Chapter Seven:

First- and Second-Order Transient Circuits

7.1 Use the differential equation approach to find $v_o(t)$ for $t > 0$ in the circuit in Fig. P7.1 and plot the response including the time interval just prior to switch action.

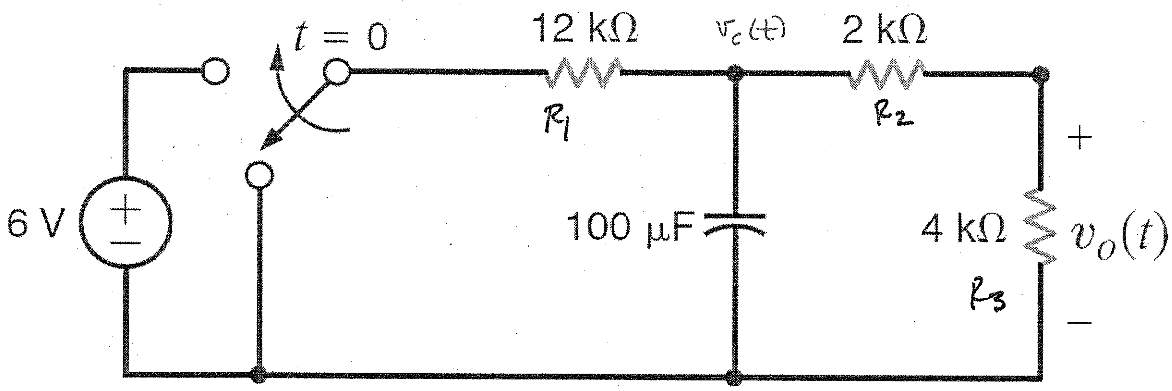


Figure P7.1

SOLUTION:

$v_c(0^-) = 0V$ for $t>0$: $\frac{6-v_c}{R_1} + \frac{v_o - v_c}{R_2} - C\frac{dv_c}{dt} = 0$ $\quad v_o = \frac{v_c R_3}{R_2+R_3} = \alpha v_c$

Multiply by $\alpha \Rightarrow$ $\frac{6\alpha}{R_1} + \frac{v_o}{R_2}\left[\alpha - 1\right] - \frac{v_o}{R_1} - \frac{C\,dv_o}{dt} = 0$

$\frac{dv_o}{dt} + v_o\left[\frac{1}{R_1C} + \frac{1-\alpha}{R_2C}\right] - \frac{6\alpha}{R_1C} = 0$ $\quad$ let $\frac{1}{R_1C} + \frac{1-\alpha}{R_2C} = B$

assume $v_o(t) = K_1 + K_2 e^{-t/\tau}$

$-\frac{K_2}{\tau}e^{-t/\tau} + K_1B + K_2Be^{-t/\tau} - \frac{6\alpha}{R_1C} = 0$

$$\left\{\begin{aligned} \tau &= 1/B = C\left[\frac{R_1(R_2+R_3)}{R_1+R_2+R_3}\right] \\ K_1 &= \frac{6\alpha}{R_1BC} = \frac{6R_3}{R_1+R_2+R_3} \end{aligned}\right.$$

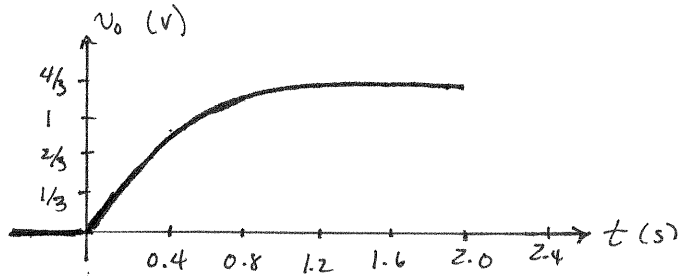
$\tau = 0.4s$ $\quad K_1 = 1.33V$

$v_o(0) = v_c(0)\alpha = 0 = K_1 + K_2 \Rightarrow K_2 = -1.33V$

$$\boxed{v_o(t) = 1.33 - 1.33e^{-2.5t}\ V}$$

$\underline{t = 0^+}$: $v_C(0^+) = 0$ $v_o(0^+) = \dfrac{6R_3}{R_1 + R_2 + R_3} = 1.33V$

$\underline{t = 0^-}$ $v_o(t) = 0$



7.2 Use the differential equation approach to find $v_C(t)$ for $t > 0$ in the circuit in Fig. P7.2 and plot the response including the time interval just prior to closing the switch.

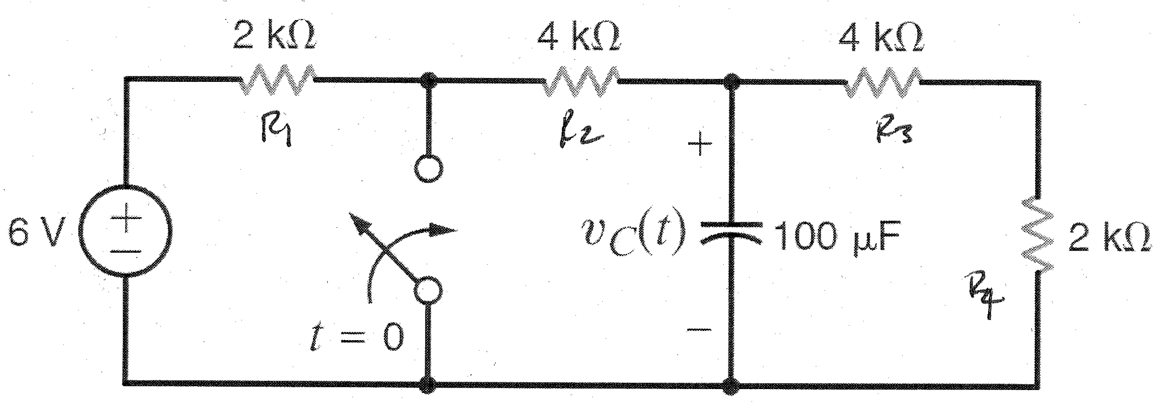


Figure P7.2

SOLUTION: $v_C(0^+) = v_C(0^-) = \dfrac{6\,(R_3+R_4)}{R_1+R_2+R_3+R_4} = 3V$

for $t>0$: $\dfrac{v_C}{R_2} + \dfrac{v_C}{R_3+R_4} + C\dfrac{dv_C}{dt} = 0$ let $v_C(t) = K_1 + K_2 e^{-t/\tau}$

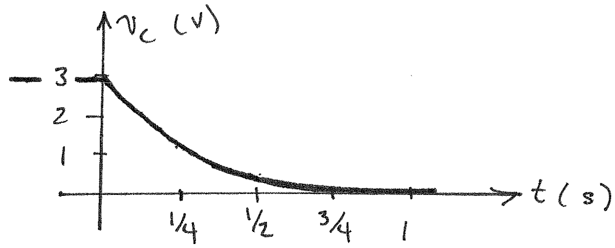
$$K_1\left(\frac{1}{R_2}+\frac{1}{R_3+R_4}\right) + K_2\left(\frac{1}{R_2}+\frac{1}{R_3+R_4}\right)e^{-t/\tau} - \frac{K_2 C}{\tau}e^{-t/\tau} = 0$$

yields $K_1 = 0$ $\tau = C\left\{\dfrac{R_2(R_3+R_4)}{R_2+R_3+R_4}\right\} = 0.24s$

$v_C(0^+) = 3 = K_1 + K_2 \Rightarrow K_2 = 3V$

$$\boxed{v_C(t) = 3e^{-t/0.24}\ V}$$

For $t > 0^-$: $v_C(0^-) = 3V$



7.3 Use the differential equation approach to find $v_C(t)$ for $t > 0$ in the circuit in Fig. P7.3. CS

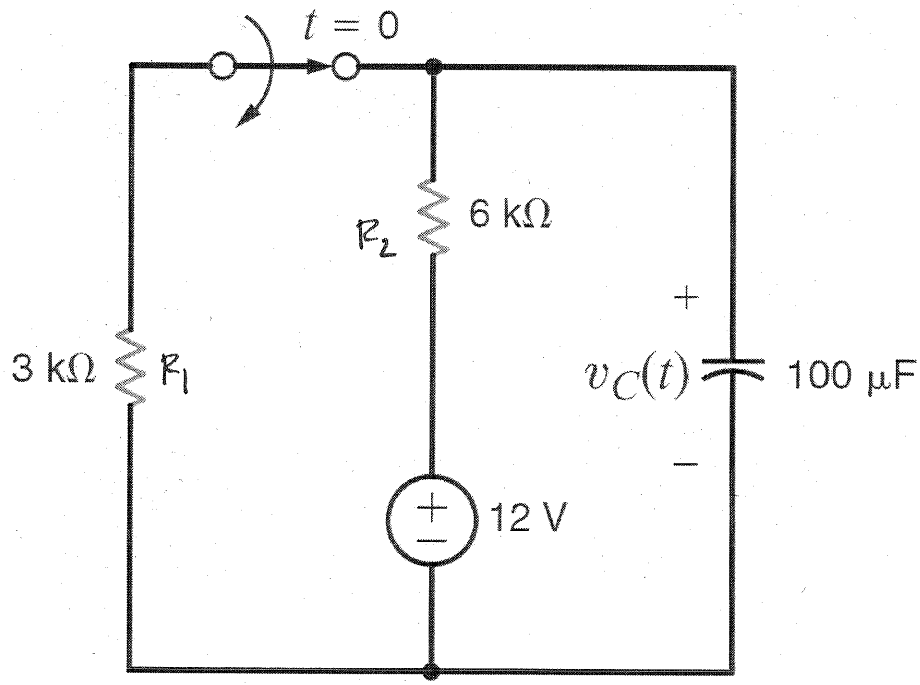


Figure P7.3

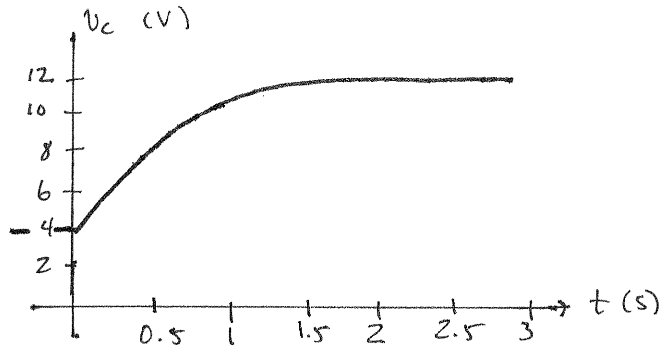
SOLUTION: $v_c(0^+) = v_c(0^-) = \frac{12(R_1)}{R_1+R_2} = 4V$ $\quad v_c(t) = K_1 + K_2 e^{-t/\tau}$

for $t>0$,

$$\frac{v_c - 12}{R_2} + C\frac{dv_c}{dt} = 0 \Rightarrow \frac{dv_c}{dt} + \frac{v_c}{R_2C} - \frac{12}{R_2C} = 0 = \frac{-K_2}{\tau}e^{-t/\tau} + \frac{K_1}{R_2C} + \frac{K_2}{R_2C}e^{-t/\tau} - \frac{12}{R_2C} = 0$$

yields: $\tau = R_2C = 0.6s$ $\quad K_1 = 12$ $\quad v_c(0^+) = 4 = K_1 + K_2 \Rightarrow K_2 = -8V$

$$\boxed{v_c(t) = 12 - 8e^{-\frac{t}{0.6}}\ V}$$



v_c (V)
12
10
8
6
4
2
0.5
1
1.5
2
2.5
3
t (s)

7.4 Use the differential equation approach to find $v_C(t)$ for $t > 0$ in the circuit in Fig. P7.4.

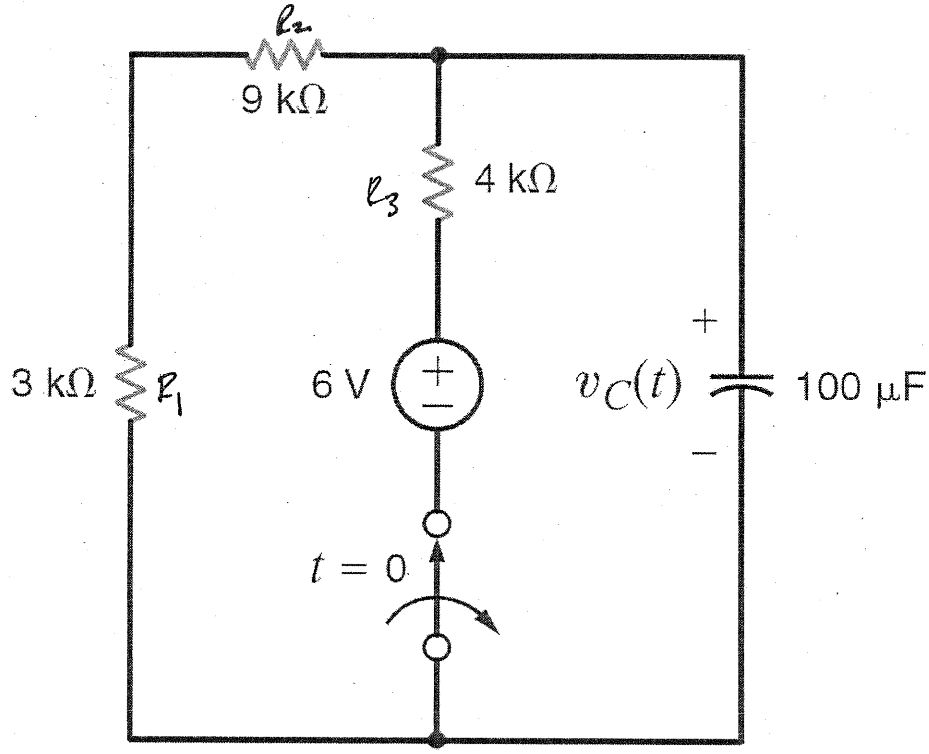


Figure P7.4

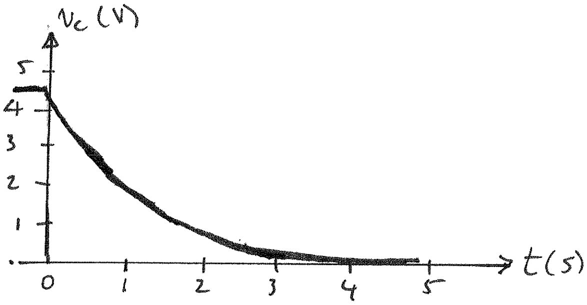
SOLUTION: $v_c(0^+) = v_c(0^-) = \dfrac{6\,(R_1+R_2)}{R_1+R_2+R_3} = 4.5\text{ V}$ $\quad v_c(t) = K_1 + K_2 e^{-t/\tau}$

for $t>0$: $\quad \dfrac{C\,dv_c(t)}{dt} + \dfrac{v_c(t)}{R_1+R_2} = 0 \Rightarrow \dfrac{dv_c}{dt} + \dfrac{v_c}{C(R_1+R_2)} = 0 = \dfrac{dv_c}{dt} + \dfrac{v_c}{\tau}$

$\tau = C\,(R_1+R_2) = 1.2\text{s}$ $\quad -\dfrac{K_2}{\tau} + \dfrac{K_1}{\tau} + \dfrac{K_2}{\tau} = 0 \Rightarrow K_1 = 0$

