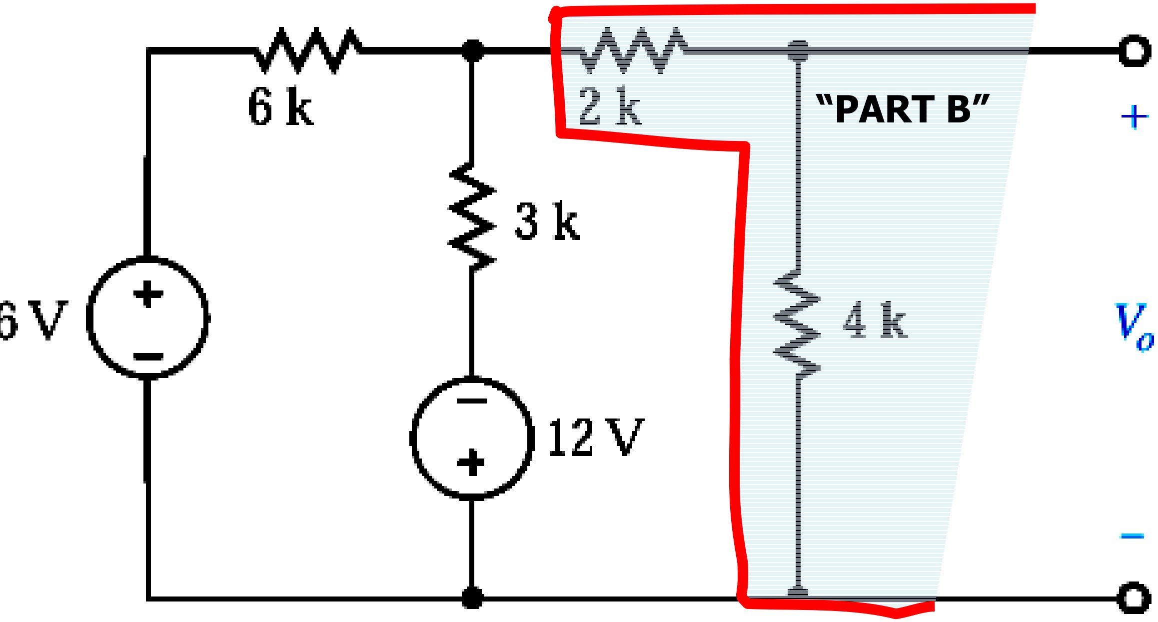
**The circuit becomes...**

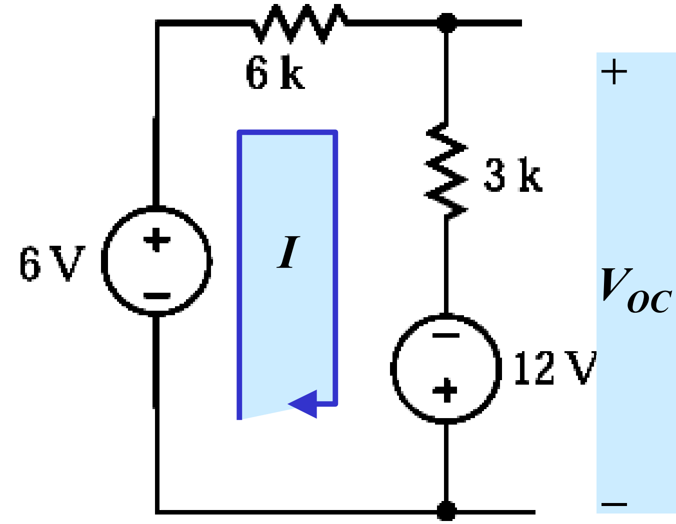



$$V_{OC} = 4k * I_1 + 2k * I_2 = 20/3 + 4V = 32/3[V]$$

GEAUX

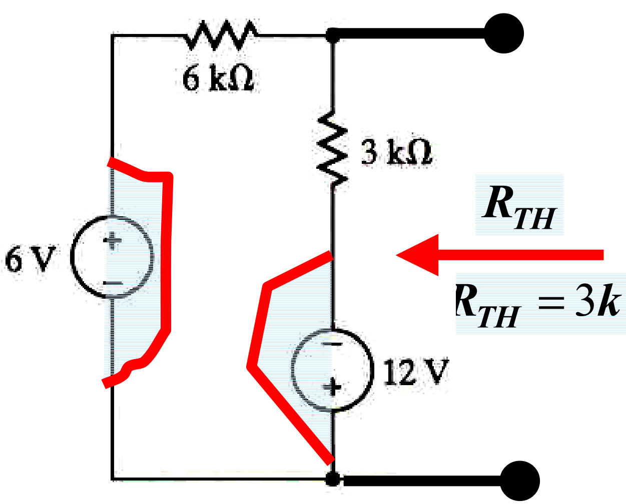
# LEARNING EXTENSION: USE THEVENIN TO COMPUTE Vo



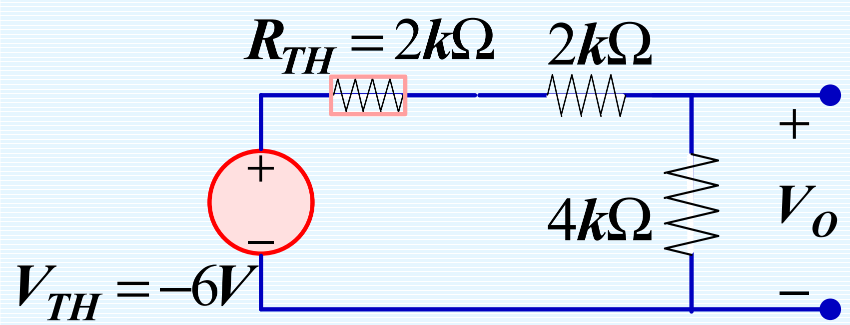

$$9kI = 18[V] \Rightarrow I = 2mA$$

$$V_{OC} = 3kI - 12 = -6[V]$$




$$R_{TH} = 3k \,||\, 6k = 2k$$

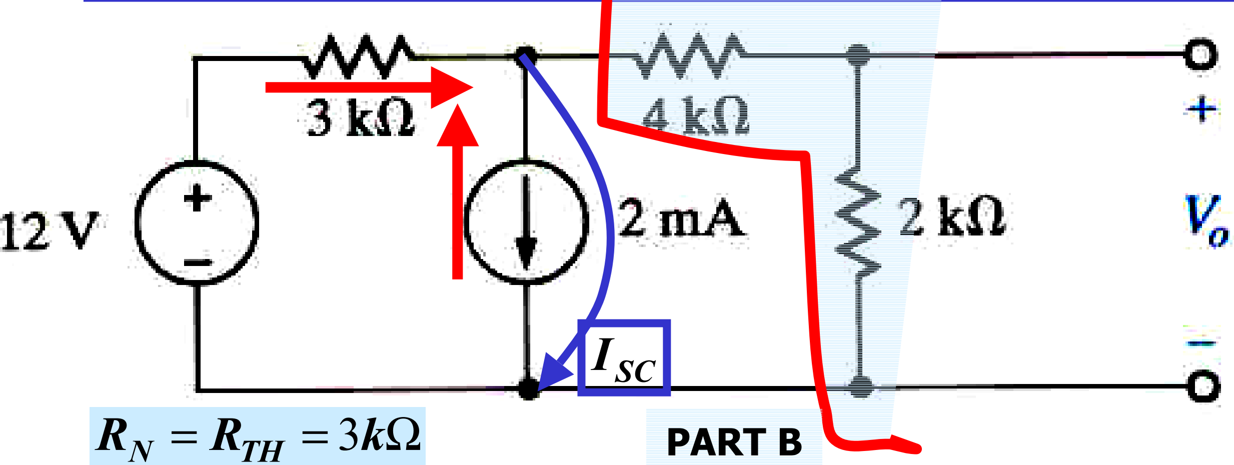
RESULTING EQUIVALENT CIRCUIT

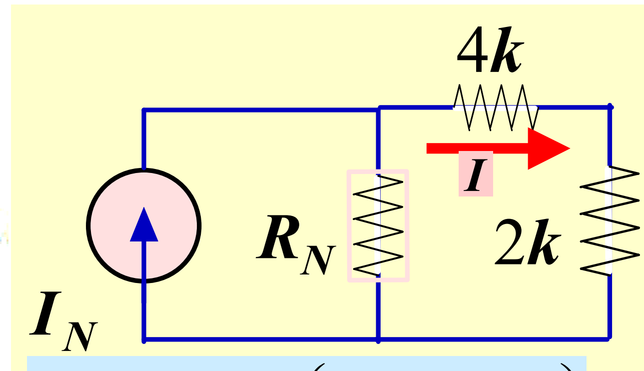



$$V_O = \frac{4}{4+4}(-6V) = -3[V]$$

GEAUX

## LEARNING EXTENSION: COMPUTE Vo USING NORTON




$$R_N = R_{TH} = 3k\Omega$$

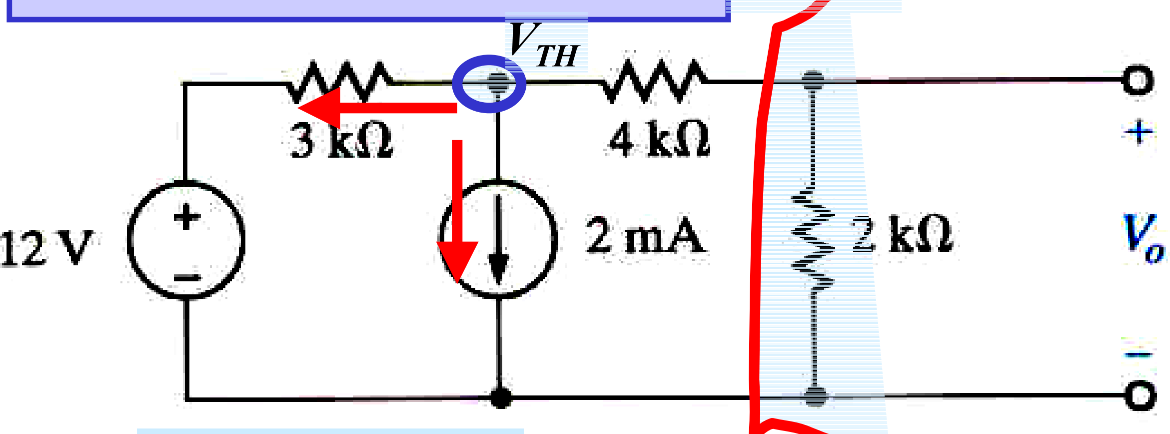
$$I_{SC} = I_N = \frac{12V}{3k} - 2mA = 2mA$$

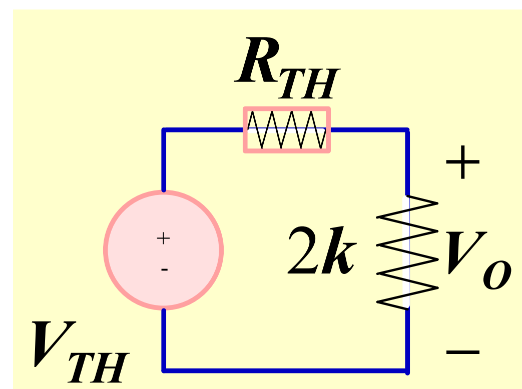
$$V_O = 2kI = 2k\left(\frac{R_N}{R_N + 6k} I_N\right)$$

$$V_O = 2\frac{3}{9}(2) = \frac{4}{3}[V]$$

## COMPUTE Vo USING THEVENIN

PART B




$$\frac{V_{TH} - 12}{3k} + 2mA = 0$$

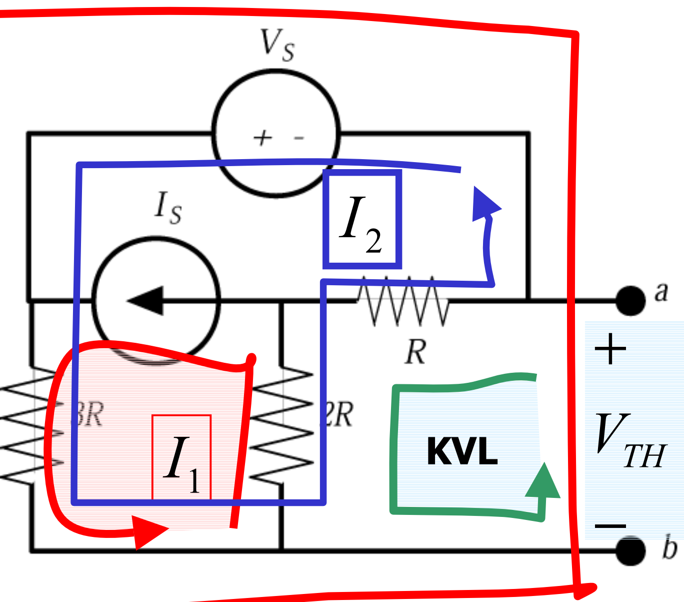
$$R_{TH} = 3k + 4k$$

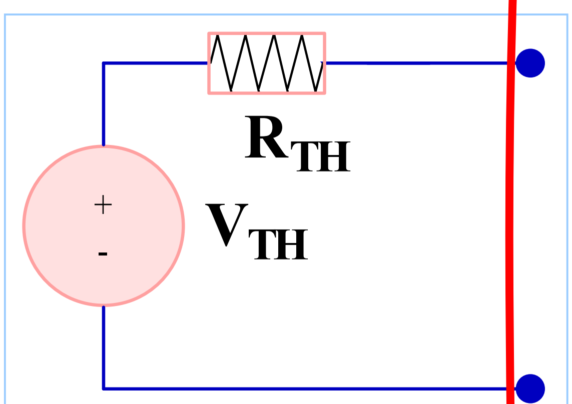
$$V_O = \frac{2}{2+7}(6V) = \frac{4}{3}[V]$$