$v_c(0^+) = 4.5 = K_1 + K_2 \Rightarrow K_2 = 4.5$

$$\boxed{v_c(t) = 4.5e^{-t/1.2}\ \text{V}}$$



v_c (V)
5
4
3
2
1
0
1
2
3
4
5
t(s)

7.5 Use the differential equation approach to find $v_C(t)$ for $t > 0$ in the circuit in Fig. P7.5 and plot the response including the time interval just prior to opening the switch. CS

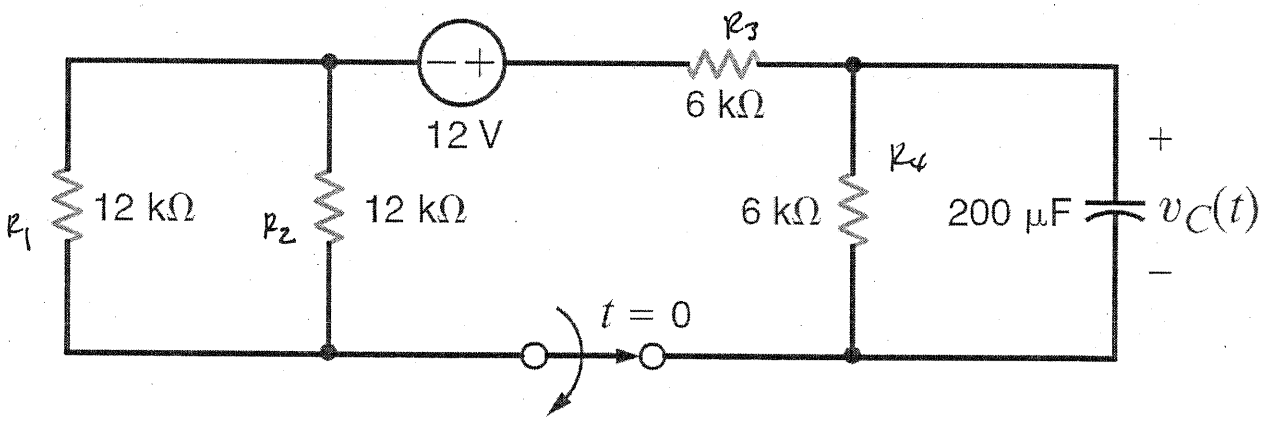


Figure P7.5

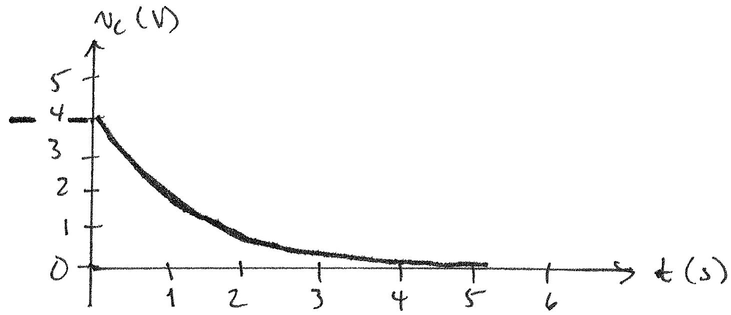
SOLUTION: $v_C(0-) = v_C(0+) = \dfrac{12\, R_4}{R_3 + R_4 + R_A}$ $\quad R_A = R_1 // R_2 = 6\,k\Omega$ $\quad v_C(0^+) = 4V$

for $t > 0$; $\quad \dfrac{v_C}{R_4} + C\dfrac{dv_C}{dt} = 0 \Rightarrow \dfrac{dv_C}{dt} + \dfrac{v_C}{R_4 C} = 0$

$v_C(t) = K_1 + K_2 e^{-t/\tau} \quad \Rightarrow \quad \dfrac{dv_C}{dt} + \dfrac{v_C}{\tau} = 0$

$\tau = R_4 C \qquad K_1 = 0 \qquad v_C(0^+) = 4 = K_1 + K_2 \Rightarrow K_2 = 4V$

$$\boxed{v_C(t) = 4e^{-t/1.2}\ V}$$



v_c (V)
5
4
3
2
1
0
1
2
3
4
5
6
t (s)

7.6 Use the differential equation approach to find $v_o(t)$ for $t > 0$ in the circuit in Fig. P7.6 and plot the response including the time interval just prior to opening the switch. **CS**

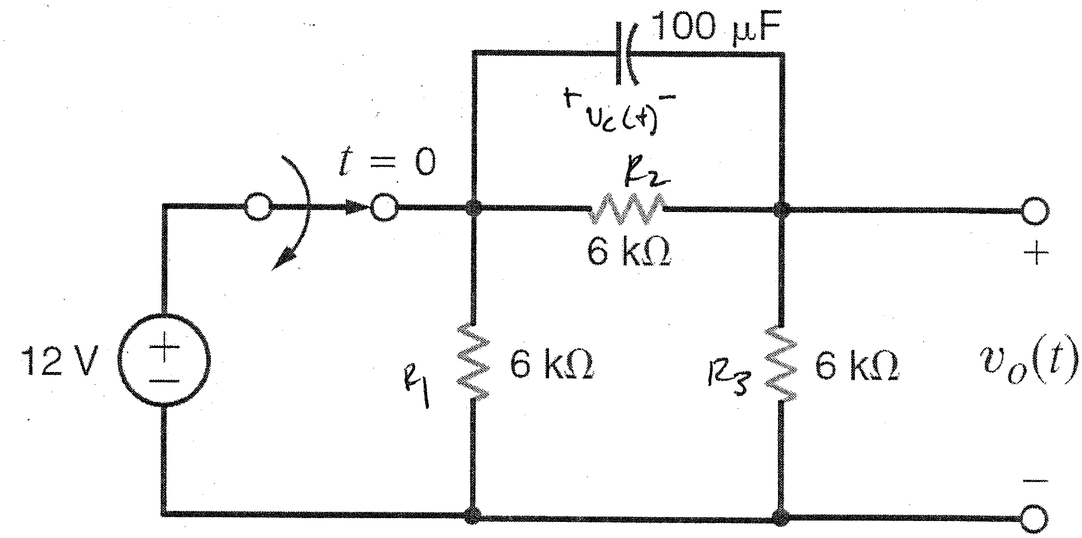


Figure P7.6

SOLUTION: $v_c(0^+) = v_c(0^-) = \frac{12\, R_2}{R_2 + R_3} = 6V$ $\quad v_o(t) = K_1 + K_2 e^{-t/\tau}$

for $t > 0$: $\quad v_o = \frac{v_c\,(-R_3)}{R_1 + R_3} = -\alpha v_c \quad \& \quad \frac{v_c}{R_2} + C\frac{dv_c}{dt} = \frac{v_o}{R_3}$.

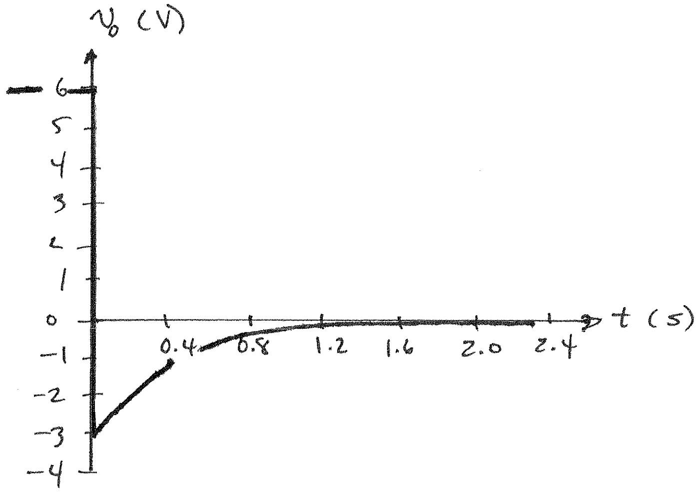
Eliminate v_c: $\quad \frac{dv_o}{dt} + v_o\left[\frac{1}{R_2 C} + \frac{\alpha}{R_3 C}\right] = 0 = \frac{dv_o}{dt} + \frac{v_o}{\tau}$

$\tau = C\left[\frac{R_2\,(R_1 + R_3)}{R_2 + R_1 + R_3}\right] = 0.4s \qquad K_1 = 0$

$v_o(0^+) = v_c(0^+)\,\alpha = -3V = K_1 + K_2 \Rightarrow K_2 = -3V$

$$\boxed{v_o(t) = -3e^{-2.5t} \text{ V}}$$

$\underline{t = 0^-}$: $v_c(0^-) = 6V$ $v_o(0^-) = 12 - v_c = 6V$



7.7 Use the differential equation approach to find $i_o(t)$ for $t > 0$ in the circuit in Fig. P7.7 and plot the response including the time interval just prior to closing the switch.

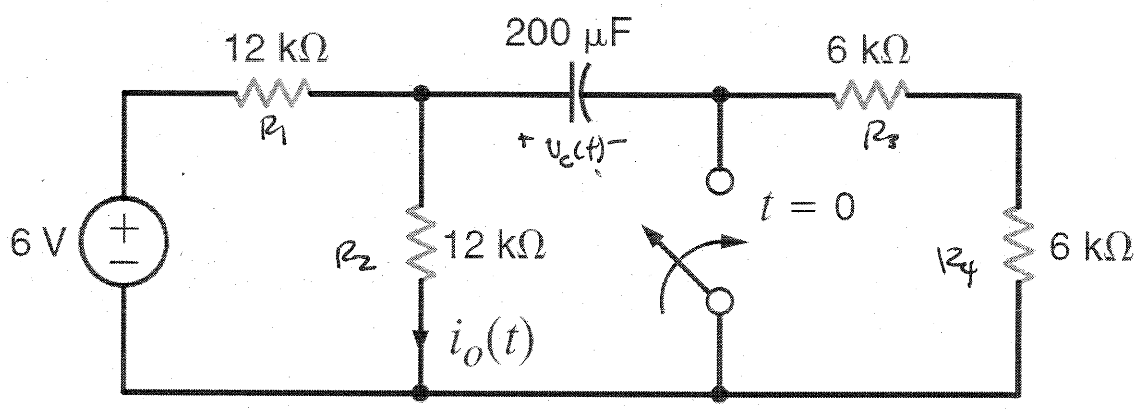


Figure P7.7

SOLUTION: $v_c(0^-) = v_c(0^+) = \dfrac{6R_2}{R_1+R_2} = 3\text{V}$ $\quad i_o(t) = \dfrac{v_c(t)}{R_2}$ for $t>0$.

for $t>0$: $\quad \dfrac{6-v_c}{R_1} = \dfrac{v_c}{R_2} + C\dfrac{dv_c}{dt} \Rightarrow \dfrac{dv_c}{dt} + v_c\left[\dfrac{1}{R_1C}+\dfrac{1}{R_2C}\right] - \dfrac{6}{R_1C} = 0$

Convert to i_o: $\quad \dfrac{di_o}{dt} + i_o\left[\dfrac{1}{R_1C}+\dfrac{1}{R_2C}\right] - \dfrac{6}{R_1R_2C} = 0$

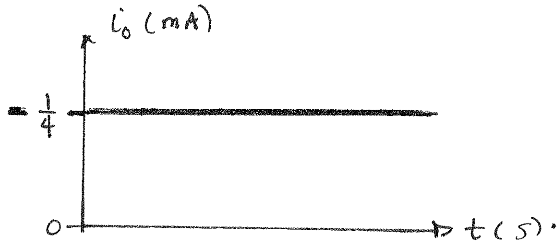
$i_o(t) = K_1 + K_2e^{-t/\tau} \Rightarrow -\dfrac{K_2}{\tau}e^{-t/\tau} + (K_1+K_2e^{-t/\tau})\left[\dfrac{1}{R_1C}+\dfrac{1}{R_2C}\right] - \dfrac{6}{R_1R_2C} = 0$

yields $\quad \tau = C\dfrac{R_1R_2}{R_1+R_2} = 1.25 \quad K_1 = \dfrac{6}{R_1+R_2} = 0.25\text{mA}$

$i_o(0^+) = K_1+K_2 = v_c(0^+)/R_2 = 0.25\text{mA} \Rightarrow K_2 = 0$

$$\boxed{i_o(t) = 0.25\text{mA}}$$

$\underline{t=0^-}$: $v_c(0^-) = 3V$ $i_c(0^-) = 0$ $i_o(0^-) = \dfrac{6}{R_1+R_2} = 0.25mA$



7.8 Use the differential equation approach to find $v_o(t)$ for $t > 0$ in the circuit in Fig. P7.8 and plot the response including the time interval just prior to closing the switch.
CS

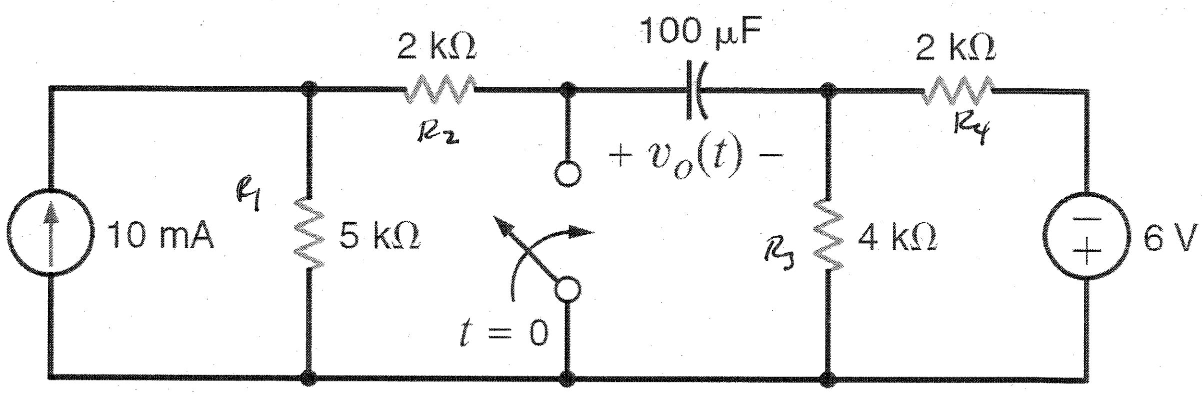
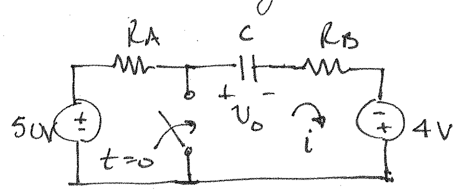


Figure P7.8

SOLUTION: Using source transformation:



$R_A = R_1 + R_2 = 7\,k\Omega \qquad R_B = R_3 // R_4 = 1.33\,k\Omega$

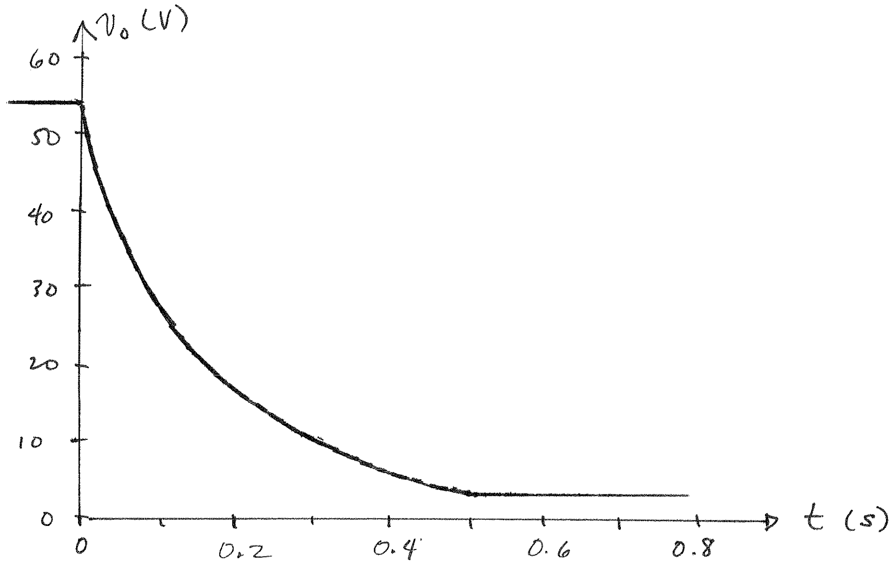
$v_o(0^+) = v_o(0^-) = 54V$

for $t>0$: $\quad 4 = v_o + iR_B \;\; \& \;\; i = C\,dv_o/dt \Rightarrow \dfrac{dv_o}{dt} + \dfrac{v_o}{R_BC} - \dfrac{4}{R_BC} = 0$

$v_o = K_1 + K_2 e^{-t/\tau} \Rightarrow \tau = R_BC \;\; \& \;\; K_1 = 4 \qquad \tau = 0.133s$

$v_o(0^+) = 54 = K_1 + K_2 \Rightarrow K_2 = 50V$

$$\boxed{v_o = 4 + 50e^{-7.5t}\ V}$$


v_o (V)
60
50
40
30
20
10
0
0
0.2
0.4
0.6
0.8
t (s)

7.9 Use the differential equation approach to find $v_C(t)$ for $t > 0$ in the circuit in Fig. P7.9 and plot the response including the time interval just prior to opening the switch.