GEAUX

# SAMPLE PROBLEM

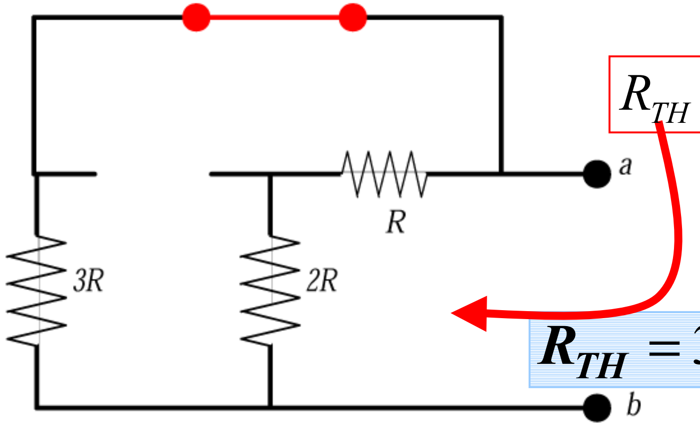
*FIND THE THEVENIN EQUIVALENT AT a - b*




**This is what we need to get**

**Equivalent Resistance: Independent sources only**




$$R_{TH} = 3R \| 3R = 1.5R$$

**Equivalent Voltage: Node, loop, superposition… Do loops**

$$I_1 = I_S$$

$$-V_S + 5R(I_1 + I_2) + RI_2 = 0$$

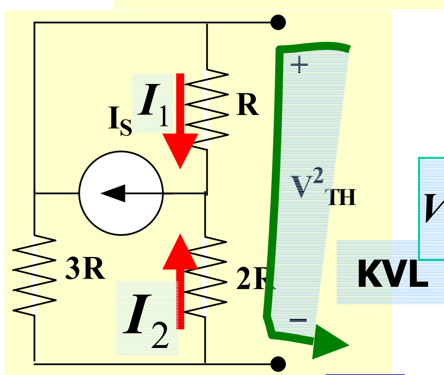
$$V_{TH} = -RI_2 - 2R(I_1 + I_2)$$

**How about source superposition?**

**Opening the current source:**

$$V_{TH}^1 = -\frac{V_S}{2}$$

**Short circuiting the voltage source**



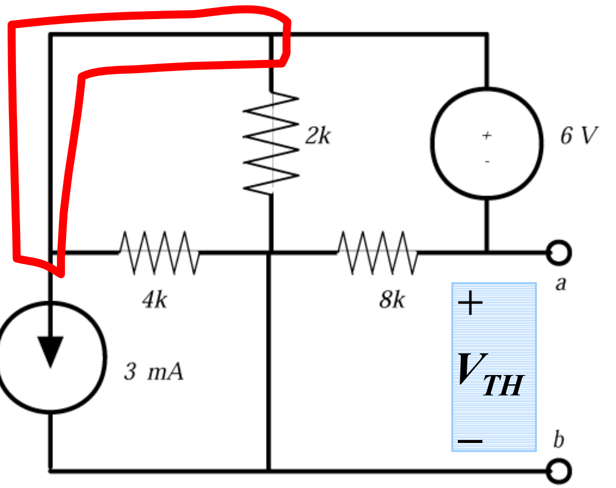

$$I_1 = \frac{5}{6} I_S \quad I_2 = \frac{1}{6} I_S$$

$$V_{TH}^2 = RI_1 - 2RI_2 = \frac{1}{2} RI_S$$

$$V_{TH} = V_{TH}^1 + V_{TH}^2$$

GEAUX

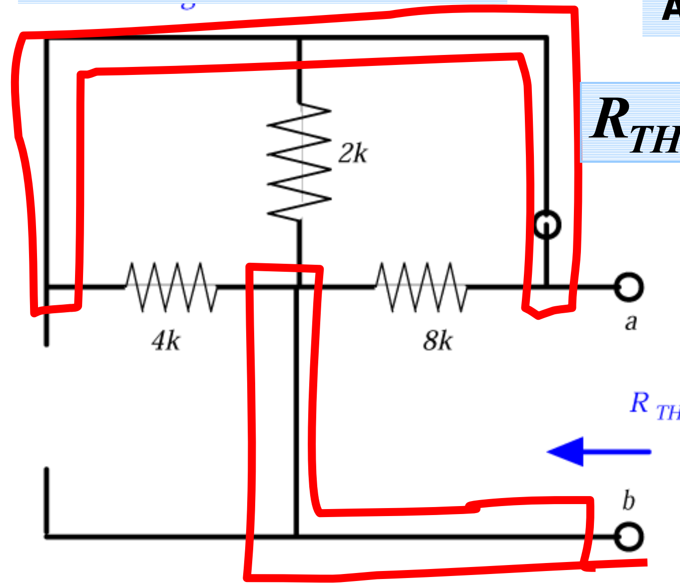
# SAMPLE PROLEM




*FIND THE THEVENIN EQUIVALENT CIRCUIT FROM TERMINALS a - b*

*SHOW ALL YOUR WORK*

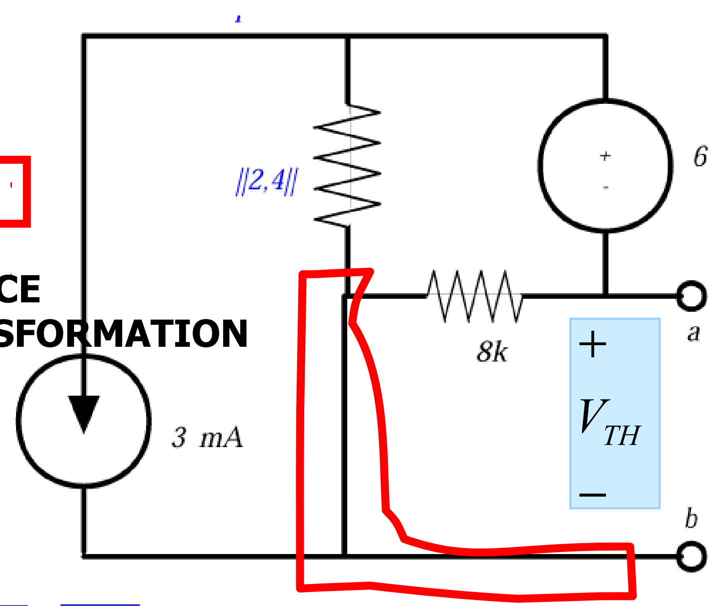
**All independent sources**




**All resistors are in parallel!!**

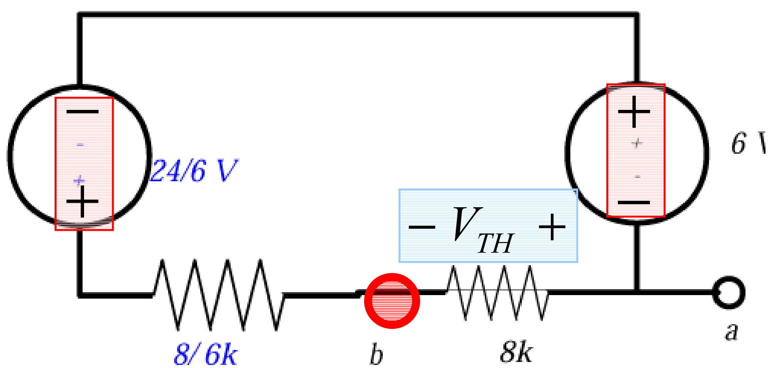
$$R_{TH} = \| 2k, 4k, 8k \| = 8/7k$$

**The circuit can be simplified**




**SOURCE TRANSFORMATION**

**,,, An to compute Equivalent Source...**

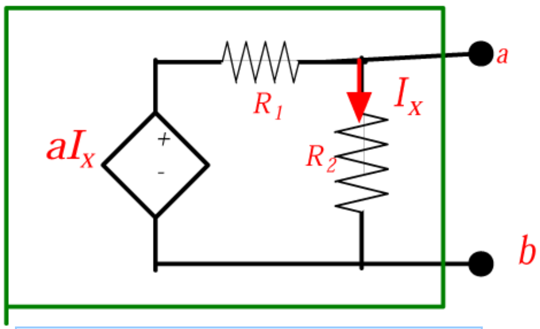



**Voltage divider**

$$V_{TH} = -\frac{8k}{8k + (8/6)k}(6 + 24/6)[V]$$

GEAUX

# THEVENIN EQUIVALENT FOR CIRCUITS WITH ONLY DEPENDENT SOURCES




$$-aI_x + (R_1 + R_2)I_X = 0$$

$$(-a + R_1 + R_2)I_x = 0$$

$$-a + R_1 + R_2 \neq 0 \Rightarrow I_x = 0$$

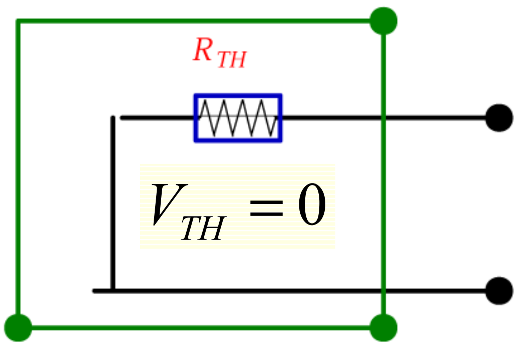
**A circuit with only dependent sources cannot self start.**

**(actually that statement has to be qualified a bit.
What happens if** $a = R_1 + R_2$ ? **)**

**FOR ANY PROPERLY DESIGNED CICUIT WITH ONLY DEPENDENT SOURCES** $V_{OC} = 0, I_{SC} = 0$

**This is a big simplification!!**

**But we need a special approach for the computation of the Thevenin equivalent resistance**



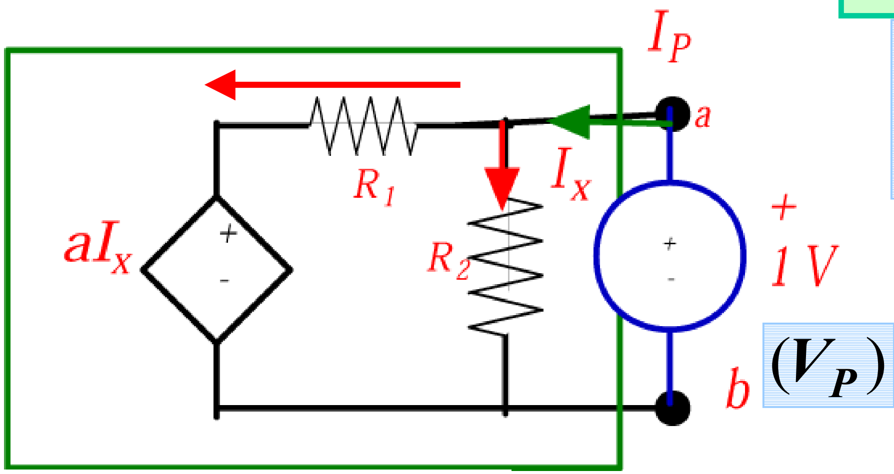

**Since the circuit cannot self start we need to probe it with an external source**

**The source can be either a voltage source or a current source and its value can be chosen arbitrarily!**

**Which one to choose is often determined by the simplicity of the resulting circuit**

GEAUX

## IF WE CHOOSE A VOLTAGE PROBE...

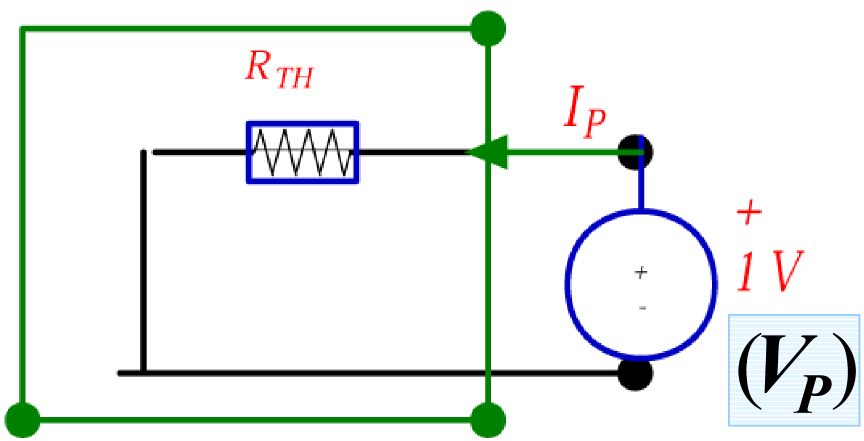



## WE MUST COMPUTE CURRENT SUPPLIED BY PROBE SOURCE

$$I_P = I_X + \frac{V_P - aI_X}{R_1}$$

$$I_X = \frac{V_P}{R_2}$$

$$I_P = \left(\frac{1}{R_2} + \frac{1}{R_1} - \frac{a}{R_1 R_2}\right) V_P$$



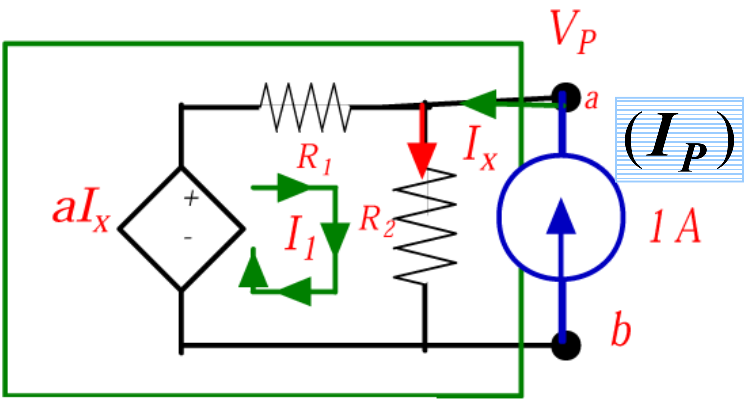

$$R_{TH} = \frac{V_P}{I_P}$$

$$R_{TH} = \frac{V_P}{\left(\frac{1}{R_2} + \frac{1}{R_1} - \frac{a}{R_1 R_2}\right) V_P}$$