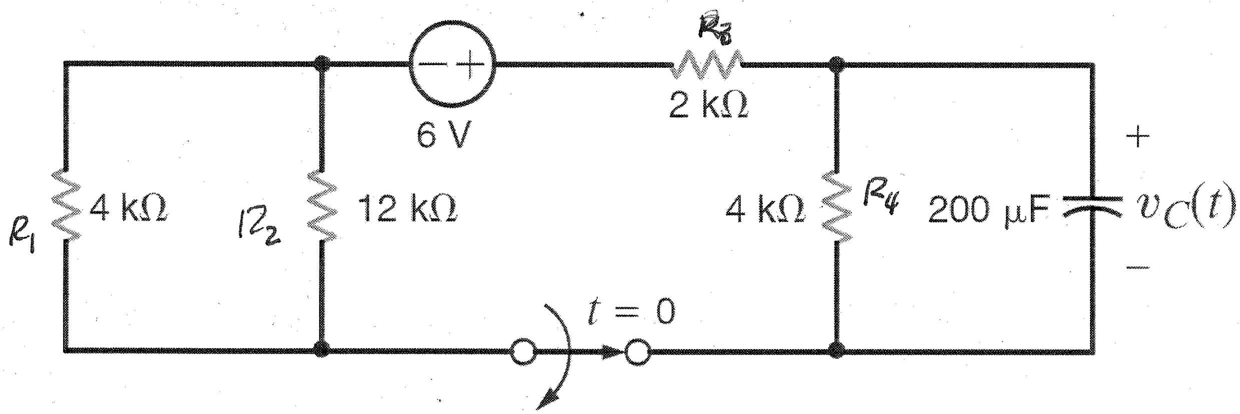
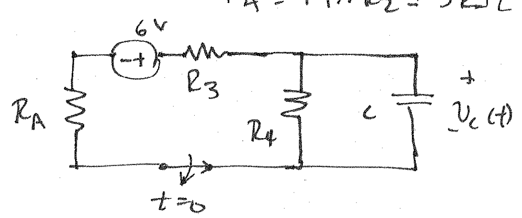


Figure P7.9

SOLUTION: $R_A = R_1 // R_2 = 3k\Omega$



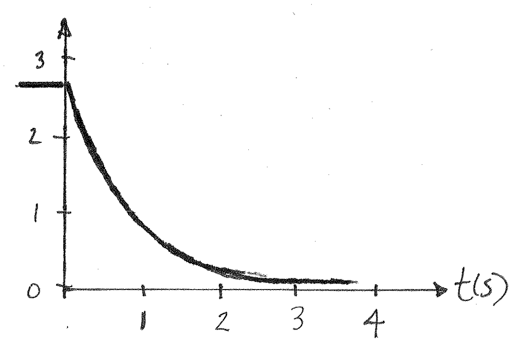
$$v_c(0^+) = v_c(0^-) = \frac{6R_4}{R_A + R_3 + R_4} = \frac{8}{3}\text{ V}$$

for $t > 0$,
$$\frac{v_c}{R_4} + C\frac{dv_c}{dt} = 0 \Rightarrow \frac{dv_c}{dt} + \frac{v_c}{R_4 C} = 0$$

$$v_c = K_1 + K_2 e^{-t/\tau} \Rightarrow \tau = R_4 C = 0.8\text{s} \qquad K_1 = 0$$

$$v_c(0^+) = \frac{8}{3} = K_1 + K_2 \Rightarrow K_2 = 2.67\text{V}$$

$$\boxed{v_c(t) = 2.67e^{-1.25t}\text{ V}}$$



7.10 Use the differential equation approach to find $i_o(t)$ for $t > 0$ in the circuit in Fig. P7.10 and plot the response including the time interval just prior to opening the switch. CS

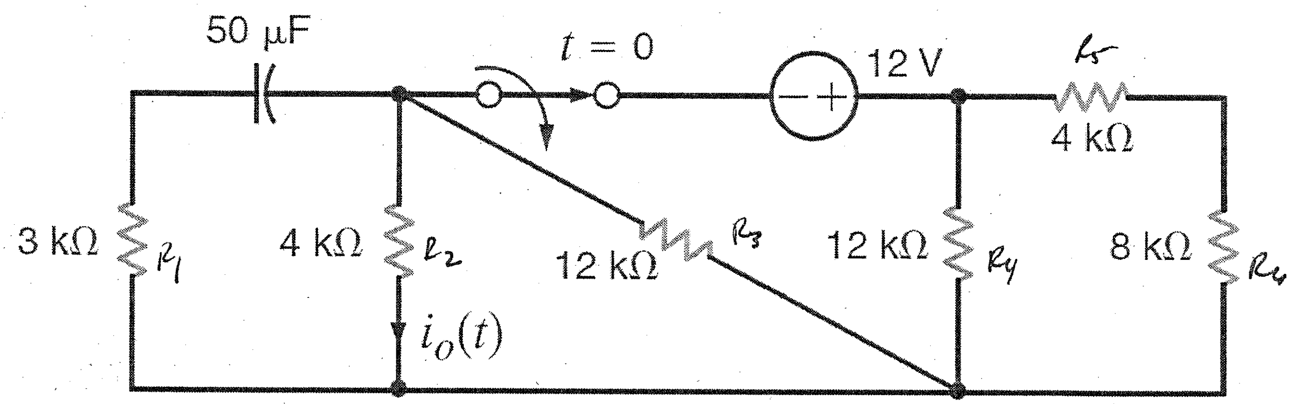
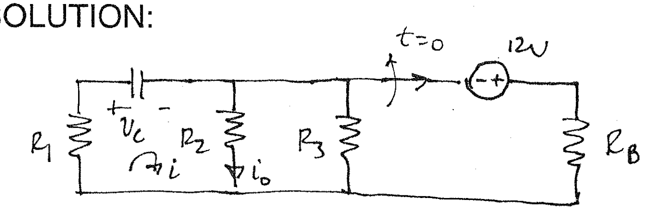


Figure P7.10

SOLUTION:



$R_B = R_4 // (R_5 + R_6) = 6\,k\Omega$

$R_A = R_2 // R_3 = 3\,k\Omega$

$v_c(0^+) = v_c(0^-) = \dfrac{12\,R_A}{R_A + R_B} = 4V$ $\qquad i_o(0^-) = \dfrac{-v_c(0^-)}{R_2} = -1mA$

For $t>0$, $v_c + i R_A + i R_1 = 0$ & $i = C\dfrac{dv_c}{dt}$ & $i_o = \dfrac{R_3}{R_2+R_3}\, i = \alpha i$

yields, $\dfrac{dv_c}{dt} + \dfrac{v_c}{C(R_1 + R_A)} = 0$

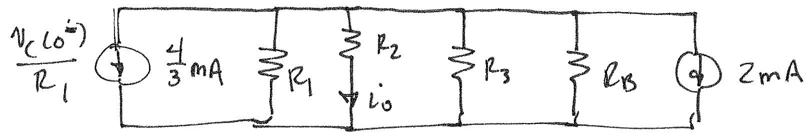
or, $\dfrac{di_o}{dt} + \dfrac{i_o}{C(R_1+R_A)} = 0$ $\qquad$ where $i_o = K_1 + K_2 e^{-t/\tau}$

yields, $\tau = C[R_1 + R_A] = 0.3s$ $\qquad K_1 = 0$

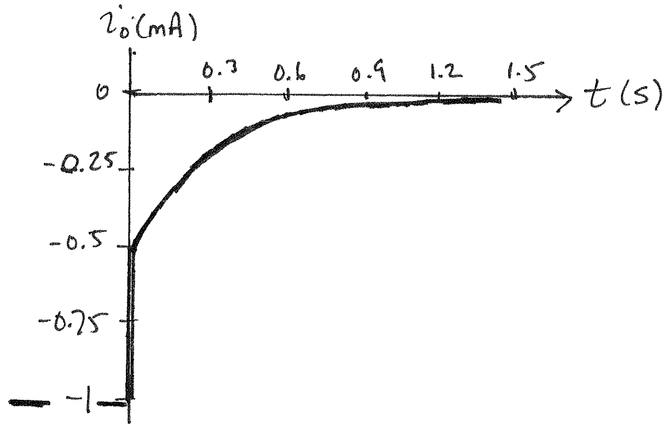
$i_o(0^+) = \dfrac{-v_c(0^+)}{R_1 + R_A}\;\dfrac{R_3}{R_3+R_2} = -0.5mA = K_1 + K_2 \Rightarrow K_2 = -0.5mA$

$$\boxed{i_o(t) = -0.5\,e^{-t/0.3}\ mA}$$

$\underline{t = 0^-}$:



$$i_o(0^-) = -\frac{(2 + 4/3)\times 10^{-3}\,(1/R_2)}{\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \frac{1}{R_B}} = -1\,\text{mA}$$



7.11 Use the differential equation approach to find $i_o(t)$ for $t > 0$ in the circuit in Fig. P7.11 and plot the response including the time interval just prior to opening the switch.

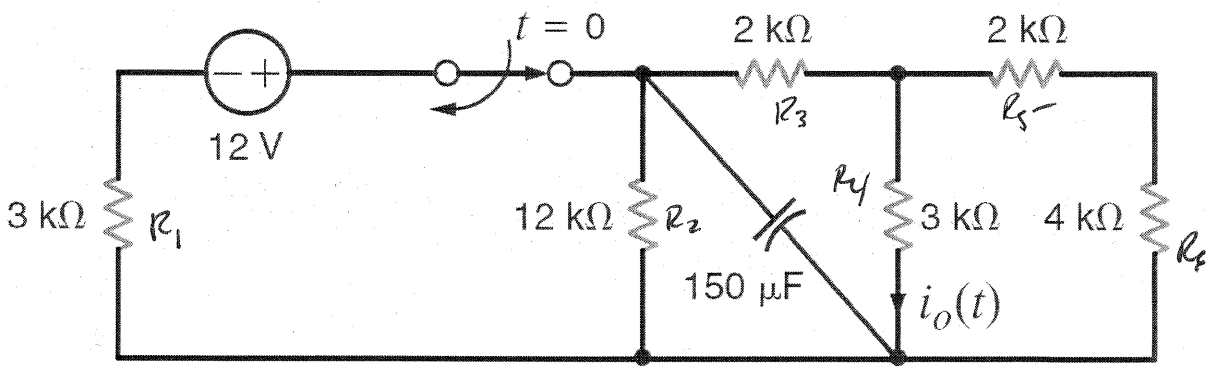
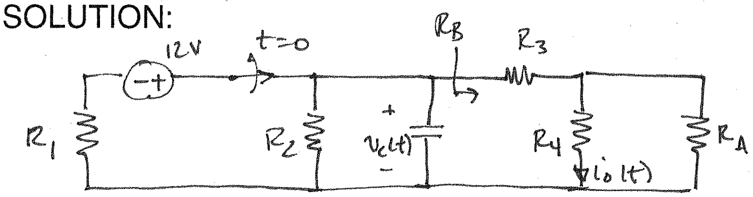


Figure P7.11

SOLUTION:



$R_A = R_5 + R_6 = 6k\Omega$

$R_B = R_3 + (R_4 // R_A) = 4k\Omega$

$R_c = R_2 // R_B = 3k\Omega$

$v_c(0^+) = v_c(0^-) = \frac{12 R_c}{R_1 + R_c} = 6V$

$i_o(t) = \frac{v_c(t)}{R_B} \frac{R_A}{R_A + R_4} = \frac{v_c(t)}{6000}$ $\quad i_o(0^-) = i_o(0^+) = 1mA$

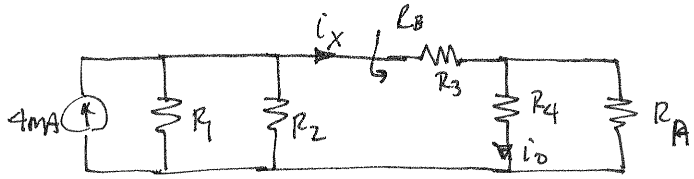
for $t > 0$, $\frac{v_c}{R_c} + C\frac{dv_c}{dt} = 0 \Rightarrow \frac{di_o}{dt} + \frac{i_o}{CR_c} = 0$

$i_o = K_1 + K_2 e^{-t/\tau} \rightarrow \tau = CR_c = 0.45s$ $\quad K_1 = 0$

$K_1 + K_2 = i_o(0^+) = 1mA \Rightarrow K_2 = 1mA$

$$\boxed{i_o(t) = e^{-t/0.45} \ mA}$$

$t = 0^-$:

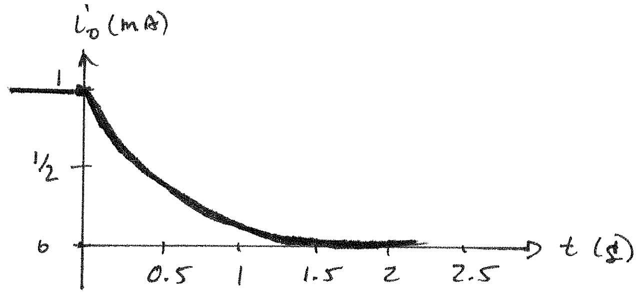


$R_B = 4k\Omega$

$R_A = 6k\Omega$

$$i_x = \frac{4\times10^{-3}\,(1/R_B)}{\frac{1}{R_B} + \frac{1}{R_1} + \frac{1}{R_2}} = 1.5\,mA$$

$$i_o = \frac{i_x R_A}{R_A + R_4} = 1\,mA$$



7.12 Use the differential equation approach to find $v_o(t)$ for $t > 0$ in the circuit in Fig. P7.12 and plot the response including the time interval just prior to opening the switch.

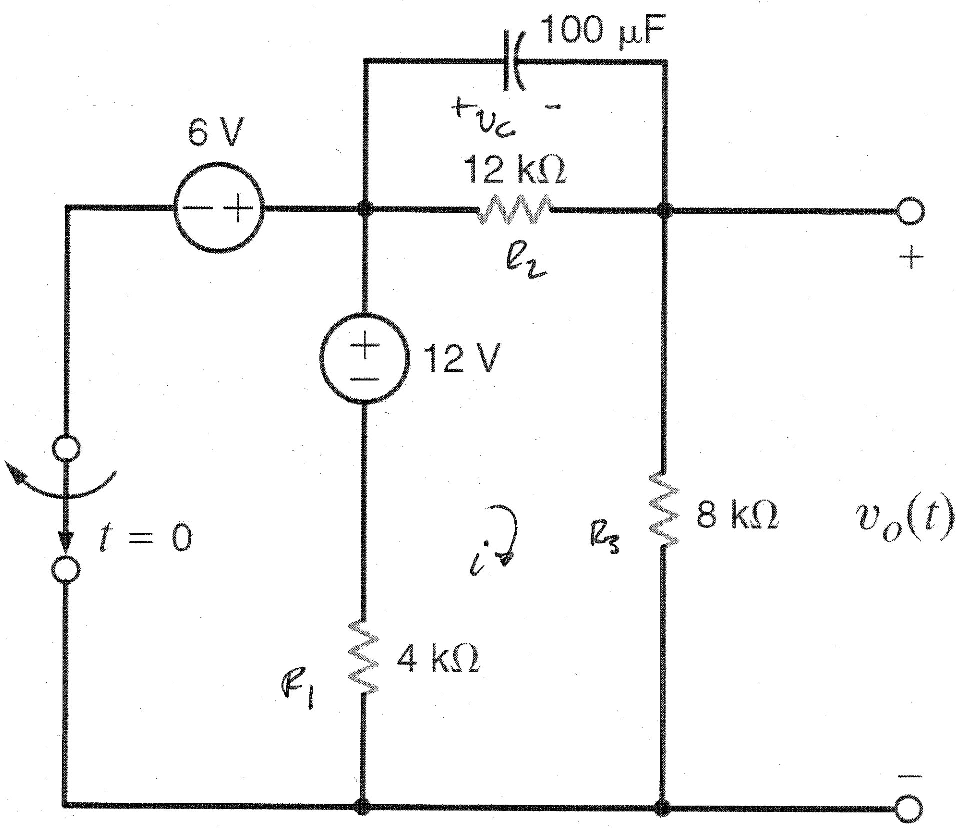


Figure P7.12

SOLUTION: $v_C(0^+) = v_C(0^-) = \dfrac{6R_2}{R_2+R_3} = 3.6\text{V}$ $\qquad v_o(0^-) = \dfrac{6R_3}{R_2+R_3} = 2.4\text{V}$

for $v_o(0^+)$: $\left[12 - v_C(0^+)\right]\dfrac{R_3}{R_1+R_3} = v_o(0^+) = 5.6\text{V}$

for $t>0$: $\qquad 12 = v_C(t) + i\,(R_1+R_3) \qquad v_o = i R_3$

and $\qquad C\dfrac{dv_C}{dt} + \dfrac{v_C}{R_2} = \dfrac{v_o}{R_3}$

yields $\qquad \dfrac{dv_o}{dt} + v_o\left[\dfrac{1}{R_2C} + \dfrac{\alpha}{R_3C}\right] - \dfrac{12\alpha}{R_2C} = 0 \qquad \alpha = \dfrac{R_3}{R_1+R_3}$

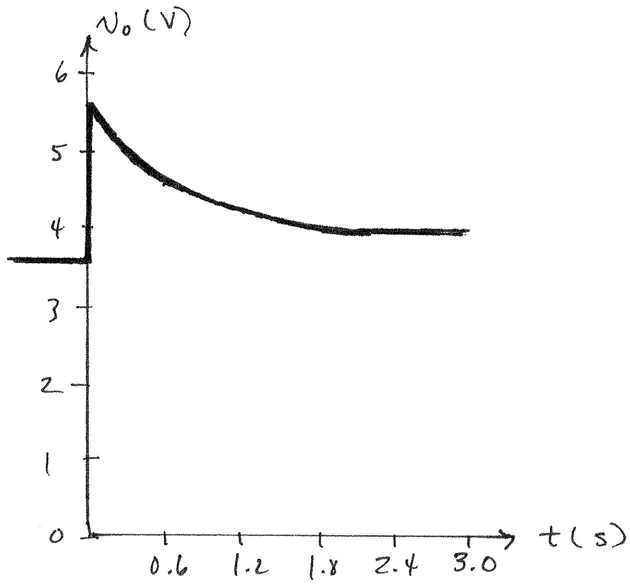
$$\tau = C\left[R_2 // (R_1+R_3)\right] = 0.6s$$

$$K_1 = \frac{12 R_3}{R_1+R_2+R_3} = 4V$$

$$v_o(0^+) = K_1 + K_2 = 5.6V \Rightarrow K_2 = 1.6V$$

$$\boxed{v_o(t) = 4 + 1.6e^{-t/0.6} \quad V}$$

$\underline{t=0^-}$: $\quad v_o(0^-) = 6 - v_c(0^-) = 3.6V$



7.13 Use the differential equation approach to find $v_o(t)$ for $t > 0$ in the circuit in Fig. P7.13 and plot the response including the time interval just prior to opening the switch.

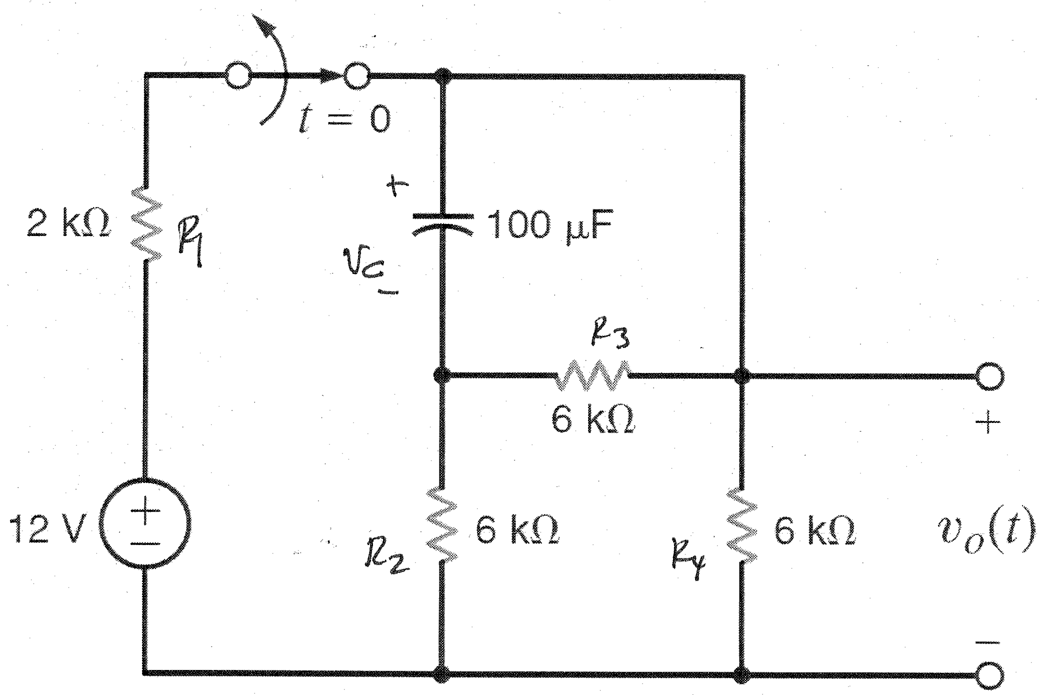
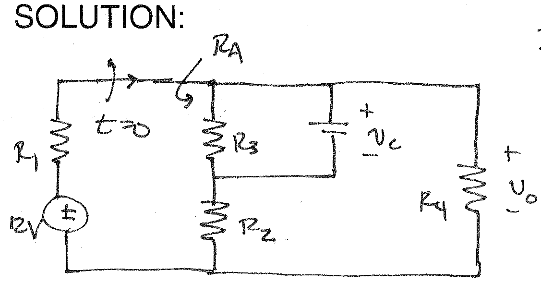


Figure P7.13

SOLUTION:



t < 0

$R_A = R_4 \,//\, (R_3+R_2) = 4k\Omega$

$V_o(0^-) = \dfrac{12 R_A}{R_A+R_1} = 8V$

$V_c(0^-) = V_c(0^+) = \dfrac{V_o(0) R_3}{R_2+R_3} = 4V$

t = 0⁺

$V_o(0^+) = \dfrac{V_c(0^+) R_4}{R_2+R_4} = 2 = K_1+K_2$

t > 0

$C\dfrac{dv_c}{dt} + \dfrac{v_c}{R_3} + \dfrac{v_o}{R_4} = 0$ and $v_o = \dfrac{v_c R_4}{R_2+R_4} = \alpha v_c$

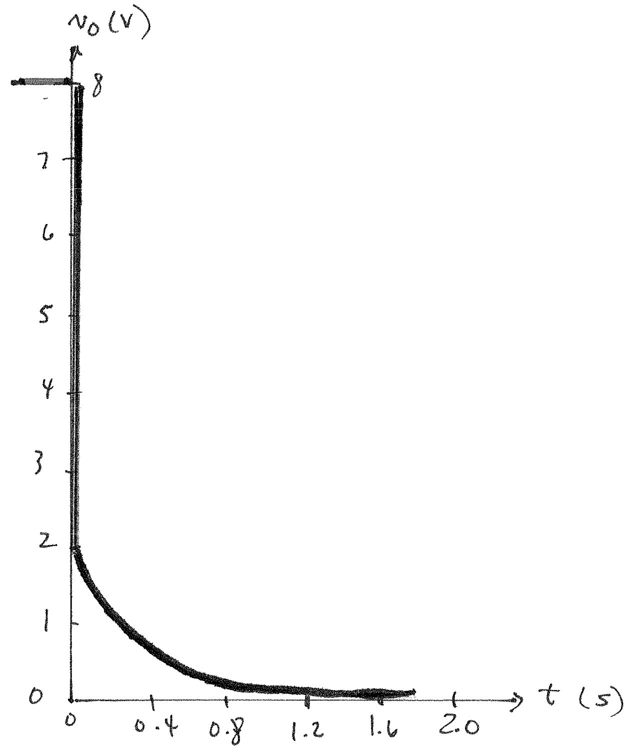
yields $\dfrac{dv_o}{dt} + v_o\left[\dfrac{1}{CR_3} + \dfrac{\alpha}{CR_4}\right] = 0$

$\tau = C\left[R_3 // (R_2 + R_4\right] = 0.4s$ $K_1 = 0$

$v_o(0+) = 2 = K_1 + K_2 \Rightarrow k_2 = 2$

$$\boxed{v_o(t) = 2e^{-2.5t} \text{ V}}$$

$\underline{t = 0^-}$: $v_o(0^-) = 8V$



7.14 Use the differential equation approach to find $v_o(t)$ for $t > 0$ in the circuit in Fig. P7.14 and plot the response including the time interval just prior to closing the switch. **PSV**

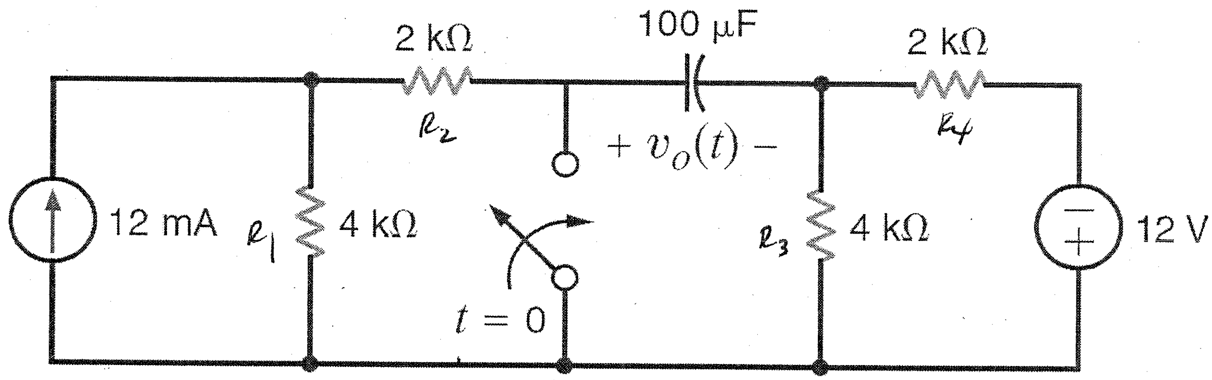
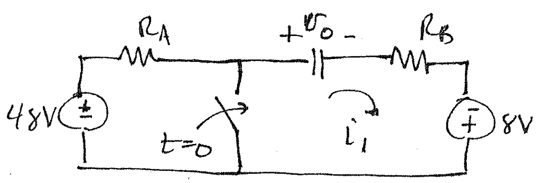


Figure P7.14

SOLUTION: By source transformation:



$R_A = R_1 + R_2 = 6 k\Omega$

$R_B = R_3 // R_4 = \frac{4}{3} k\Omega$

$\underline{t<0}$ $\quad v_o(0^-) = v_o(0^+) = 56V$ $\quad$ Note: $v_o(t) = v_c(t)$

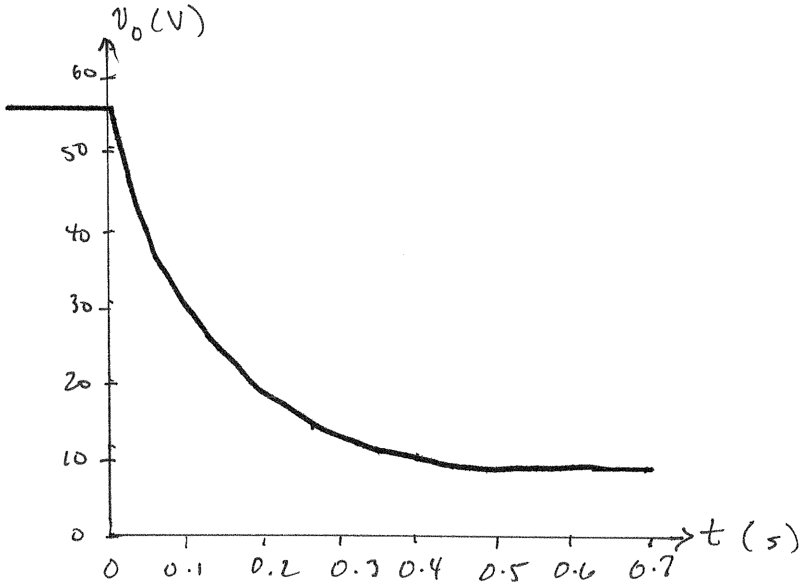
$\underline{t=0^+}$ $\quad v_o(0^+) = 56V$

$\underline{t>0}$ $\quad 8 = v_o + i_1 R_B$ & $i_1 = C\frac{dv_c}{dt} \Rightarrow \frac{dv_o}{dt} + \frac{v_o}{R_B C} - \frac{8}{R_B C} = 0$

$v_o = K_1 + K_2 e^{-t/\tau} \rightarrow \tau = R_B C = 0.133 s$ $\quad K_1 = 8V$