**The value chosen for the probe voltage is irrelevant. Oftentimes we simply set it to one**

GEAUX

# IF WE CHOOSE A CURRENT SOURCE PROBE



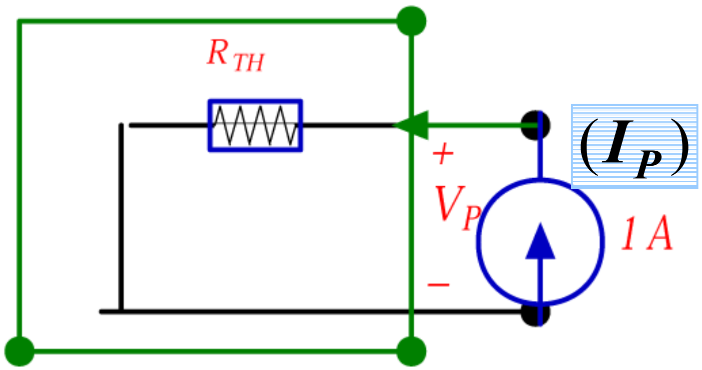

**We must compute the node voltage V_p**

*KCL*

$$\frac{V_P}{R_2} + \frac{V_P - aI_X}{R_1} - I_P = 0$$

$$I_X = \frac{V_P}{R_2}$$

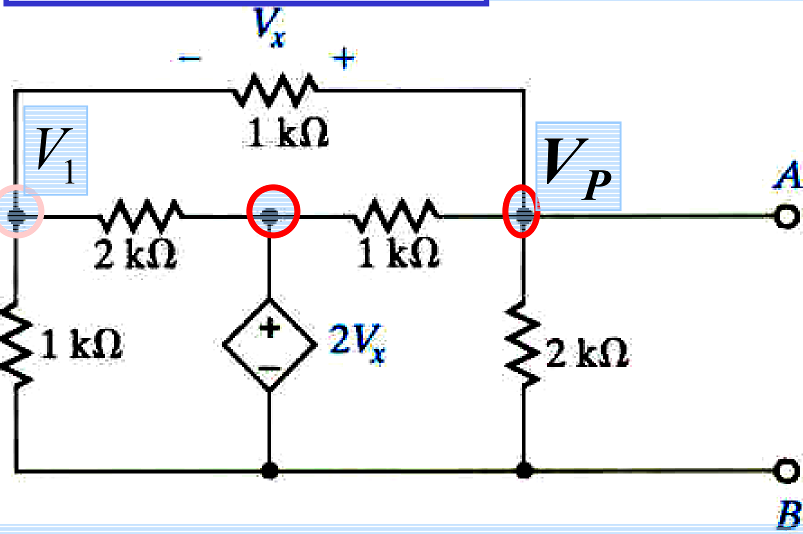
$$\left(\frac{1}{R_2} + \frac{1}{R_1} - \frac{a}{R_1 R_2}\right) V_P = I_P$$




$$R_{TH} = \frac{V_P}{I_P}$$

**The value of the probe current is irrelevant. For simplicity it is often choosen as one.**

## LEARNING EXAMPLE — FIND THE THEVENIN EQUIVALENT




$$KCL@V_1: \frac{V_1}{1k} + \frac{V_1 - 2V_X}{2k} + \frac{V_1 - V_P}{1k} = 0$$

**Controlling variable:** $V_X = V_P - V_1$

SOLVING THE EQUATIONS

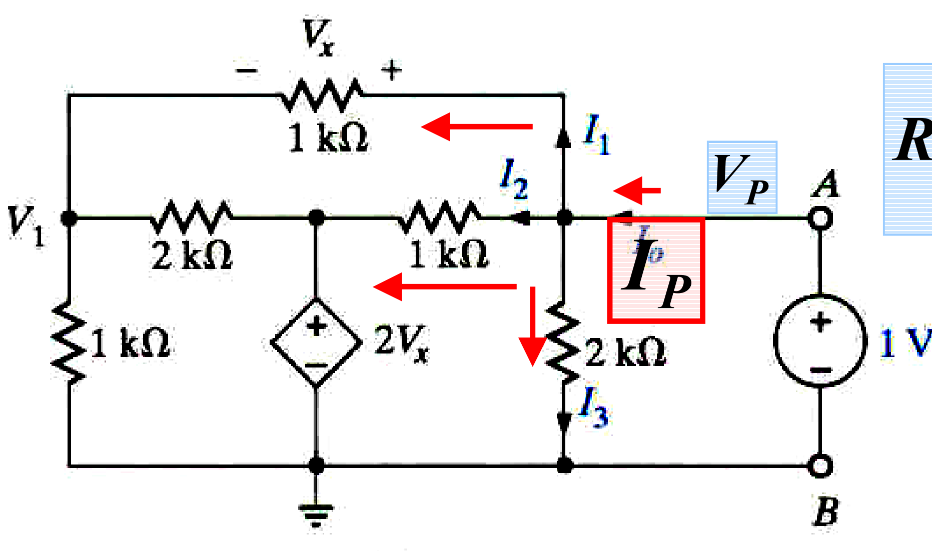
$$V_1 = \frac{4}{7}V_P, \qquad V_X = \frac{3}{7}V_P$$

$$I_P = \frac{V_P}{2k} + \frac{V_P - 2V_X}{1k} + \frac{V_X}{1k}$$

$$I_P = \frac{15V_P}{14k}$$

**Do we use current probe or voltage probe?**

**If we use voltage probe there is only one node not connected through source**

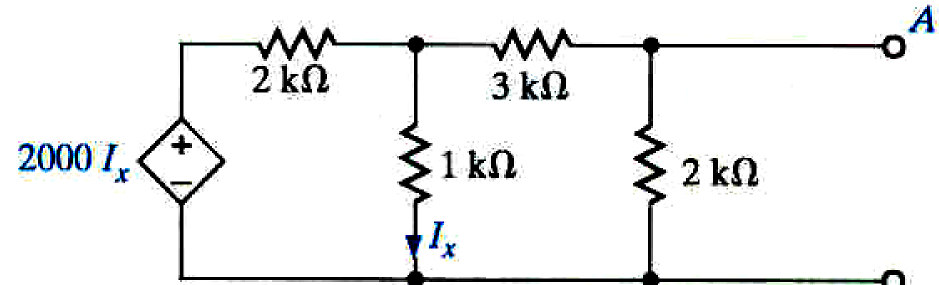



$$R_{TH} = \frac{V_P}{I_P} = \frac{14}{15}k\Omega$$

**Using voltage probe. Must compute current supplied**

GEAUX

**LEARNING EXAMPLE** Find the Thevenin Equivalent circuit at A - B

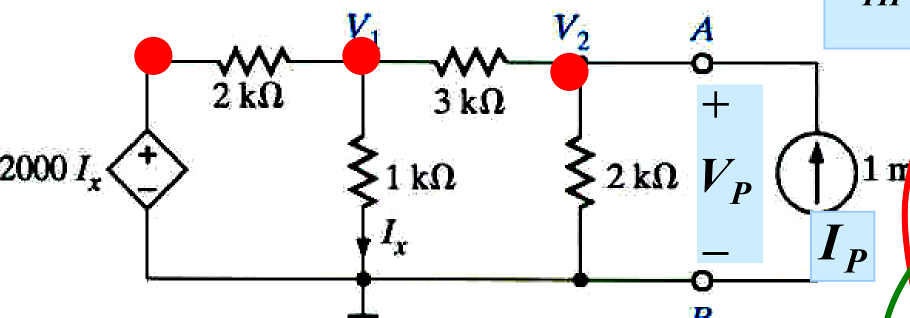



**Only dependent sources. Hence V_th = 0**

**To compute the equivalent resistance we must apply an external probe**

**We choose to apply a current probe**

$$R_{TH} = \frac{V_P}{I_P}$$




**@V_1**

$$\frac{V_1 - 2000I_x}{2k} + \frac{V_1}{1k} + \frac{V_1 - V_2}{3k} = 0$$

**@V_2**

$$\frac{V_2 - V_1}{3k} + \frac{V_2}{2k} = 1 \times 10^{-3} \quad (I_P)$$

**Controlling variable**

$$I_x = \frac{V_1}{1k}$$

**"Conventional" circuit with dependent sources - use node analysis**

$$3(V_1 - 2V_1) + 6V_1 + 2(V_1 - V_2) = 0$$

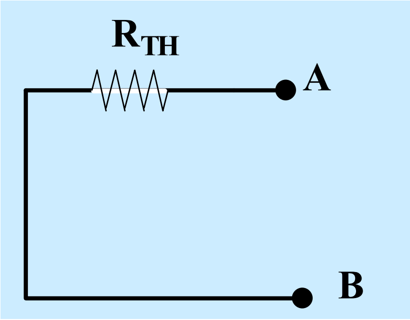
$$2(V_2 - V_1) + 3V_2 = 6[V]$$

$$5V_1 - 2V_2 = 0 \quad * / 2$$

$$-2V_1 + 5V_2 = 6 \quad * / 5$$

$$V_2 = \frac{30}{21} = \frac{10}{7}$$

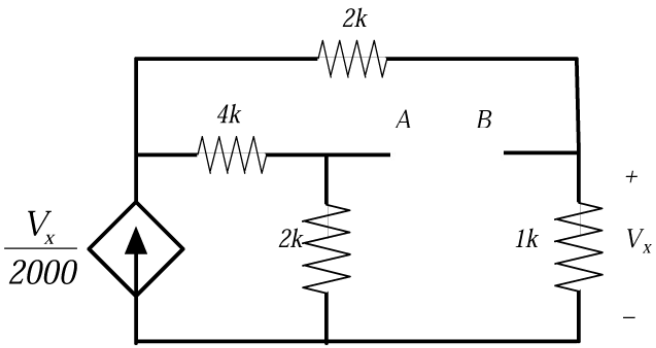
$$(V_P = V_2) \cap (I_P = 1mA) \Rightarrow R_{TH} = \frac{V_2}{1mA} = (10/7)k\Omega$$



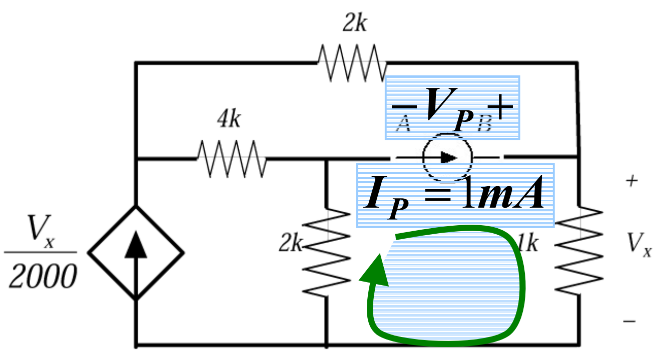

**Thevenin equivalent**

GEAUX

# SAMPLE PROBLEM



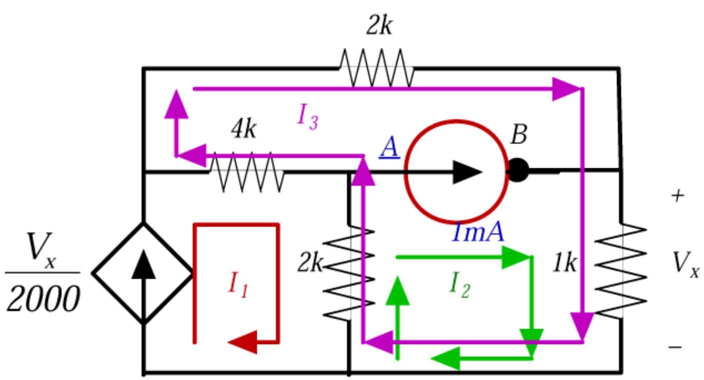

FIND THE THEVENIN EQUIVALENT AT TERMINALS A - B USING A 1mA SOURCE

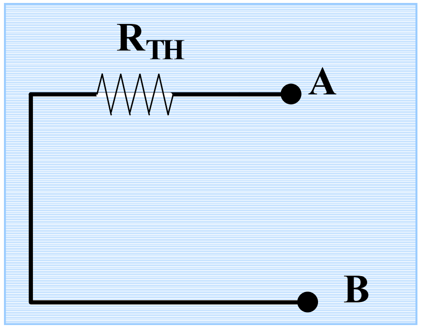



| |
|---|
| I_1 = I_p/2 |
| I_3=0 |
| R_th = 2kOhms |

MUST FIND $V_{AB} = V_P$. METHOD?

**Loop analysis**




**Thevenin equivalent**

$$R_{TH} = \frac{V_P}{I_P} = \frac{V_P}{1mA}$$

**The resistance is numerically equal to V_p but with units of KOhm**

$$I_1 = \frac{V_X}{2000};\ I_2 = I_P$$

$$2k * I_3 + 1k * (I_2 + I_3) + 2k * (I_3 + I_2 - I_1) + 4k * (I_3 - I_1) = 0$$

**Controlling variable** $V_X = 1k * (I_3 + I_2)$

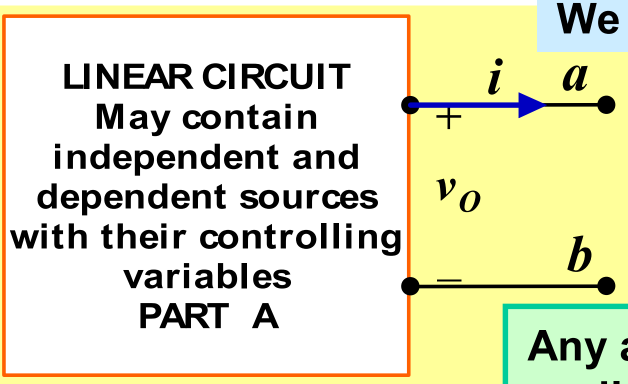
**Voltage across current probe**

$$-V_P + 1k * (I_3 + I_2) + 2k * (I_3 + I_2 - I_1) = 0$$

GEAUX

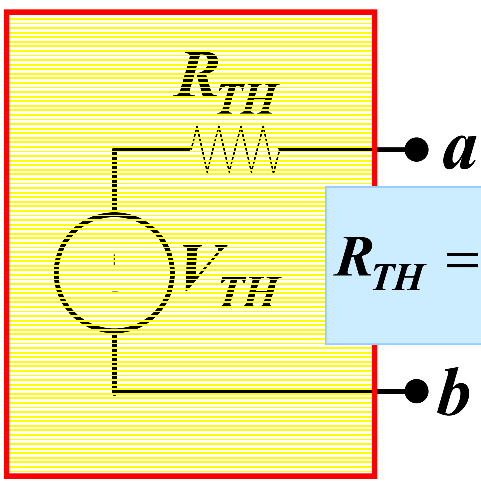
# Thevenin Equivalent
# Circuits with both Dependent and Independent Sources

**We will compute open circuit voltage and short circuit current**




**For each determination of a Thevenin equivalent we will solve two circuits**

**Any and all the techniques discussed should be readily available; e.g.,**
**KCL, KVL,combination series/parallel, node, loop analysis, source superposition, source transformation, homogeneity**



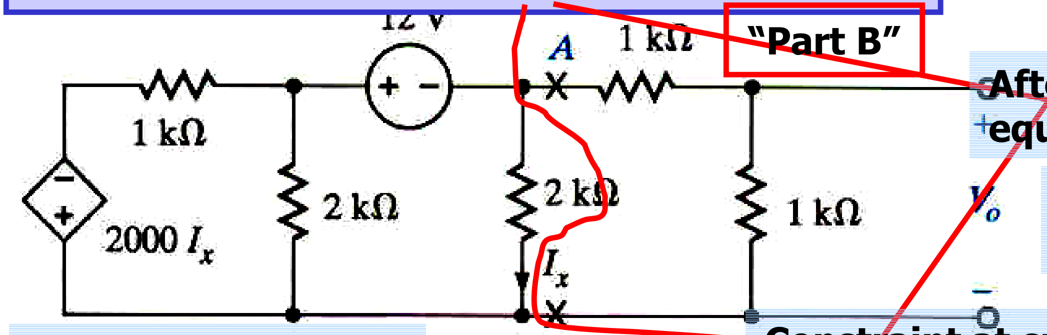

$$R_{TH} = \frac{V_{OC}}{I_{SC}}$$

$$V_{TH} = V_{OC}$$

**The approach of setting to zero all sources and then combining resistances to determine the Thevenin resistance is in general not applicable!!**

GEAUX

# EXAMPLE Use Thevenin to determine Vo




**Guidelines to partition:**

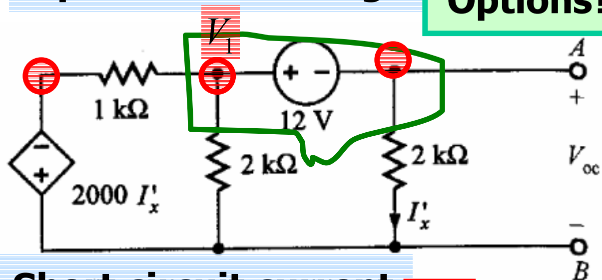
"Part A" should be as simple as possible.

After "Part A" is replaced by the Thevenin equivalent we should have a very simple circuit

The dependent sources and their controlling variables must remain together

**Open circuit voltage**

**Options???**




Constraint at super node $V_1 - V_{OC} = 12 \Rightarrow V_1 = 12 + V_{OC}$

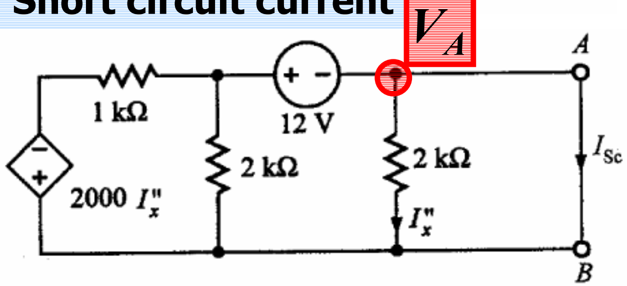
KCL at super node $\frac{(12+V_{OC}) - (-aI_X^1)}{1k} + \frac{12+V_{OC}}{2k} + \frac{V_{OC}}{2k} = 0$

Equation for controlling variable $I_X^1 = \frac{V_{OC}}{2k}$

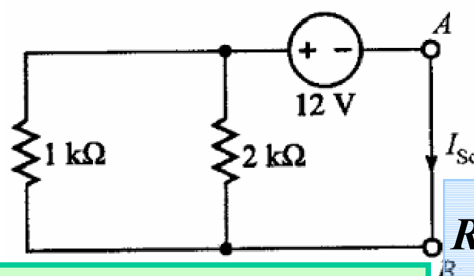
Solve $V_{OC} = -\frac{36}{4 + (a/1k)}$

**Short circuit current**

$V_A$




$$I_X'' = \frac{V_A}{2k} = 0$$



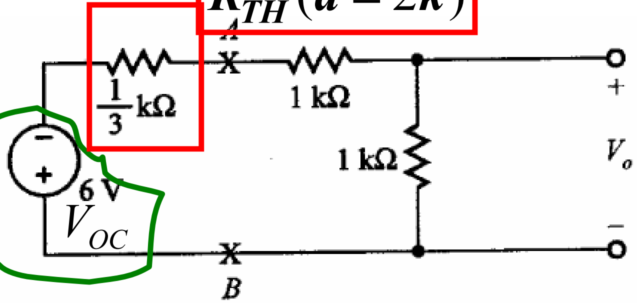

$$I_{SC} = -\frac{12V}{1k \| 2k} = -18mA$$

$$R_{TH} = \frac{V_{OC}}{I_{SC}} = \frac{2}{4 + (a/1k)}[k\Omega]$$

**Negative resistances for some "a's"**

**Solution to the problem**

$R_{TH}(a = 2k)$

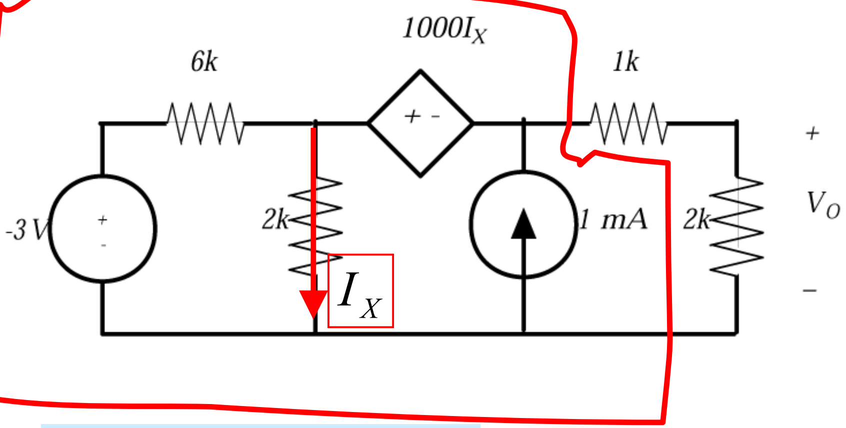



Setting all sources to zero and combining resistances will yield an incorrect value!!!!

$$V_0 = \frac{1k}{1k + 1k + R_{TH}} V_{TH}$$

GEAUX

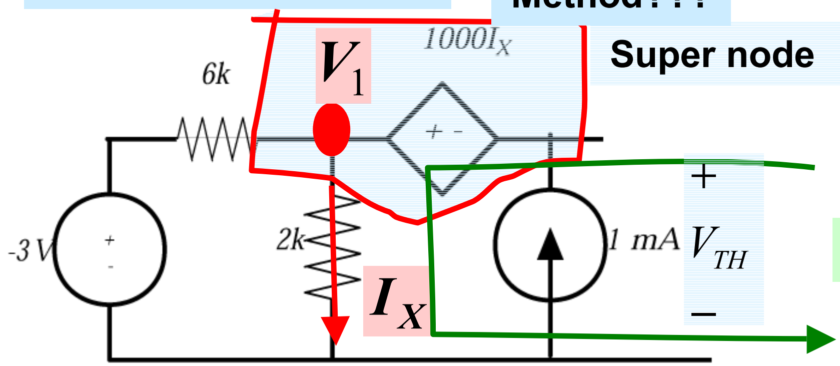
# Find Vo using Thevenin




Open circuit voltage

Method???

Super node




KVL

$$\frac{V_1}{2k} - 1mA + \frac{V_1 - (-3V)}{6k} = 0$$

$$V_1 = (3/4)[V]$$

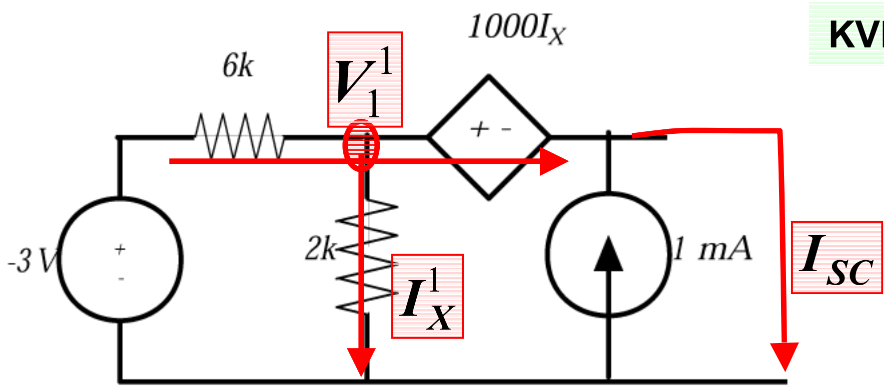
KVL $-V_{TH} - 1000 I_X + V_1 = 0$

Controlling variable

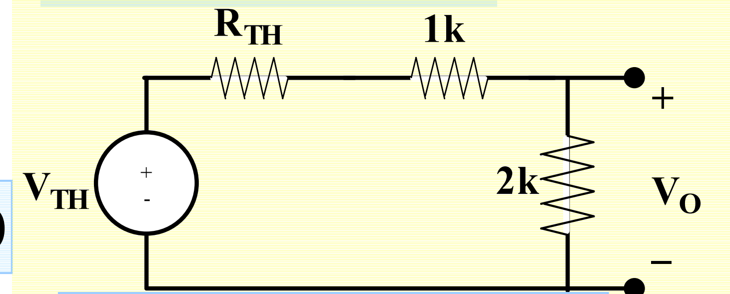
$$I_X = \frac{V_1}{2k}$$

$$V_{TH} = (3/8)[V]$$

Short Circuit Current




The equivalent circuit




$$V_1^1 = 1000 I_X^1$$

$$I_X^1 = \frac{V_1^1}{2k} \Rightarrow V_1^1 = 0 \Rightarrow I_X^1 = 0$$

KCL

$$I_{SC} = 1mA + (-3V)/(6k) = 0.5mA$$

$$R_{TH} = \frac{V_{OC}}{I_{SC}} = (3/4)k$$

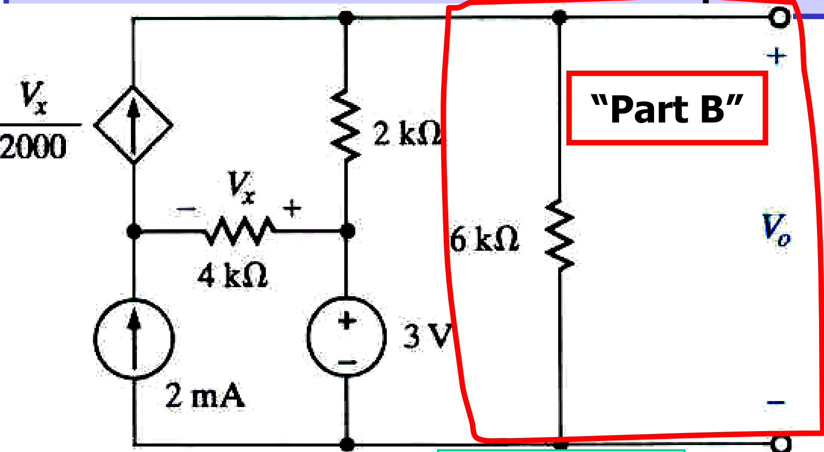
$$V_0 = \frac{2}{2 + 1 + (3/4)}(3/8)[V]$$