$K_2 = v_o(0^+) - K_1 = 48V$

$$\boxed{v_o(t) = 8 + 48 e^{-7.5t} V}$$



v_0 (V)
60
50
40
30
20
10
0
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
t (s)

7.15 Use the differential equation approach to find $i(t)$ for $t > 0$ in the network in Fig. P7.15.

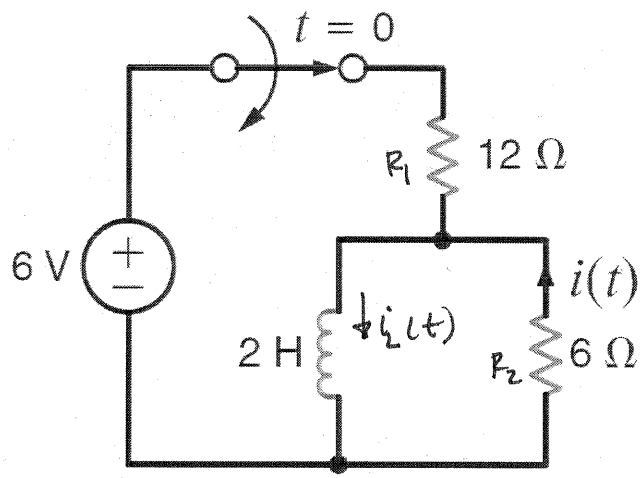


Figure P7.15

SOLUTION: $\underline{t=0^-}$: $i_L(0^-) = 6/R_1 = 0.5\text{ A} = i_L(0^+)$

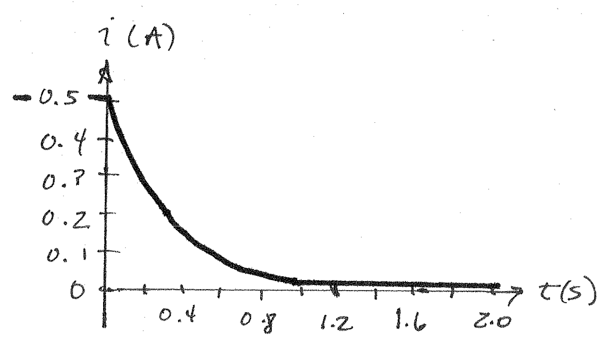
$\underline{t=0^+}$: $i(0^+) = i_L(0^+) = 0.5\text{ A} = K_1 + K_2$

$\underline{t>0}$ $\quad L\dfrac{di}{dt} + R_2 i = 0 \Rightarrow \dfrac{di}{dt} + \left(\dfrac{R_2}{L}\right) i = 0$

$i = K_1 + K_2 e^{-t/\tau} \Rightarrow \tau = \dfrac{L}{R_2} = \dfrac{1}{3}\text{ s} \qquad K_1 = 0$

$K_2 = i(0^+) - K_1 = 0.5\text{A}$

$$\boxed{i(t) = 0.5e^{-3t}\text{ A}}$$



7.16 Use the differential equation approach to find $i(t)$ for $t > 0$ in the circuit in Fig. P7.16 and plot the response including the time interval just prior to switch movement.

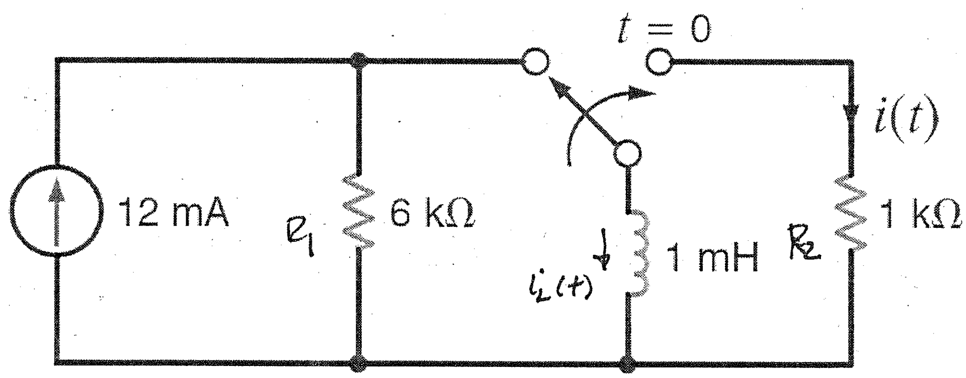


Figure P7.16

SOLUTION:

$t=0^-$ $\quad i_L(0^-) = 12mA = i_L(0^+)$

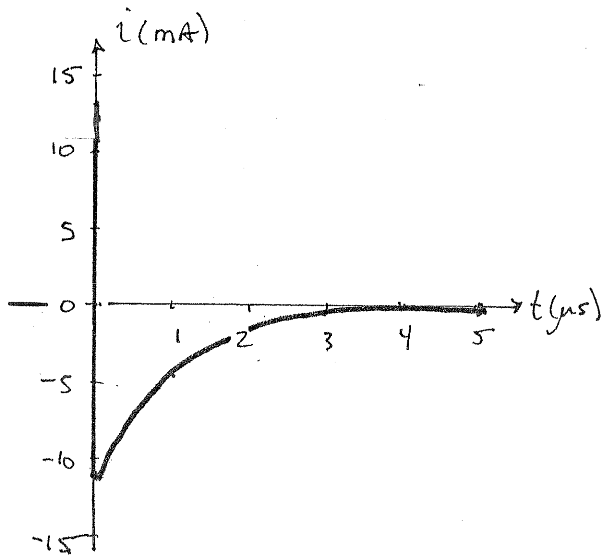
$t=0^+$ $\quad i = -i_L = -12mA$

$t>0$ $\quad L\dfrac{di_L}{dt} = R_2 i$ & $i = -i_L \Rightarrow \dfrac{di}{dt} + \left(\dfrac{R_2}{L}\right) i = 0$

$i = K_1 + K_2 e^{-t/\tau} \Rightarrow \tau = \dfrac{L}{R_2} = 1\mu s \quad K_1 = 0$

$K_2 = i(0^+) - K_1 = -12mA$

$$\boxed{i(t) = -12e^{-10^6 t}\ mA}$$



7.17 In the circuit in Fig. 7.17, find $i_o(t)$ for $t > 0$ using the differential equation approach.

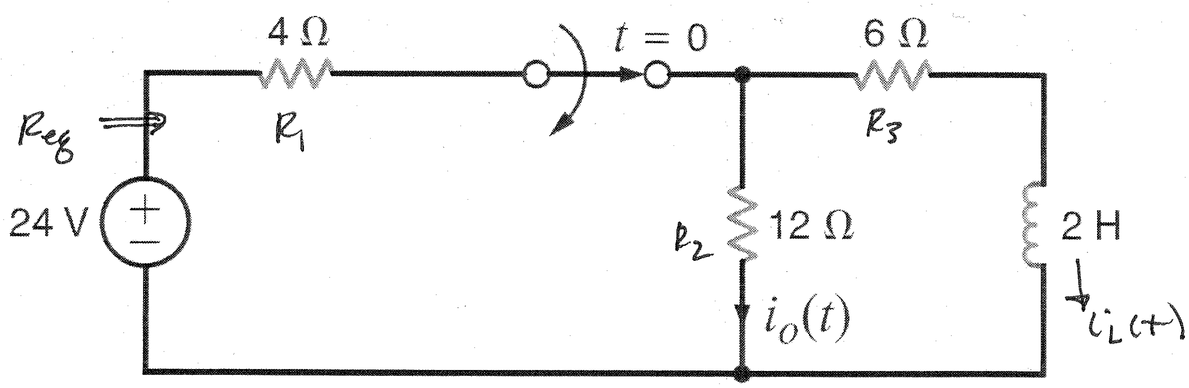


Figure P7.17

SOLUTION:

$\underline{t=0^-}$ $\quad i_o + i_L = \frac{24}{R_{EQ}} = 3 \qquad R_{EQ} = R_1 + [R_2 /\!/ R_3] = 8\Omega$

$\frac{i_o}{i_L} = \frac{R_3}{R_2} = \frac{1}{2} \Rightarrow i_L(0^-) = 2A = i_L(0^+)$

$\underline{t=0^+}$ $\quad i_L(0^+) = 2A = -i_o(0^+)$

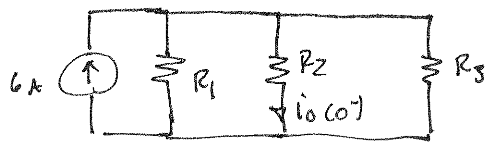
$\underline{t>0}$ $\quad L\frac{di_L}{dt} = i_o(R_2+R_3) \quad + \; i_o = -i_L \Rightarrow \frac{di_o}{dt} + \frac{i_o(R_2+R_3)}{L} = 0$

$i_o = K_1 + K_2 e^{-t/\tau} \Rightarrow \tau = \frac{L}{R_2+R_3} = \frac{1}{9}s \qquad K_1 = 0$

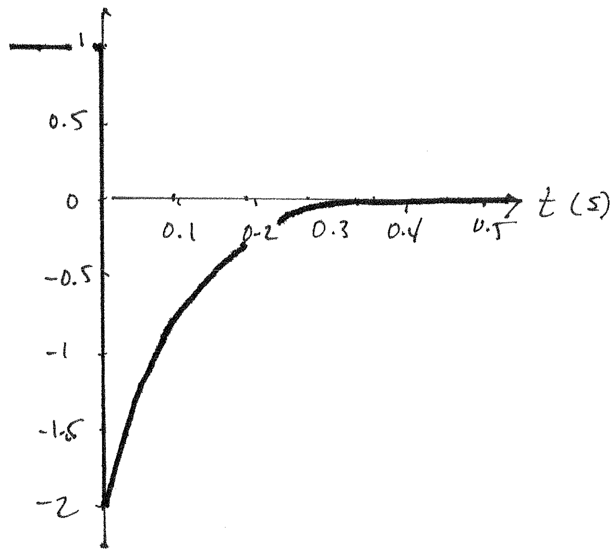
$K_2 = i_o(0^+) - K_1 = -2A$

$$\boxed{i_o(t) = -2e^{-9t} \; A}$$

$\underline{t=0^-}$: $i_L(0^-) = 2A$



$$i_o(0^-) = \frac{6\left(\frac{1}{R_2}\right)}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} = 1A$$



7.18 Use the differential equation approach to find $i(t)$ for $t > 0$ in the circuit in Fig. P7.18 and plot the response including the time interval just prior to opening the switch.

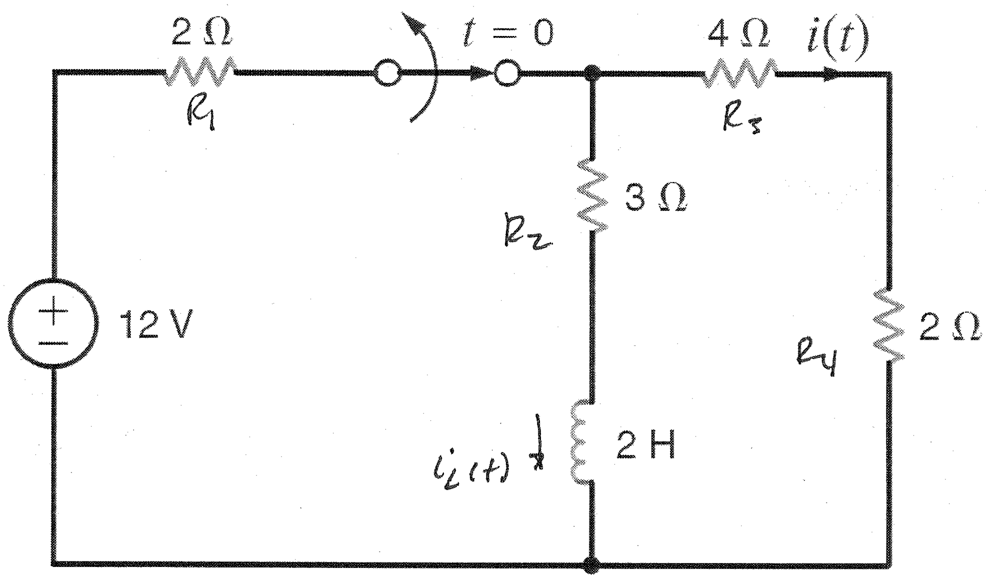
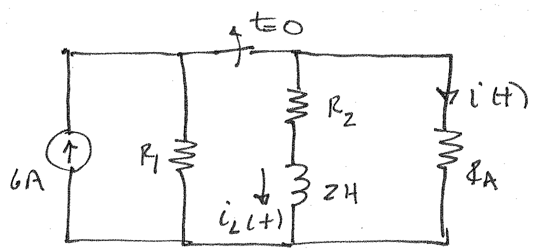


Figure P7.18

SOLUTION: By source transformation:



$R_A = R_3 + R_4 = 6\Omega$

$\underline{t=0^-}$: $i_L = \dfrac{6\left(1/R_2\right)}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_A}} = 2A$

$i(0^-) = \dfrac{6\left(1/R_A\right)}{\frac{1}{R_A}+\frac{1}{R_1}+\frac{1}{R_2}} = 1A$

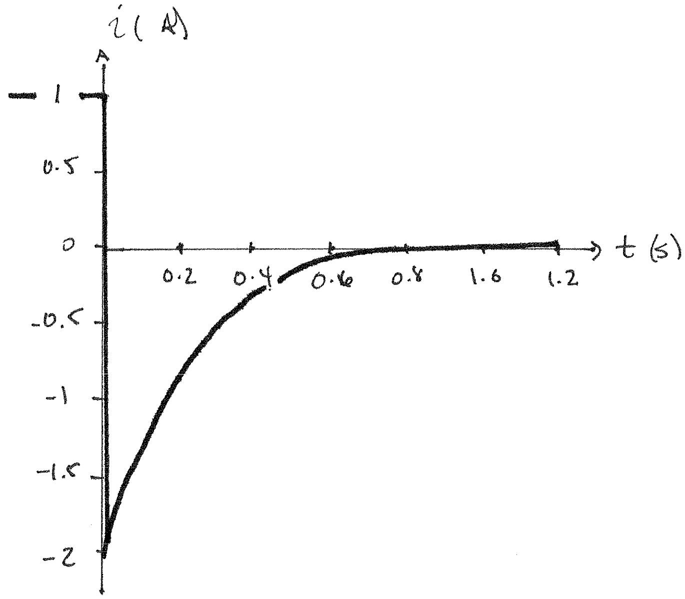
$\underline{t=0^+}$ $i_L(0^+) = i_L(0^-) = 2A$

$i(0^+) = -i_L(0^+) = -2A$

$\underline{t>0}$: $L\left(di_L/dt\right) = i\left(R_A+R_2\right)$ & $i = -i_L \Rightarrow \dfrac{di}{dt} + \left(\dfrac{R_A+R_2}{L}\right)i = 0$

$i = K_1 + K_2 e^{-t/\tau}$

yields $\tau = \dfrac{L}{R_A+R_2} = \dfrac{2}{9}s$ $K_1 = 0$ $K_2 = -2$ $\boxed{i = -2e^{-4.5t} A}$


i(A)
1
0.5
0
-0.5
-1
-1.5
-2
0.2
0.4
0.6
0.8
1.6
1.2
t (s)