The equivalent resistance cannot be obtained by short circuiting the sources and determining the resistance of the resulting interconnection of resistors

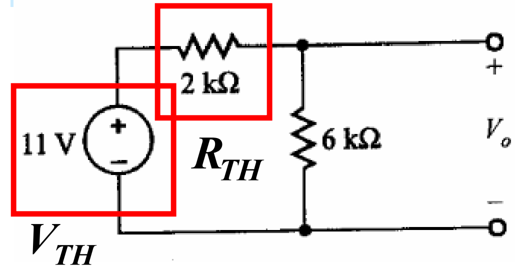
# EXAMPLE: Use Thevenin to compute Vo

**DON'T PANIC!!**

**Select your partition**

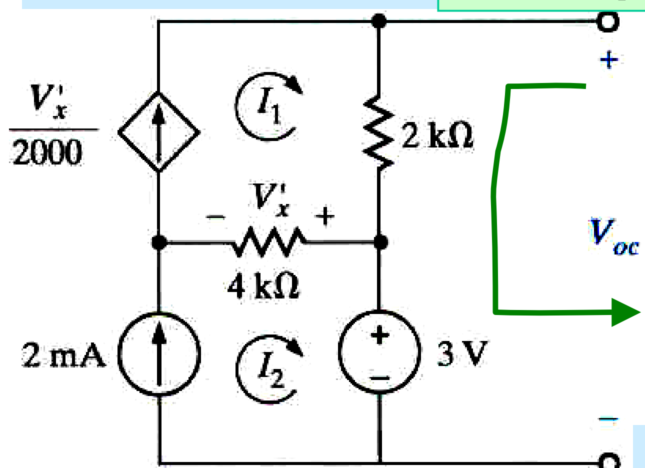



**Now compute V_0 using the Thevenin equivalent**




$$V_0 = \frac{6k}{6k+8k} 11[V]$$

**Open Circuit Voltage** **Use loops**




**KVL for V_oc**

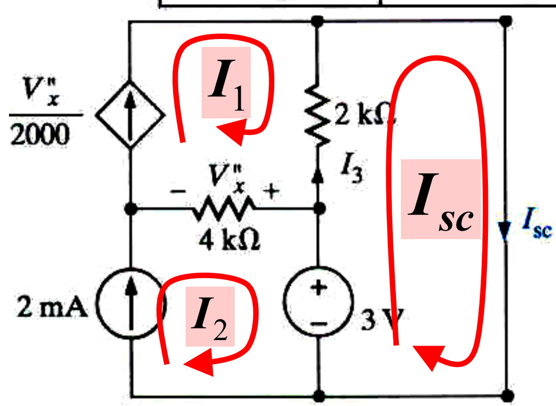
**Loop equations** $I_1 = \frac{V_X^1}{2000}; \; I_2 = 2mA$

**Controlling variable** $V_X^1 = 4k(I_1 - I_2)$

$$V_X^1 = 2kI_1 \Rightarrow 2kI_1 = 4k(I_1 - I_2) \Rightarrow I_1 = 4mA$$

$$V_{OC} = 2k * I_1 + 3[V] = 2k * 4mA + 3V = 11V$$

**Short circuit current**




$$I_1 = \frac{V''_x}{2000}; \; I_2 = 2mA$$

$$-3V + 2k(I_{SC} - I_1) = 0$$

**Controlling variable**

$$V''_X = 4k * (I_1 - I_2)$$

**Loop equations**

$I_1 = 4mA$ **Same as before**

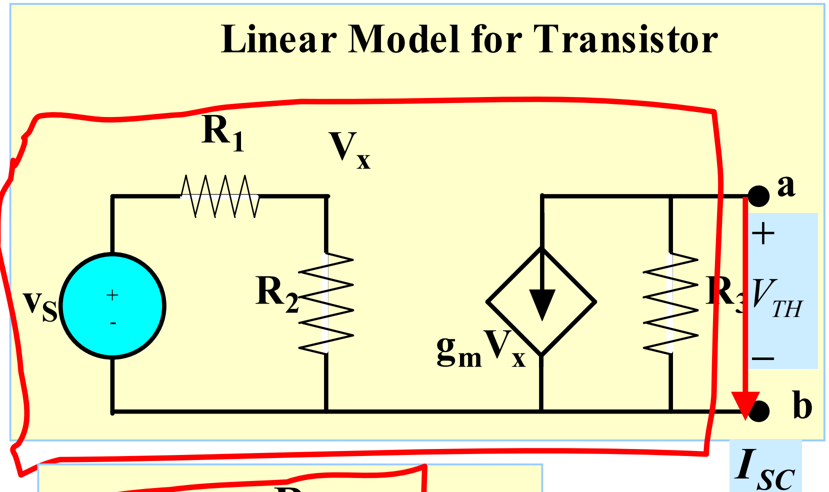
$$I_{SC} = \frac{3V + 2k * I_1}{2k} = \frac{11}{2} mA$$

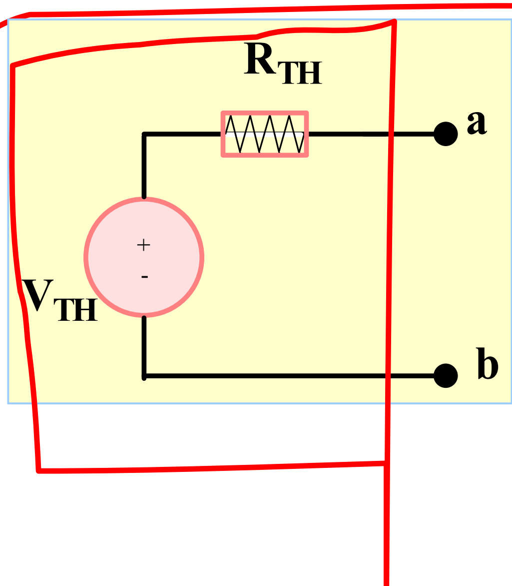
**Thevenin resistance**

$$R_{TH} = \frac{V_{OC}}{I_{SC}} = \frac{11[V]}{(11/2)mA} = 2k\Omega$$

GEAUX

**EXAMPLE**




**The alternative for mixed sources**

$$V_{TH} = V_{OC}, \quad R_{TH} = \frac{V_{OC}}{I_{SC}}$$

**Open circuit voltage**

$$V_{TH} = -g_m R_3 V_x$$

$$V_x = \frac{R_2}{R_1 + R_2} v_S \Rightarrow V_{TH} = -g_m \frac{R_3 R_2}{R_1 + R_2} v_S$$

**Short circuit current**

$$I_{SC} = -g_m V_x = -g_m \frac{R_2}{R_1 + R_2} v_S$$

**Equivalent Resistance**

$$R_{TH} = \frac{V_{OC}}{I_{SC}} = R_3$$

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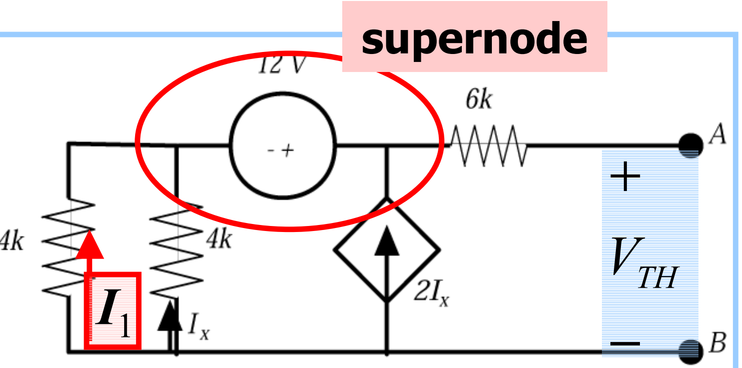
## SAMPLE PROBLEM

**Mixed sources. Must compute Voc and Isc**




*FIND THE THEVENIN EQUIVALENT AT THE TERMINALS A - B*

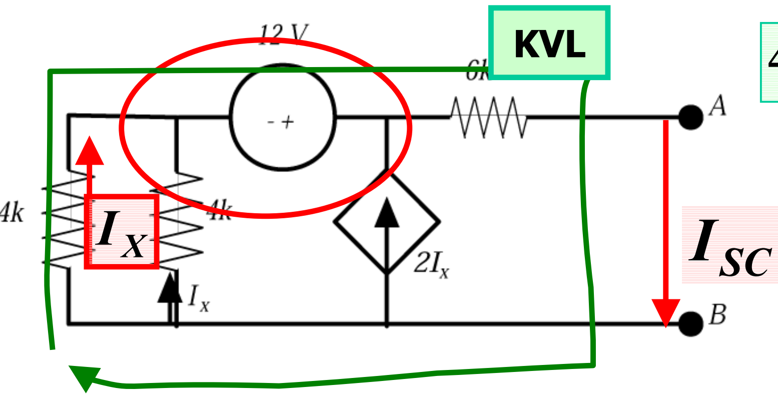
**Open circuit voltage**

**KCL at super node** $I_1 + I_X + 2I_X = 0$

**The two 4k resistors are in parallel** $I_1 = I_X$

$$I_X = 0 \Rightarrow V_{TH} = 12[V]$$

**Short circuit current**




**KCL at supernode**

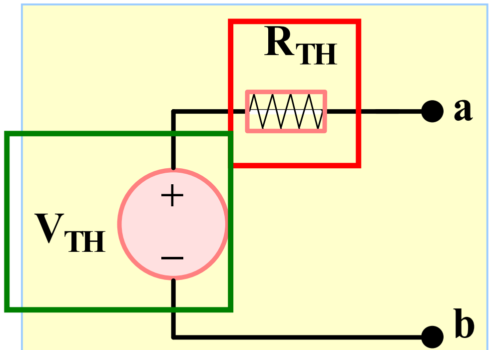
$$I_{SC} = 4I_X$$

$$4k * (I_{SC} / 4) - 12[V] + 6k * I_{SC} = 0$$

$$I_{SC} = \frac{12}{7} mA$$

$$R_{TH} = \frac{V_{TH}}{I_{SC}} = \frac{12V}{(12/7)mA} = 7k\Omega$$

**FINAL ANSWER**

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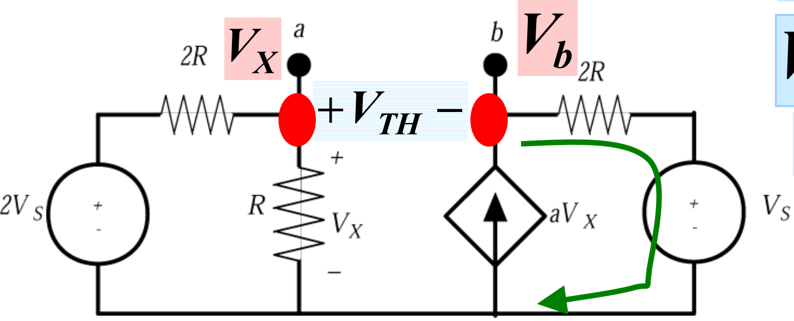
## SAMPLE PROBLEM

*FIND THE THEVENIN EQUIVALENT AT a - b*




**Mixed sources! Must compute open loop voltage and short circuit current**

**Open circuit voltage**

$$V_{TH} = V_X - V_b$$

**For Vx use voltage divider**

$$V_X = \frac{R}{R+2R}(2V_S) = \frac{2}{3}V_S$$

**For Vb use KVL**

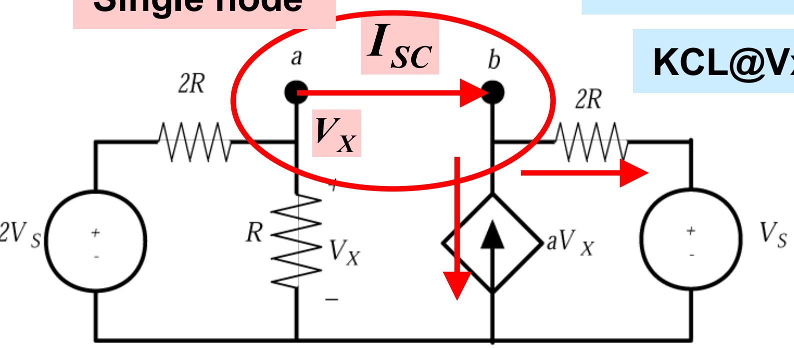
$$V_b = 2R(aV_X) + V_S = (1 + 4aR/3)V_S$$

$$V_{TH} = V_X - (2RaV_X + V_S) = (1-2Ra)(2V_S/3) - V_S$$

$$V_{TH} = -\frac{1+4aR}{3}V_S$$

**Short circuit current**

**Single node**




**We need to compute V_x**

**KCL@Vx**

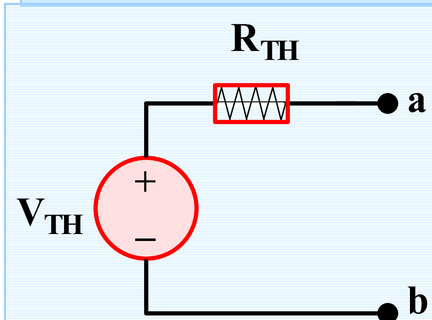
$$\frac{V_x^1 - 2V_S}{2R} + \frac{V_X^1}{R} - aV_X^1 + \frac{V_X^1 - V_s}{2R} = 0 \Rightarrow V_X^1 = \frac{3V_S}{4-2aR}$$

**KCL again can give the short circuit current**

$$I_{SC} = -aV_X^1 + \frac{V_X^1 - V_S}{2R}$$

$$I_{SC} = -\frac{1+4aR}{4R(1-2aR)}V_S$$

$$R_{TH} = \frac{V_{OC}}{I_{SC}} = \frac{V_{TH}}{I_{SC}} = \frac{4R(1-2aR)}{3}$$




**FINAL ANSWER**

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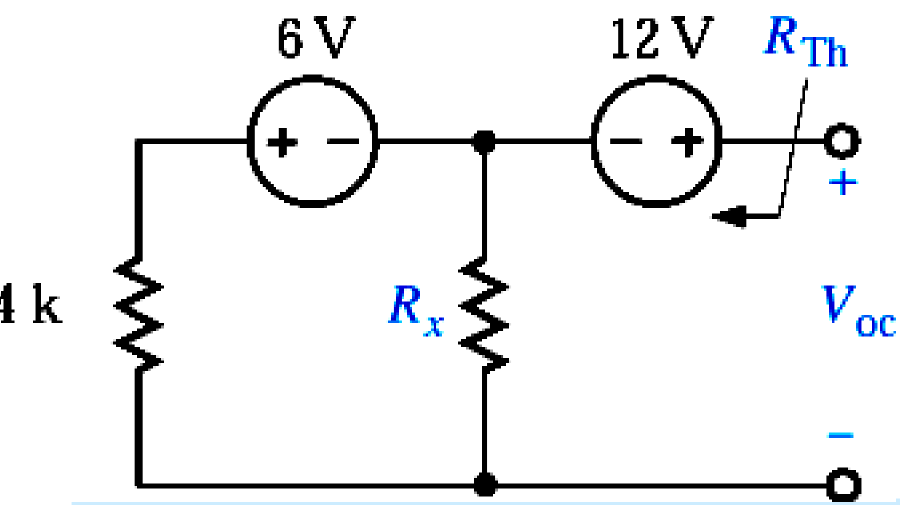
## LEARNING EXAMPLE

FIND AND PLOT $R_{TH}, V_{OC}$, WHEN $0 \le R_X \le 10k\Omega$



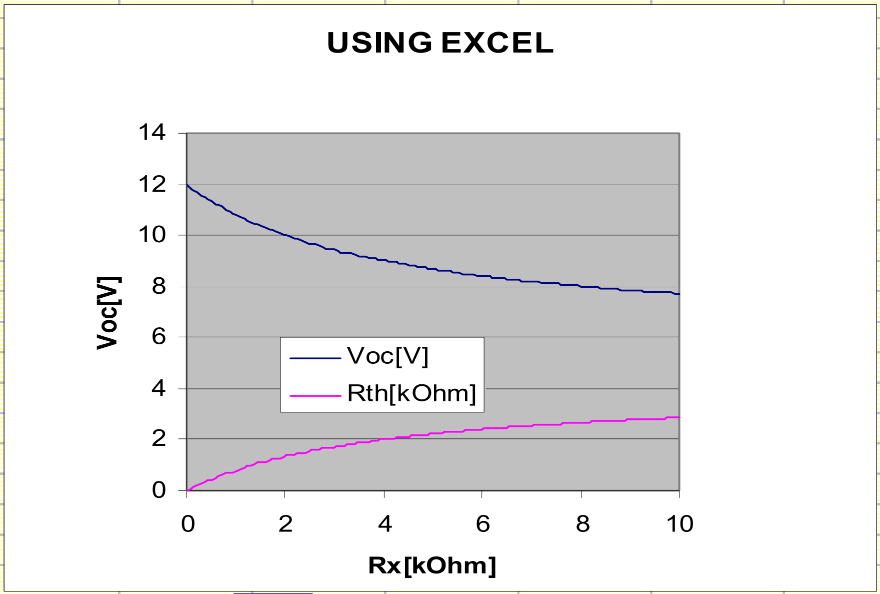

### DATA TO BE PLOTTED

$$R_{TH} = 4k \parallel R_X = \frac{4R_X}{4+R_X} \qquad V_{OC} = 12 - 6\left[\frac{R_X}{4k+R_X}\right]$$

### Using EXCEL to generate and plot data

**THEVENIN EQUIVALENT EXAMPLE**

| Rx[kOhm] | Voc[V] | Rth[kOhm] |
|---|---|---|
| 0 | 12 | 0 |
| 0.1 | 11.8537 | 0.097560976 |
| 0.2 | 11.7143 | 0.19047619 |
| 0.3 | 11.5814 | 0.279069767 |
| 0.4 | 11.4545 | 0.363636364 |
| 0.5 | 11.3333 | 0.444444444 |
| 0.6 | 11.2174 | 0.52173913 |
| 0.7 | 11.1064 | 0.595744681 |
| 0.8 | 11 | 0.666666667 |
| 0.9 | 10.898 | 0.734693878 |
| 1 | 10.8 | 0.8 |
| 1.1 | 10.7059 | 0.862745098 |
| 1.2 | 10.6154 | 0.923076923 |
| 1.3 | 10.5283 | 0.981132075 |
| 1.4 | 10.4444 | 1.037037037 |
| 1.5 | 10.3636 | 1.090909091 |
| 1.6 | 10.2857 | 1.142857143 |
| 1.7 | 10.2105 | 1.192982456 |
| 1.8 | 10.1379 | 1.24137931 |
| 1.9 | 10.0678 | 1.288135593 |

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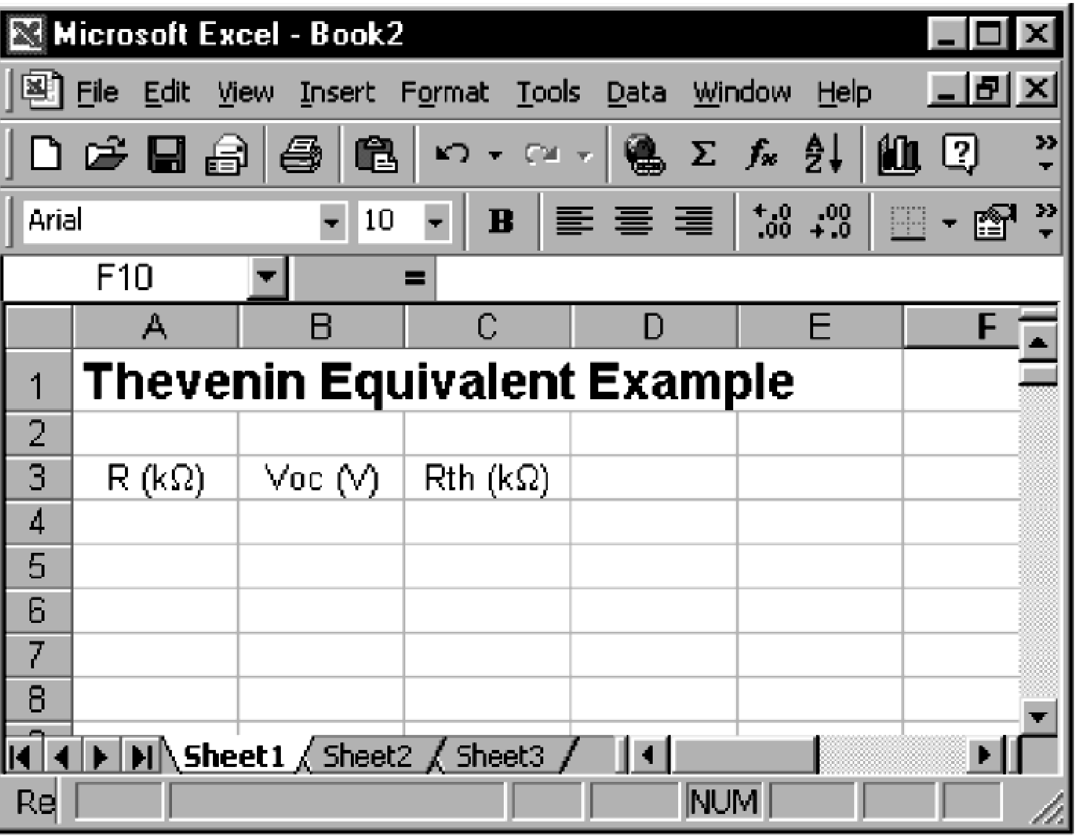
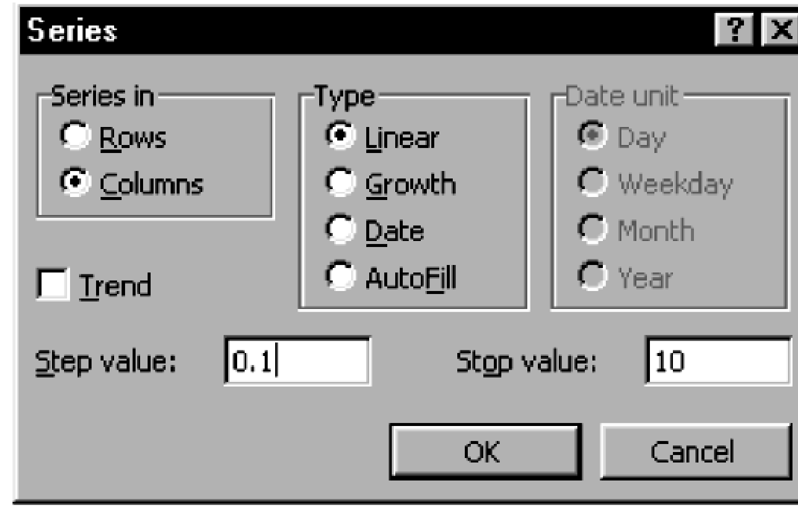
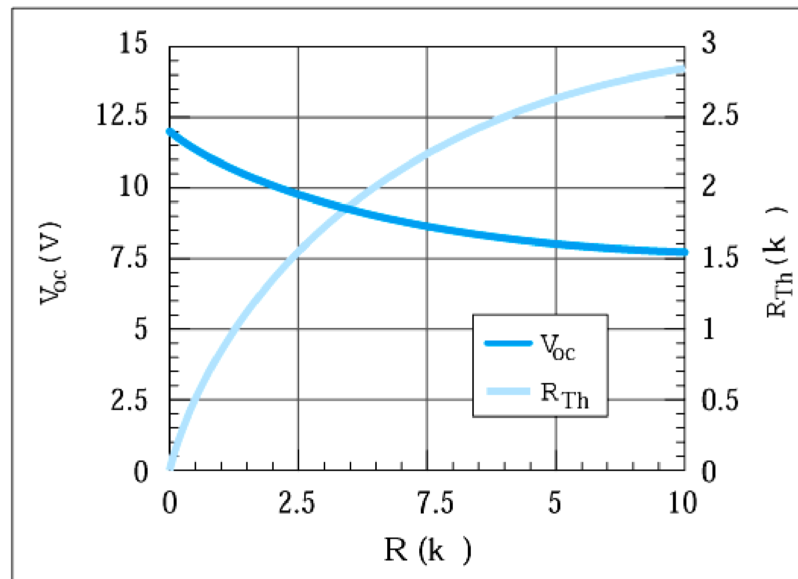

Microsoft Excel - Book2
File Edit View Insert Format Tools Data Window Help
Arial
10
F10
A B C D E F
Thevenin Equivalent Example
R (kΩ)
Voc (V)
Rth (kΩ)
Sheet1
Sheet2
Sheet3
Re
NUM

(a)


Series
Series in
Rows
Columns
Type
Linear
Growth
Date
AutoFill
Date unit
Day
Weekday
Month
Year
Trend
Step value:
0.1
Stop value:
10
OK
Cancel