7.19 In the network in Fig. 7.19, find $i_o(t)$ for $t > 0$ using the differential equation approach. CS

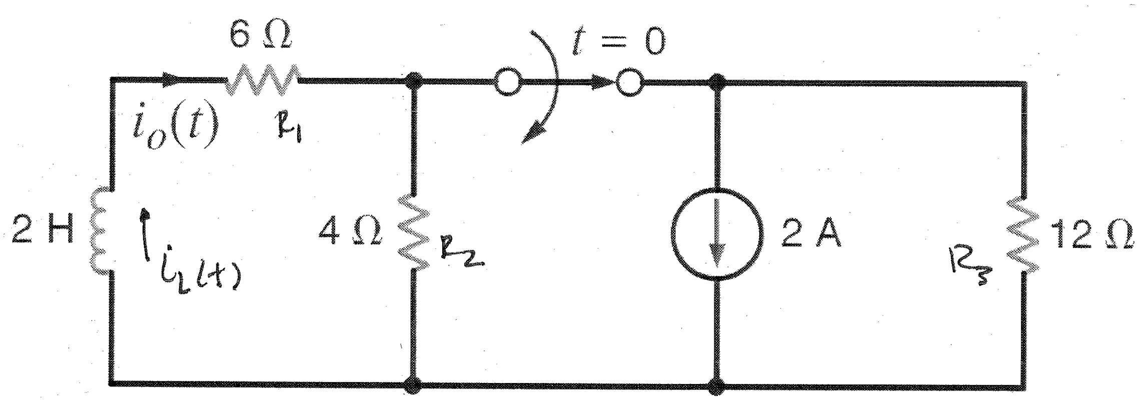


Figure P7.19

SOLUTION: $\underline{t=0^-}$: $i_L(0^-) = \dfrac{2\,(1/R_1)}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} = \dfrac{2}{3}$ A

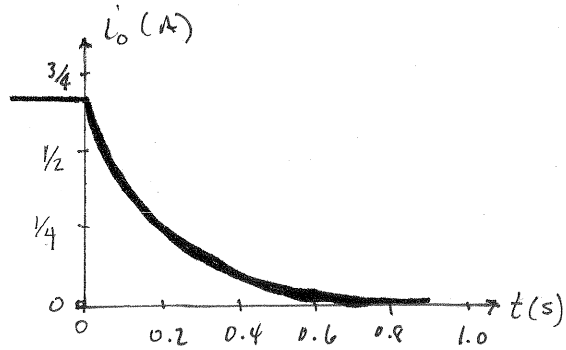
$\underline{t=0^+}$ $i_o = i_L = \frac{2}{3}$ A

$\underline{t>0}$ $L\dfrac{di_L}{dt} + i_o(R_1+R_2) = 0$ & $i_L = i_o \Rightarrow \dfrac{di_o}{dt} + \dfrac{(R_1+R_2)}{L}i_o = 0$

$i_o = K_1 + K_2 e^{-t/\tau} \Rightarrow \tau = \dfrac{L}{R_1+R_2} = \dfrac{1}{5}$ s $\quad K_1 = 0$

$K_2 = i_o(0^+) - K_1 = \frac{2}{3}$ A

$$\boxed{i_o(t) = 0.67e^{-5t} \text{ A}}$$



7.20 Use the differential equation approach to find $i(t)$ for $t > 0$ in the circuit in Fig. P7.20 and plot the response including the time interval just prior to switch movement. **PSV**

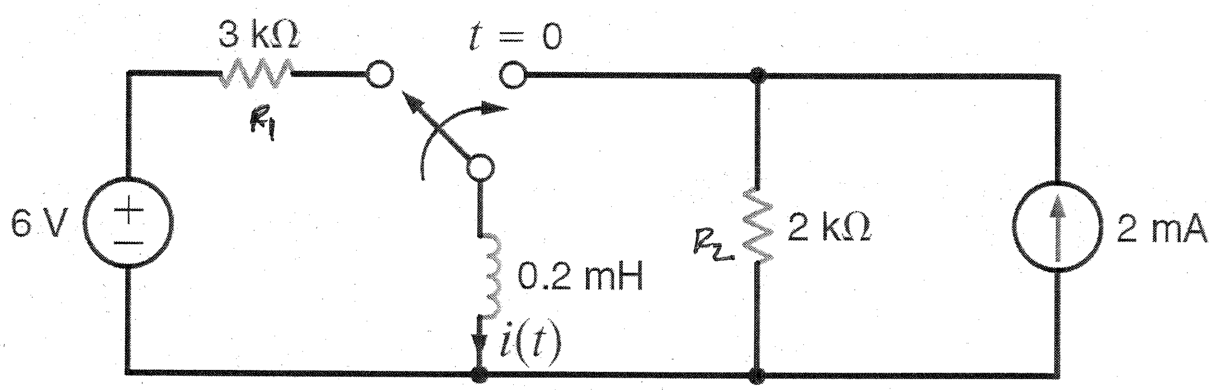
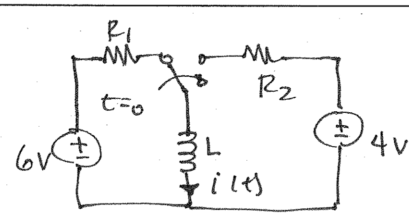


Figure P7.20

SOLUTION:



$\underline{t=0^-}$: $i(0^-) = \frac{6}{R_1} = 2mA = i(0^+)$

$\underline{t=0^+}$ $i(0^+) = 2mA$

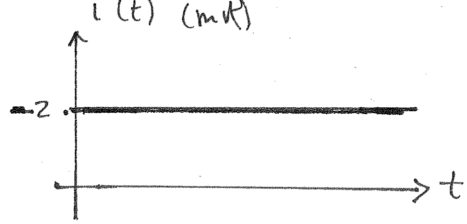
$\underline{t>0}$ $4 = R_2 i + L\, di/dt$

$di/dt + \frac{R_2}{L} i - \frac{4}{L} = 0$ $i = K_1 + K_2 e^{-t/\tau}$

$\tau = L/R_2 = 0.1 \mu s$ $K_1 = 4/R_2 = 2mA$

$K_2 = i(0^+) - K_1 = 0$

$\boxed{i(t) = 2mA}$



7.21 Use the differential equation approach to find $i_L(t)$ for $t > 0$ in the circuit in Fig. P7.21 and plot the response including the time interval just prior to opening the switch.

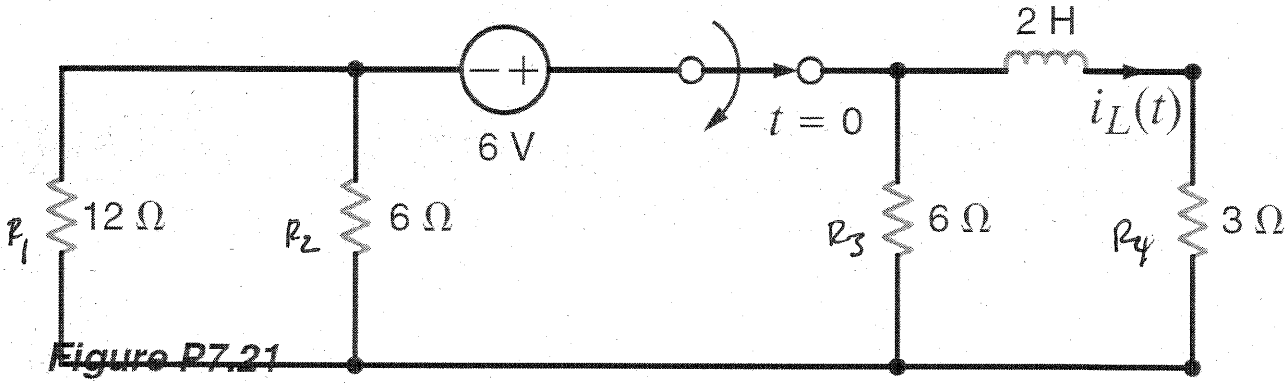
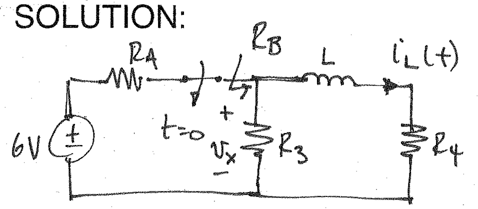


Figure P7.21

SOLUTION:



$R_A = R_1 // R_2 = 4\Omega$

$R_B = R_3 // R_4 = 2\Omega$

$\underline{t=0^-}$: $i_L(0^-) = v_x(0^-)/R_4$ $\quad v_x = \frac{6 R_B}{R_B + R_A} = 2V$ $\quad i_L(0^-) = \frac{2}{3}A$

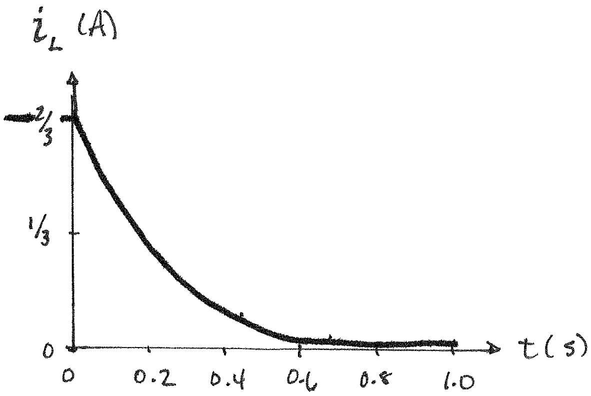
$\underline{t=0^+}$ $i_L(0^+) = i_L(0^-) = 2/3\,A$

$\underline{t>0}$: $L\frac{di_L}{dt} + i_L(R_3 + R_4) = 0 \Rightarrow \frac{di_L}{dt} + \left(\frac{R_3+R_4}{L}\right)i = 0$

$i = K_1 + K_2 e^{-t/\tau} \Rightarrow \tau = \frac{L}{R_3 + R_4} = \frac{2}{9}\,s \qquad K_1 = 0$

$K_2 = i_L(0^+) - K_1 = \frac{2}{3}A$

$$\boxed{i_L(t) = 0.67\,e^{-4.5t}\,A}$$



i_L (A)
2/3
1/3
0
0
0.2
0.4
0.6
0.8
1.0
t(s)

7.22 Use the differential equation approach to find $i_o(t)$ for $t > 0$ in the circuit in Fig. P7.22 and plot the response including the time interval just prior to opening the switch.

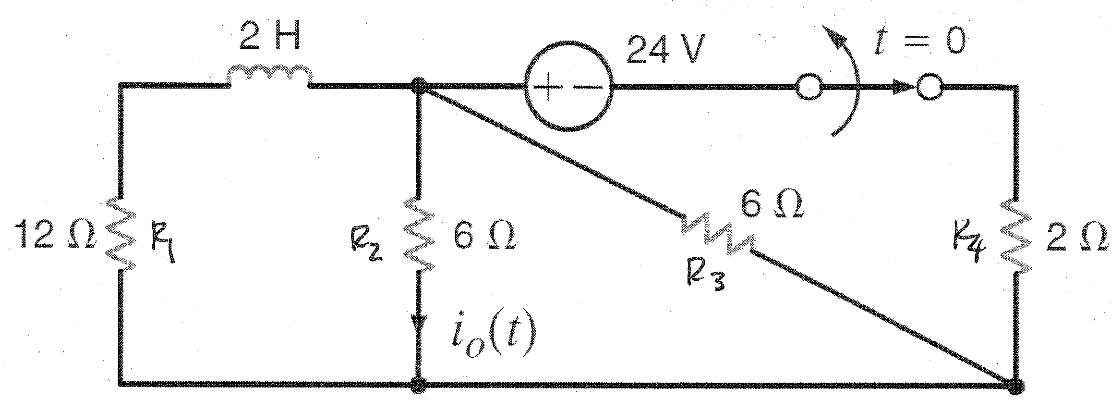
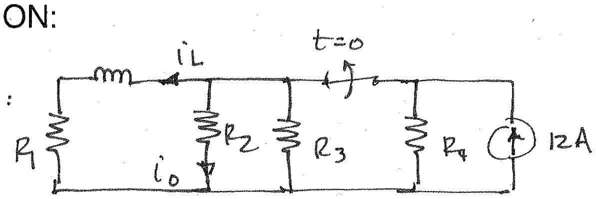


Figure P7.22

SOLUTION:

$t = 0^-$:



$$i_L(0^-) = \frac{12\,(1/R_1)}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}} = \frac{12}{11}\text{ A}$$

$$i_o(0^-) = \frac{12\text{A}\,(1/R_2)}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_4}} = \frac{24}{11}\text{ A}$$

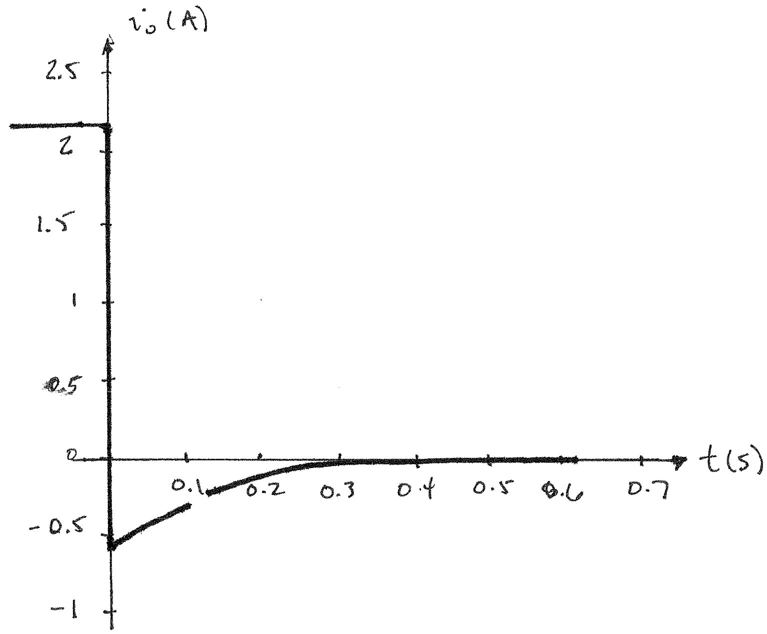
$t = 0^+$ $\quad i_L(0^+) = \frac{12}{11}\text{ A} \qquad i_o(0^+) = \frac{-i_L(0^+)R_3}{R_2+R_3} = -\frac{6}{11}\text{ A}$

$t > 0$: $\quad L\frac{di_L}{dt} + i_L(R_1+R_B) = 0 \qquad R_B = R_2 // R_3 \qquad i_o = \frac{-i_L R_3}{R_2+R_3}$

$$\frac{di_o}{dt} + \left(\frac{R_1+R_B}{L}\right) i_o = 0 \qquad \& \quad i_o(t) = K_1 + K_2 e^{-t/\tau}$$

$$\tau = \frac{L}{R_1+R_B} = \frac{2}{15}\text{ s} \qquad K_1 = 0 \qquad K_2 = i_o(0^+) - K_1 = -\frac{6}{11}\text{ A}$$

$$\boxed{i_o(t) = -0.545 e^{-7.5t}\text{ A}}$$



i_o (A)
2.5
2
1.5
1
0.5
0
-0.5
-1
0.1
0.2
0.3
0.4
0.5
0.6
0.7
t(s)

7.23 Using the differential equation approach, find $i_o(t)$ for $t > 0$ in the circuit in Fig. P7.23 and plot the response including the time interval just prior to opening the switch.