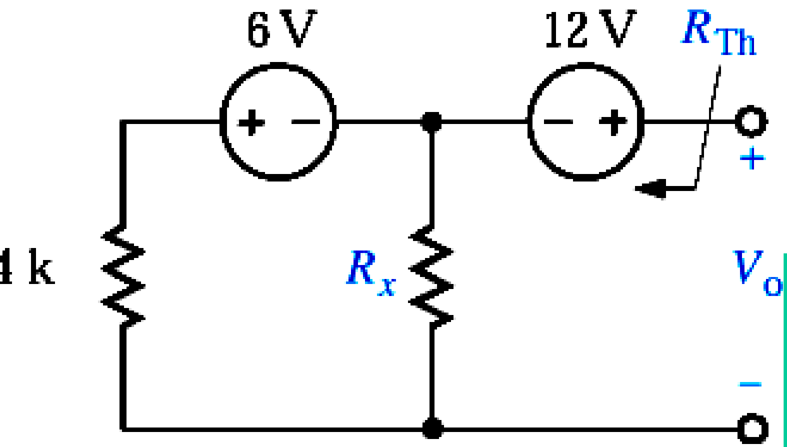
(b)


$V_{oc}$ (V)
$R_{Th}$ (k )
R (k )
$V_{oc}$
$R_{Th}$
0
2.5
5
7.5
10
12.5
15
0.5
1
1.5
2
2.5
3

(c)

## LEARNING EXAMPLE

FIND AND PLOT $R_{TH}, V_{OC}$, WHEN $0 \le R_X \le 10k\Omega$

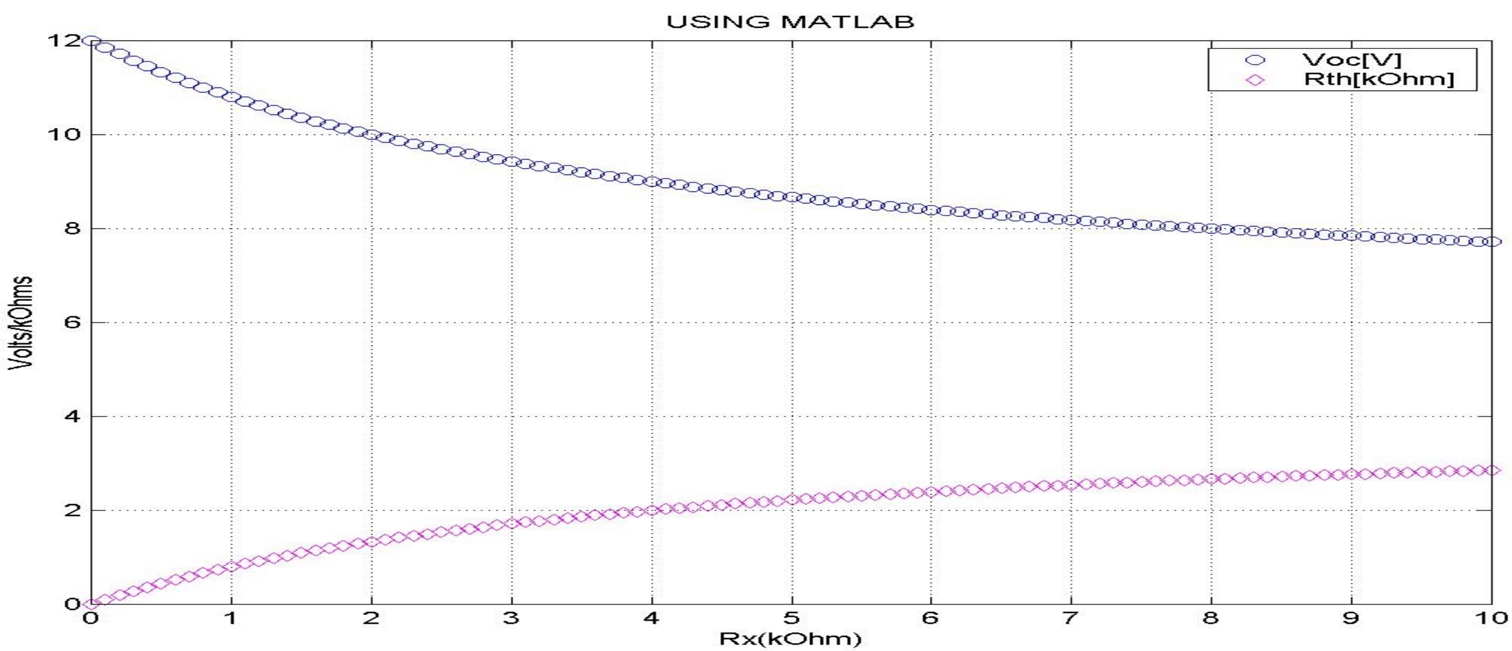



**DATA TO BE PLOTTED**

$$R_{TH} = 4k \parallel R_X = \frac{4R_X}{4+R_X} \qquad V_{OC} = 12 - 6\left[\frac{R_X}{4k+R_X}\right]$$

**Using MATLAB to generate and plot data**

```
» Rx=[0:0.1:10]'; %define the range of resistors to use
» Voc=12-6*Rx./(Rx+4); %the formula for Voc. Notice "./"
» Rth=4*Rx./(4+Rx); %formula for Thevenin resistance.
» plot(Rx,Voc,'bo', Rx,Rth,'md')
» title('USING MATLAB'), %proper graphing tools
» grid, xlabel('Rx(kOhm)'), ylabel('Volts/kOhms')
» legend('Voc[V]','Rth[kOhm]')
```

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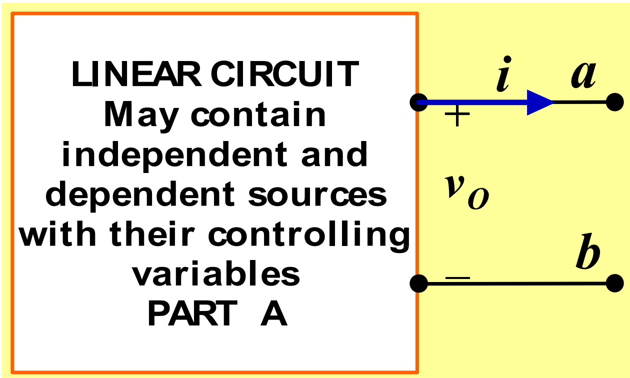
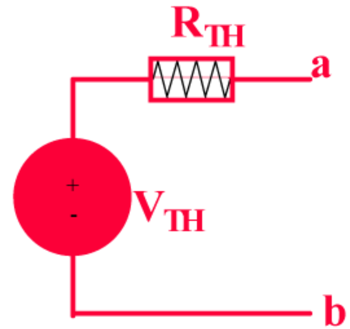
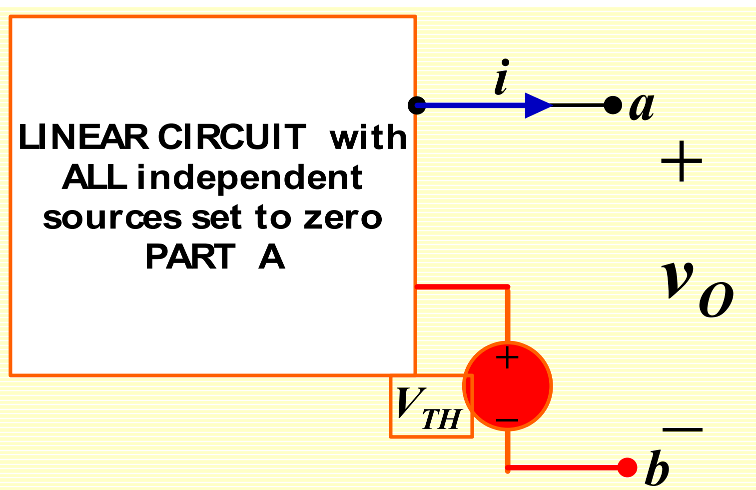
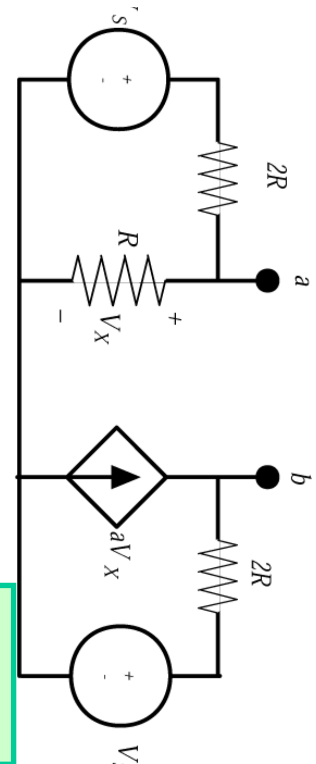
# A MORE GENERAL VIEW OF THEVENIN THEOREM

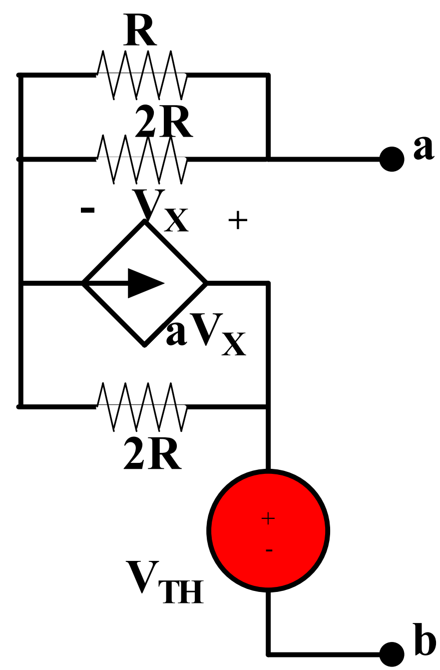



**USUAL INTERPRETATION**



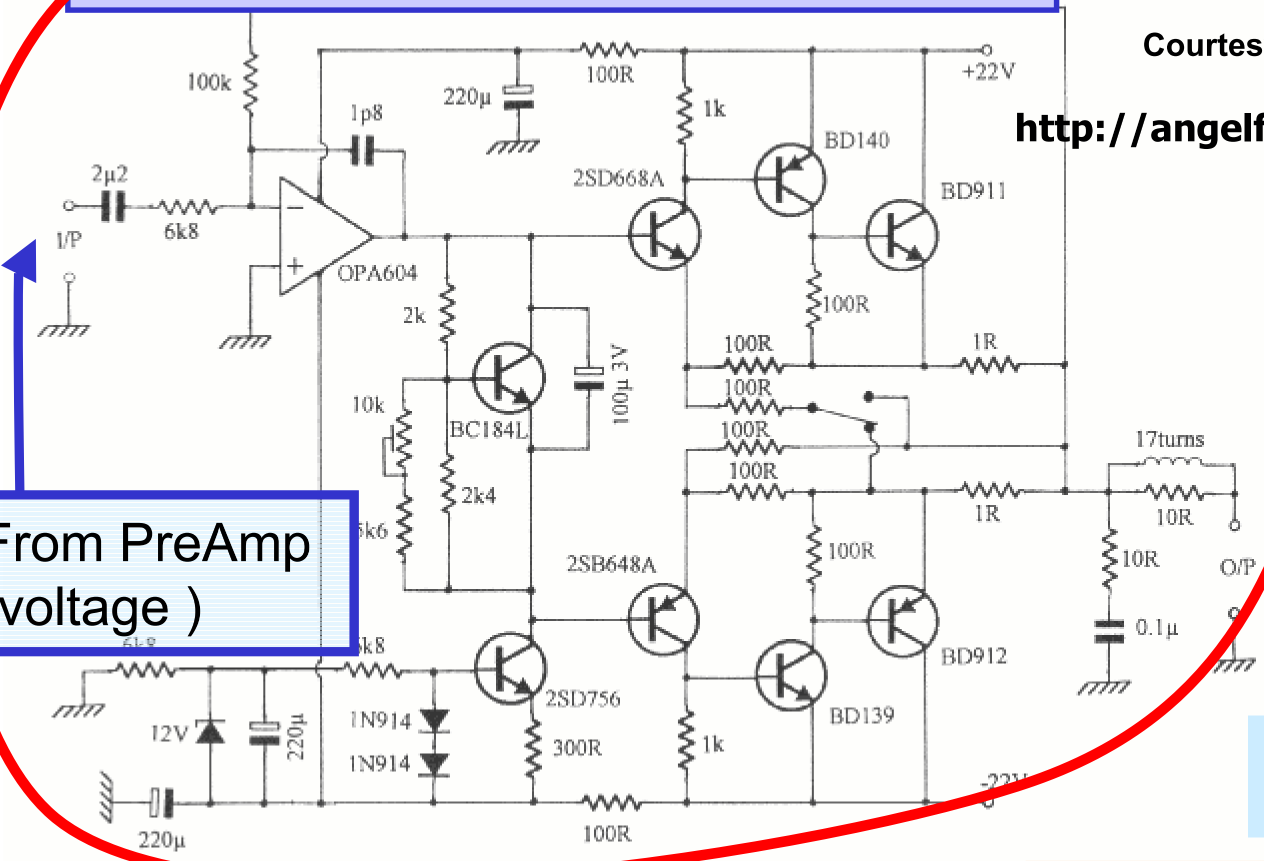

**Thevenin**

**THIS INTERPRETATION APPLIES EVEN WHEN THE PASSIVE ELEMENTS INCLUDE INDUCTORS AND CAPACITORS**

# MAXIMUM POWER TRANSFER

Courtesy of M.J. Renardson

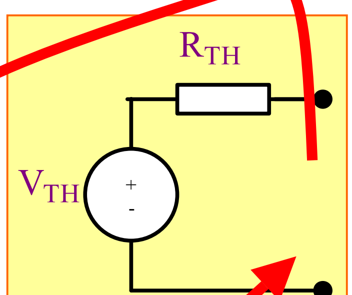
http://angelfire.com/ab3/mjramp/index.html

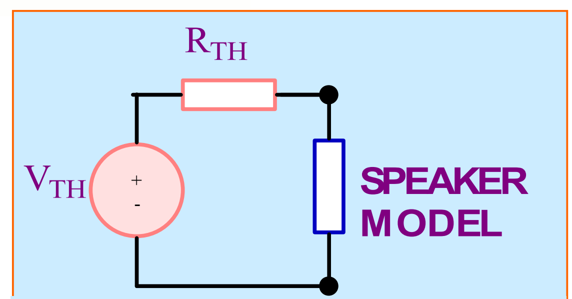



From PreAmp (voltage )

To speakers

The simplest model for a speaker is a resistance...




**BASIC MODEL FOR THE ANALYSIS OF POWER TRANSFER**

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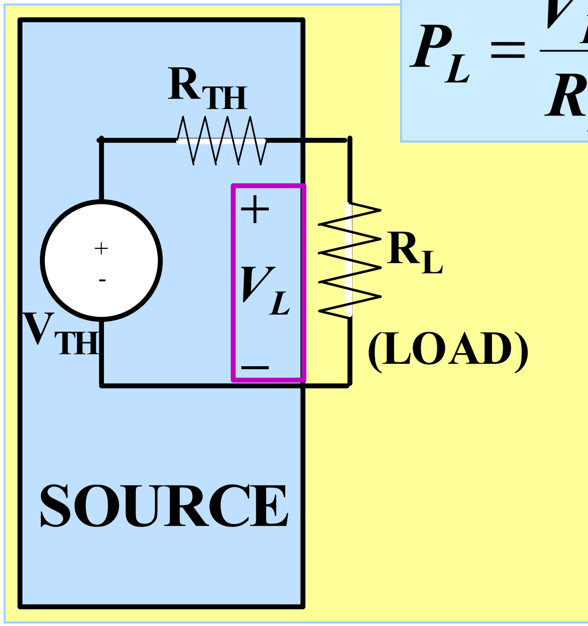
# MAXIMUM POWER TRANSFER




$$P_L = \frac{V_L^2}{R_L};\ V_L = \frac{R_L}{R_{TH}+R_L}V_{TH}$$

$$P_L = \frac{R_L}{(R_{TH}+R_L)^2}V_{TH}^2$$

**For every choice of R_L we have a different power. How do we find the maximum value?**

**Consider P_L as a function of R_L and find the maximum of such function**

$$\frac{dP_L}{dR_L} = V_{TH}^2\left(\frac{(R_{TH}+R_L)^2 - 2R_L(R_{TH}+R_L)}{(R_{TH}+R_L)^4}\right)$$

**3**

**Set the derivative to zero to find extreme points. For this case we need to set to zero the numerator**

$$R_{TH} + R_L - 2R_L = 0 \Rightarrow R_L^* = R_{TH}$$

**The maximum power transfer theorem**

**Technically we need to verify that it is indeed a maximum**

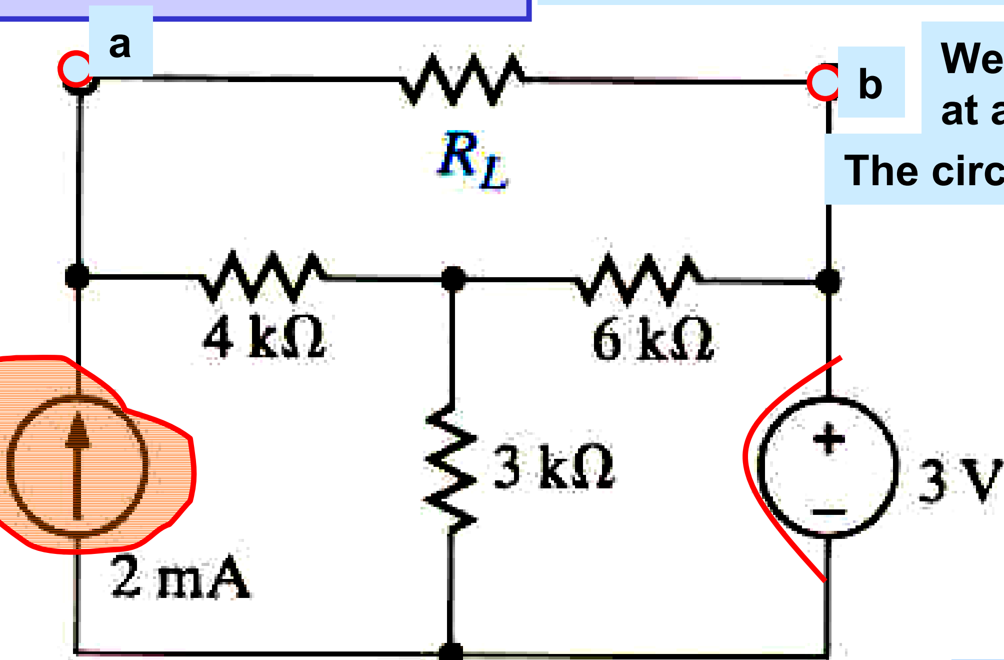
**The load that maximizes the power transfer for a circuit is equal to the Thevenin equivalent resistance of the circuit.**

**The value of the maximum power that can be transferred is**

$$P_L(\max) = \frac{V_{TH}^2}{4R_{TH}}$$

**ONLY IN THIS CASE WE NEED TO COMPUTE THE THEVENIN VOLTAGE**

**LEARNING EXAMPLE** DETERMINE $R_L$ FOR MAXIMUM POWER TRANSFER



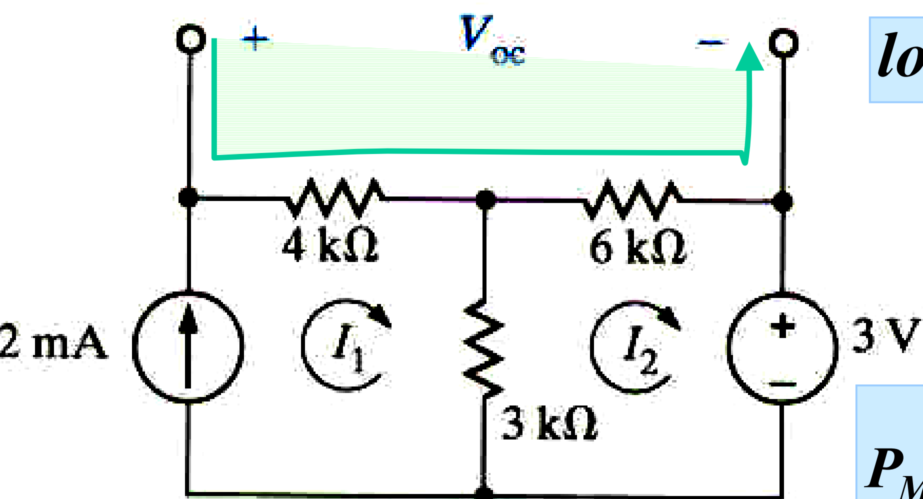

We need to find the Thevenin resistance at a - b.

The circuit contains only independent sources ....

$$R_{TH} = 4k + || 3k, 6k || = 6k$$

Resistance for maximum power transfer

If we MUST find the value of the power that can be transferred THEN we need the Thevenin voltage!!!




$$loop1 : I_1 = 2mA$$

$$loop2 : 3k * (I_2 - I_1) + 6k * I_2 + 3[V] = 0$$

$$I_2 = -\frac{3[V]}{9k} + \frac{1}{3} I_1 = \frac{1}{3}[mA]$$

$$KVL : V_{OC} = 4k * I_1 + 6k * I_2 = 10[V]$$

$$P_{MX} = \frac{V_{TH}^2}{4R_{TH}} \Rightarrow P_{MX} = \frac{100[V^2]}{4 * 6k} = \frac{25}{6}[mW]$$

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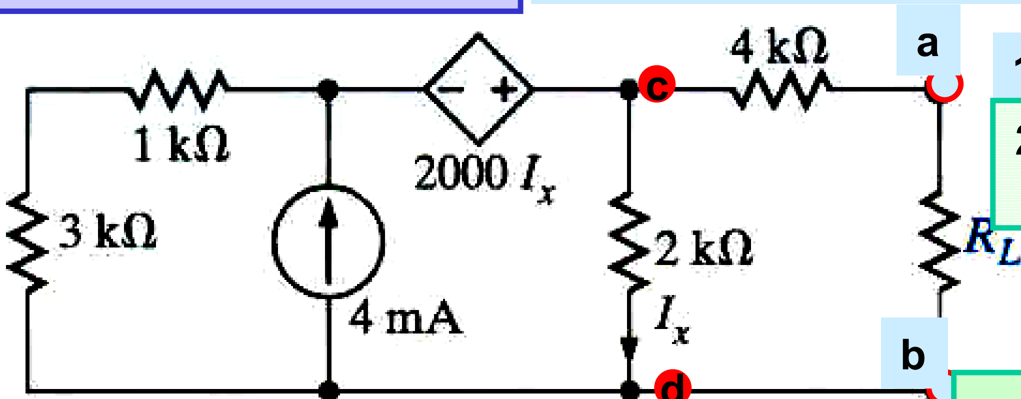
**LEARNING EXAMPLE**

## DETERMINE $R_L$ AND MAXIMUM POWER TRANSFERRED

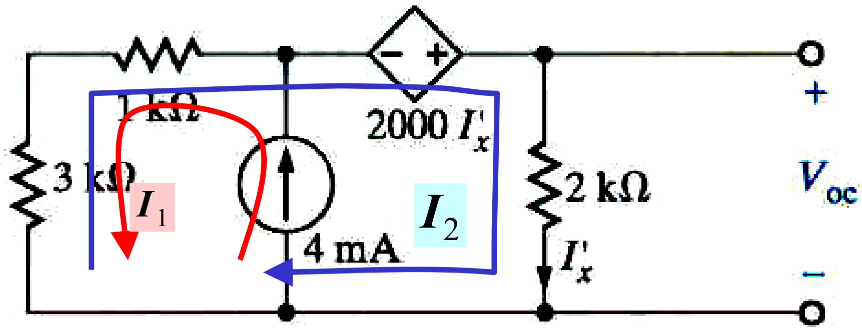



1. Find the Thevenin equivalent at a - b

2. Remember that for maximum power transfer $R_L = R_{TH}$ $P_{MX} = \frac{V_{TH}^2}{4R_{TH}}$

This is a mixed sources problem

.... And it is simpler if we do Thevenin at c - d and account for the 4k at the end




$loop1: I_1 = 4mA$

$loop2: -2kI'_X + 2kI_2 + 4k(I_2 - I_1) = 0$

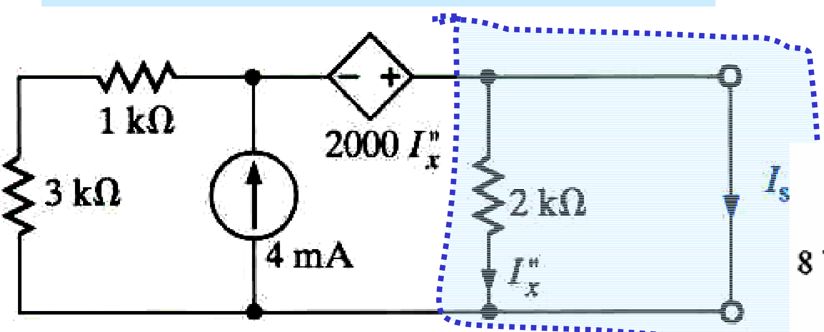
Controlling variable: $I'_X = I_2$

$I_2 = I_1 = 4mA \Rightarrow V_{OC} = 8[V]$

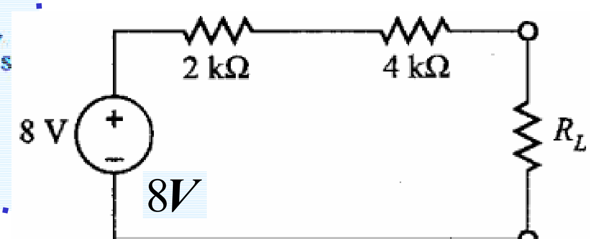
$R_{TH} = 2k$

Now the short circuit current

$I''_X = 0 \Rightarrow I_{SC} = 4mA$




Remember now where the partition was made




$R_L = 6k$

$P_{MX} = \frac{8^2}{4*6}[mW] = \frac{8}{3}[mW]$

STOP