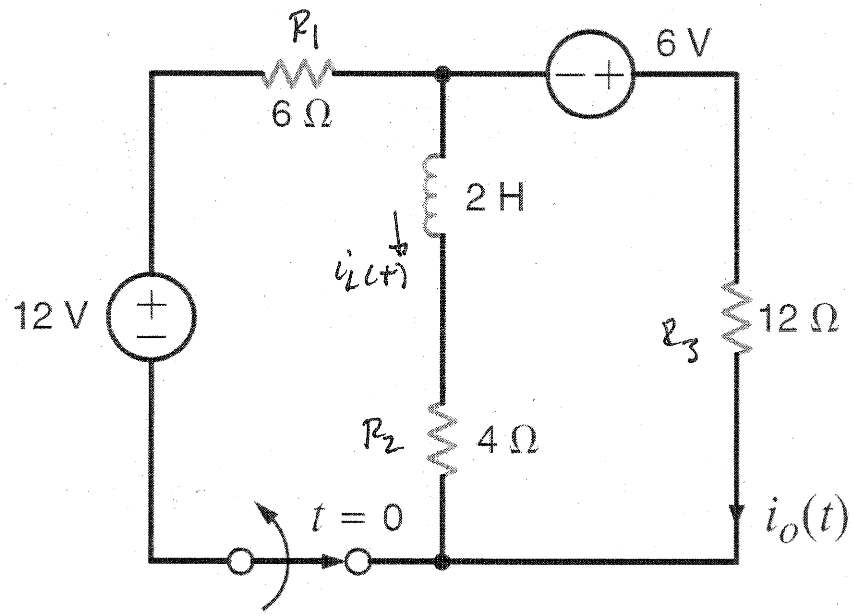
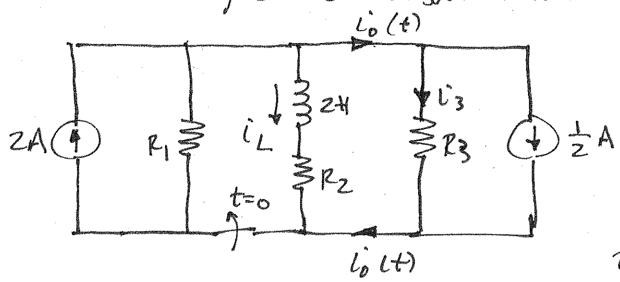


Figure P7.23

SOLUTION: By source transformation:



$\underline{t=0^-}$ $\quad i_L(0^-) = \dfrac{(2-0.5)\ 1/R_2}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} = \frac{3}{4}$ A

$i_o(0^-) = i_3(0^-) + 0.5$

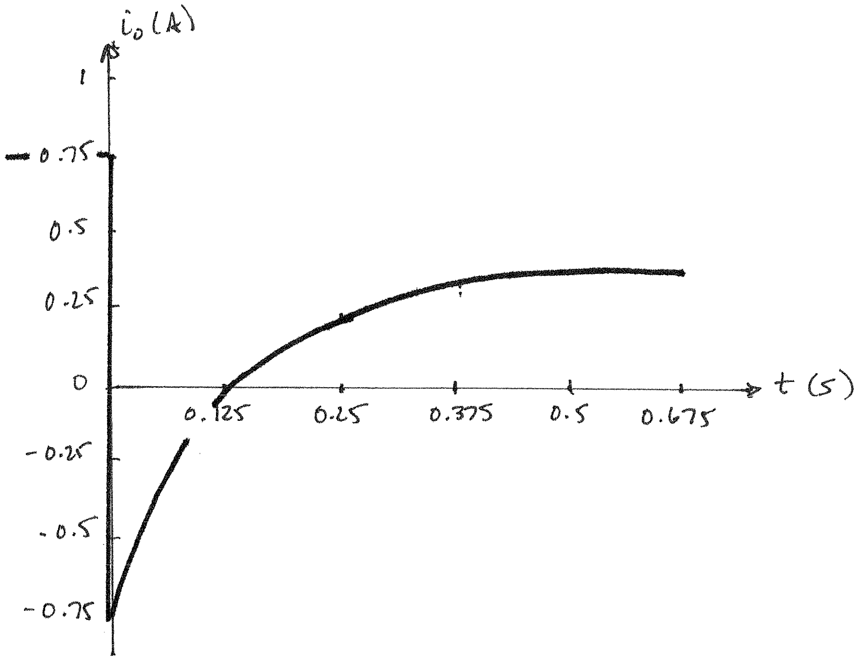
$i_3(0^-) = \dfrac{1.5\,(1/R_3)}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}} = \frac{1}{4}$ A $\qquad i_o(0^-) = \frac{3}{4}$ A

$\underline{t=0^+}$: $\quad i_o(0^+) = -i_L(0^+) = -\frac{3}{4}$ A

$\underline{t>0}$: $\quad 6 = R_3 i_o + R_2 i_o + L\dfrac{di_o}{dt} \Rightarrow \dfrac{di_o}{dt} + \dfrac{i_o(R_2+R_3)}{L} - \dfrac{6}{L} = 0$

$i_o = K_1 + K_2 e^{-t/\tau} \Rightarrow \tau = \dfrac{L}{R_2+R_3} = \dfrac{1}{8}$ s $\qquad K_1 = \dfrac{6}{R_2+R_3} = \dfrac{3}{8}$ A

$K_2 = i_o(0^+) - K_1 = -9/8$ A $\qquad \boxed{i_o(t) = 0.375 - 1.125e^{-8t}\ \text{A}}$



i_o (A)
1
0.75
0.5
0.25
0
-0.25
-0.5
-0.75
0.125
0.25
0.375
0.5
0.675
t (s)

7.24 Use the differential equation approach to find $v_o(t)$ for $t > 0$ in the circuit in Fig. P7.24 and plot the response including the time interval just prior to opening the switch.

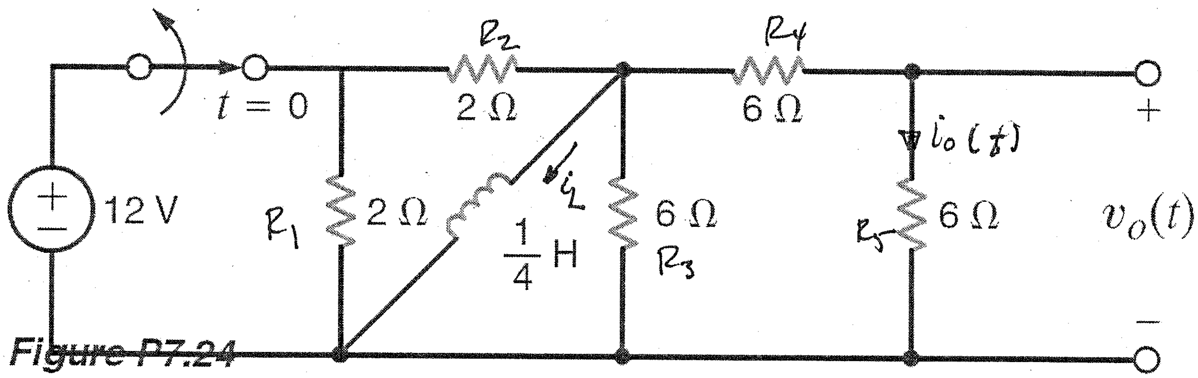


Figure P7.24

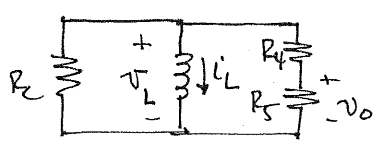
SOLUTION:

$\underline{t=0^-}$: $i_L(0^-) = {}^{12}/_{R_2} = 6A$ $\quad v_o(0^-) = 0V$

$\underline{t=0^+}$ $\quad i_L(0^+) = 6A \quad v_o(0^+) = i_o R_5 \quad i_o = \dfrac{-i_L R_C}{R_A + R_C}$ $\quad \left\{ \begin{array}{l} R_A = R_4 + R_5 = 12\Omega \\ R_C = R_3 // [R_1 + R_2] \\ R_C = 2.4\Omega \end{array} \right.$

$v_o(0^+) = -6V$

$\underline{t>0}$



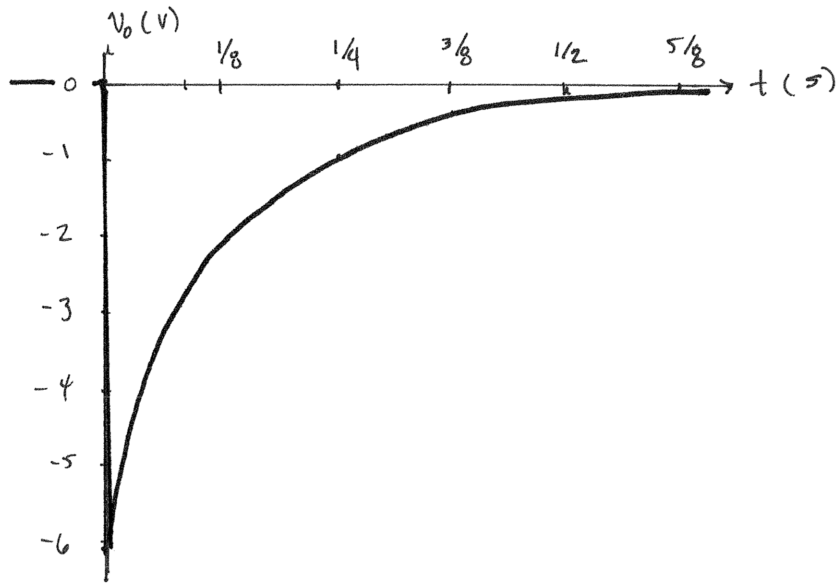
$$i_L + \frac{v_L}{R_C} + \frac{v_L}{R_A} = 0 \quad \& \quad v_L = L\frac{di_L}{dt}$$

$$\frac{di_L}{dt} + \frac{R_A R_C}{(R_A + R_C)L} i_L = 0$$

But $v_o = -\left(\dfrac{i_L R_C}{R_C + R_A}\right) R_5 \rightarrow \dfrac{dv_o}{dt} + \dfrac{R_A R_C}{(R_A+R_C)L} v_o = 0$

$v_o = K_1 + K_2 e^{-t/\tau} \rightarrow \tau = \dfrac{L(R_A + R_C)}{R_A R_C} = \dfrac{1}{8}\,s \qquad K_1 = 0$

$K_2 = v_o(0^+) - K_1 = -6V \qquad \boxed{v_o = -6e^{-8t}\ V}$



v_o (V)
t (s)
1/8
1/4
3/8
1/2
5/8
0
-1
-2
-3
-4
-5
-6

7.25 Use the differential equation approach to find $i(t)$ for $t > 0$ in the circuit in Fig. P7.25 and plot the response including the time interval just prior to opening the switch.

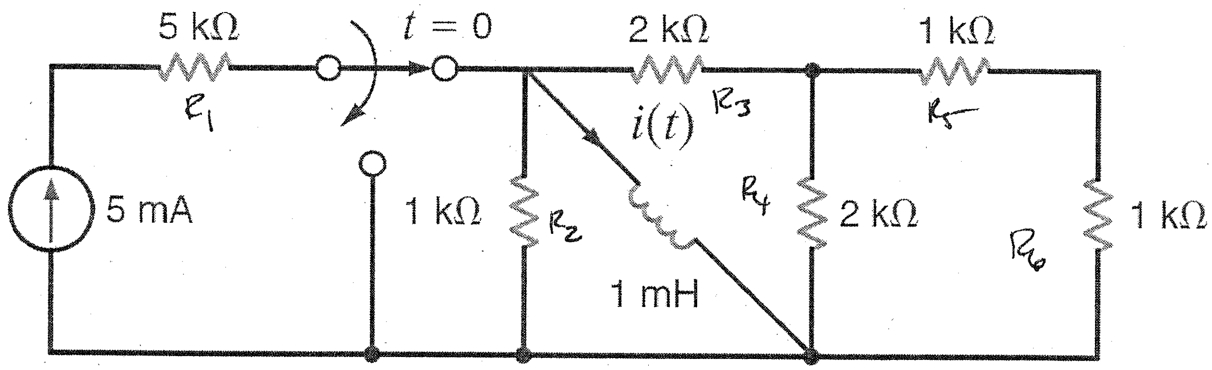


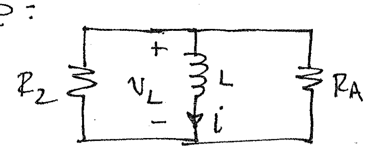
Figure P7.25

SOLUTION:

$\underline{t=0^-}$: $i(0^-) = 5\,mA$

$\underline{t=0^+}$: $i(0^+) = i(0^-) = 5\,mA$

$\underline{t>0}$:



$R_A = R_3 + \{R_4 // [R_5 + R_6]\}$

$R_A = 3\,k\Omega$

$\tau = \dfrac{L(R_A + R_2)}{R_A R_2} = \dfrac{4}{3}\,\mu s$

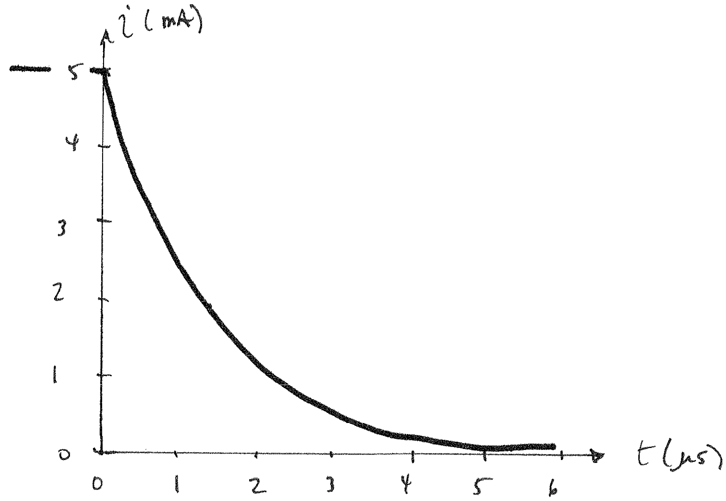
$$\boxed{i(t) = 5e^{-7.5\times10^5 t}\ mA}$$

$i + \dfrac{V_L}{R_2} + \dfrac{V_L}{R_A} = 0$ & $V_L = L\dfrac{di}{dt}$

$\dfrac{di}{dt} + \dfrac{R_A R_2}{(R_A + R_2)L}\,i = 0$

$i = K_1 + K_2 e^{-t/\tau}$

$K_1 = 0$ $K_2 = i(0^+) - K_1 = 5\,mA$



i (mA)
5
4
3
2
1
0
0
1
2
3
4
5
6
t (μs)

7.26 Find $v_C(t)$ for $t > 0$ in the network in Fig. P7.26 using the step-by-step method. CS

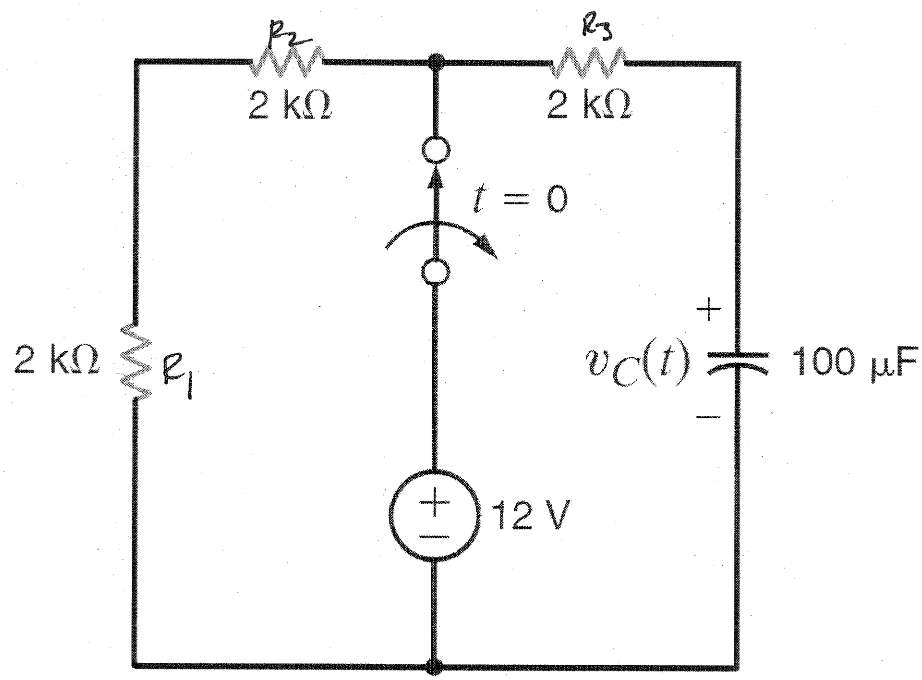
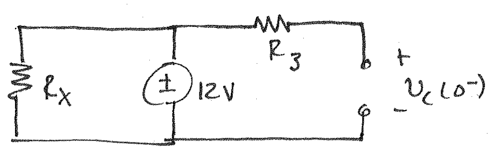


Figure P7.26

SOLUTION:

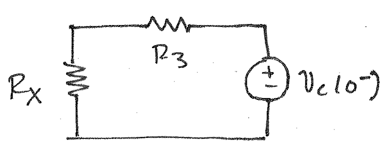
$t = 0^-$



$R_X = R_1 + R_2 = 4k\Omega$

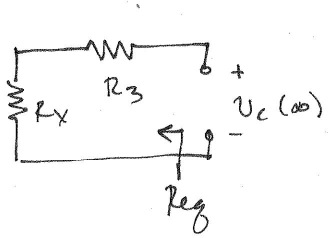
$v_c(0^-) = 12\text{V}$

$t = 0^+$



$v_c(0^+) = v_c(0^-) = 12 = K_1 + K_2$

$t \to \infty$



$v_c(\infty) = 0 = K_1$

$\tau = R_{eq} C = (R_X + R_3) C = 0.6\text{s}$

$v_c(t) = K_1 + K_2 e^{-t/\tau}$

$\boxed{v_c(t) = 12e^{-1.67t} \text{ V}}$

7.27 Use the step-by-step method to find $i_o(t)$ for $t > 0$ in the circuit in Fig. P7.27.

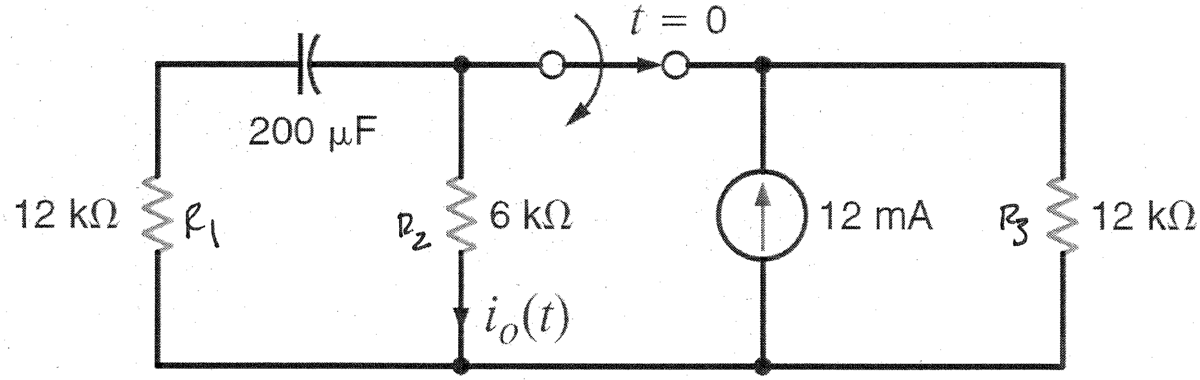
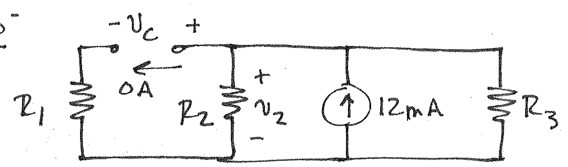


Figure P7.27

SOLUTION: $i_o(t) = K_1 + K_2 e^{-t/\tau}$

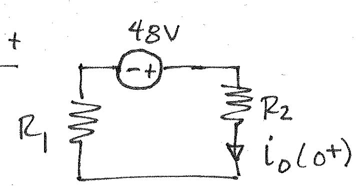
$t = 0^-$



$v_2 = v_c = 12\times10^{-3}\left[\frac{R_3 R_2}{R_3+R_2}\right]$

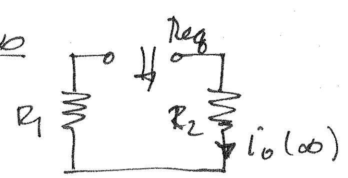
$v_c(0^-) = 48\text{V}$

$t = 0^+$



$i_o(0^+) = \frac{48}{R_1+R_2} = 2.67 \text{ mA} = K_1 + K_2$

$t = \infty$



$i_o(\infty) = 0 = K_1$

$\tau = C R_{eq} \qquad R_{eq} = R_1 + R_2 = 18 k\Omega$

$\tau = 3.6 \text{ s}$

$$\boxed{i_o(t) = 2.67 e^{-t/3.6} \text{ mA}}$$

7.28 Use the step-by-step technique to find $i_o(t)$ for $t > 0$ in the network in Fig. P7.28. CS

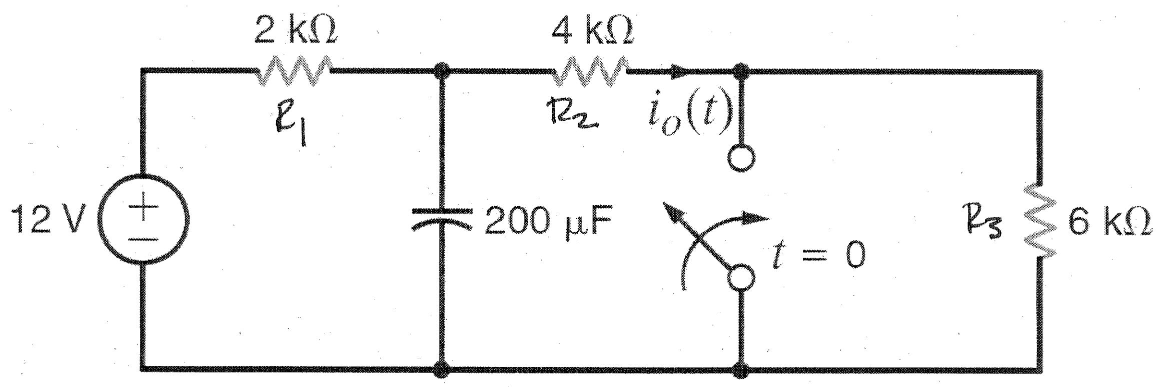
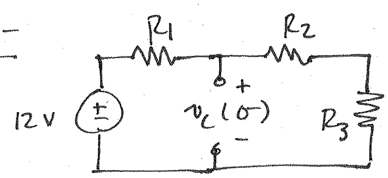


Figure P7.28

SOLUTION: $i_o(t) = K_1 + K_2 e^{-t/\tau}$

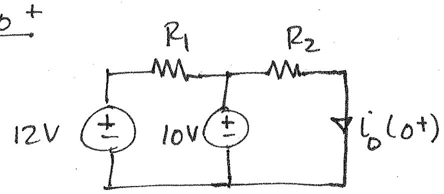
$t = 0^-$



By voltage division: $v_c(0^-) = 12 \dfrac{(R_2+R_3)}{R_1+R_2+R_3}$

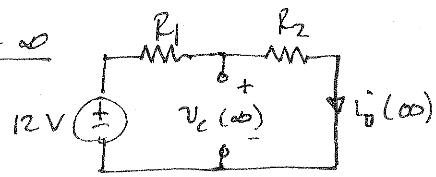
$v_c(0^-) = 10\text{V}$

$t = 0^+$



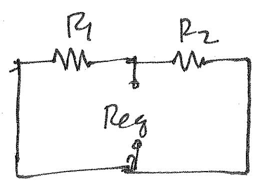
$i_o(0^+) = 10/R_2 = 2.5\text{mA} = K_1 + K_2$

$t = \infty$



$i_o(\infty) = \dfrac{12}{R_1+R_2} = 2\text{mA} = K_1$

$\tau = ?$



$\tau = C R_{eq}$ $R_{eq} = R_1 // R_2 = \frac{4}{3}\,k\Omega$

$\tau = 0.267\text{ s}$

$$\boxed{i_o(t) = 2 + 0.5e^{-3.75t} \text{ mA}}$$

7.29 Use the step-by-step method to find $v_o(t)$ for $t > 0$ in the network in Fig. P7.29.

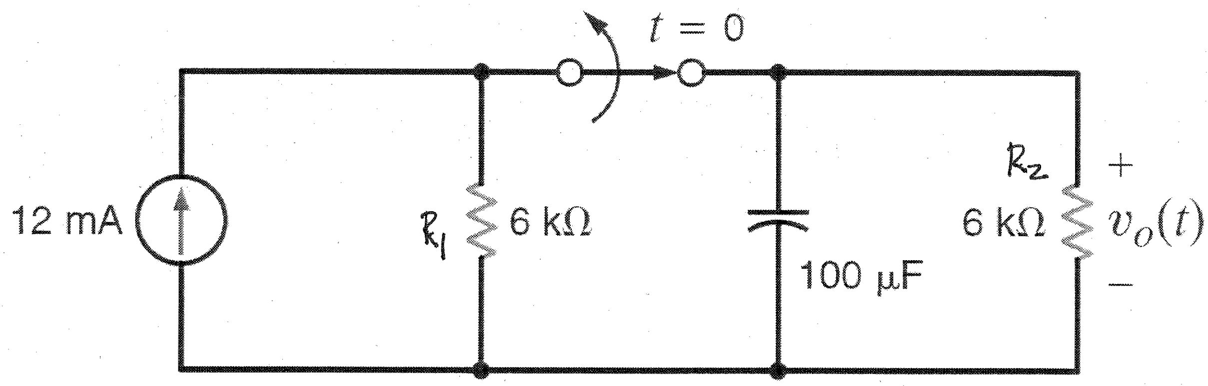
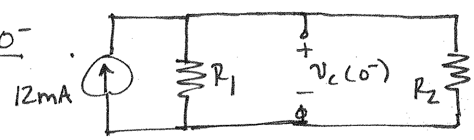


Figure P7.29

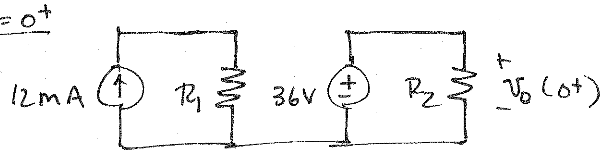
SOLUTION: $v_o(t) = K_1 + K_2 e^{-t/\tau}$

$\underline{t=0^-}$



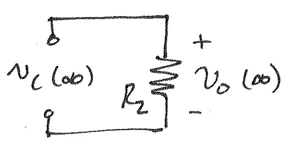
$v_c(0^-) = 12\times10^{-3} \dfrac{(R_1 R_2)}{R_1+R_2} = 36\text{V}$

$\underline{t=0^+}$



$v_o(0^+) = 36 = K_1 + K_2$

$\underline{t=\infty}$



$v_o(\infty) = 0 = K_1$

$\underline{\tau = ?}$. $\tau = R_{eq} C$ $\quad R_{eq} = R_2 = 6\text{k}\Omega$ $\quad \tau = 0.6\text{s}$

$$\boxed{v_o(t) = 36\, e^{-t/0.6} \text{ V}}$$

7.30 Use the step-by-step method to find $i_o(t)$ for $t > 0$ in the circuit in Fig. P7.30.

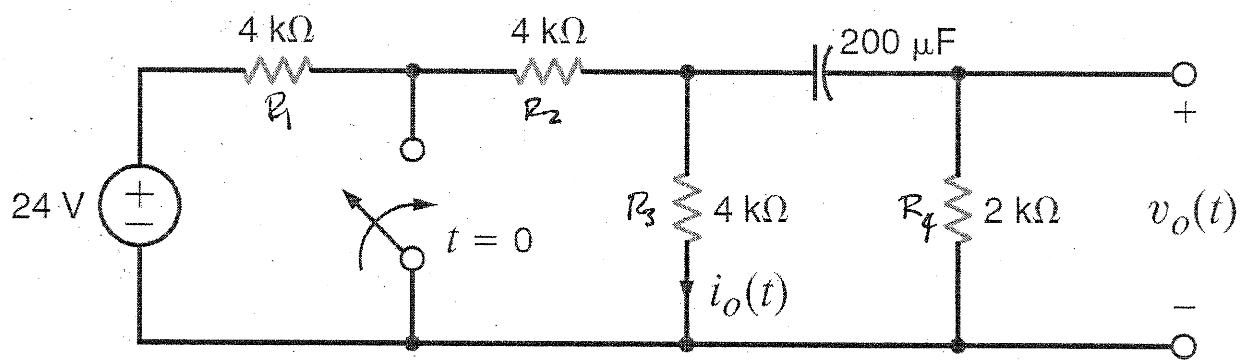
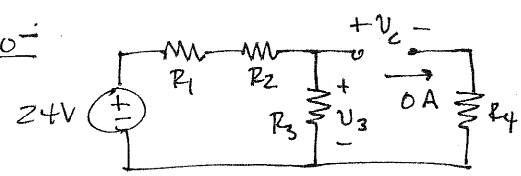


Figure P7.30

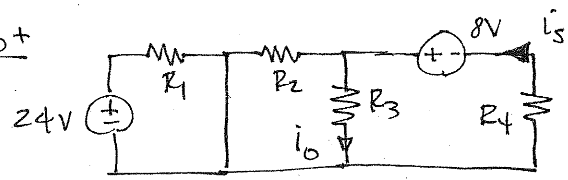
SOLUTION: $i_o(t) = K_1 + K_2 e^{-t/\tau}$

$t = 0^-$



$v_c = v_3 = \dfrac{24 R_3}{R_1 + R_2 + R_3} = 8V$

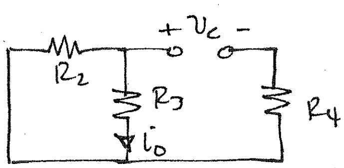
$t = 0^+$



$i_s = \dfrac{8}{R_4 + R_x}$ $\quad R_x = \dfrac{R_2 R_3}{R_2 + R_3} = 2k\Omega$

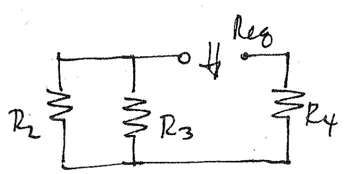
$i_o = \dfrac{i_s R_2}{R_2 + R_3} = 1mA = K_1 + K_2$

$t = \infty$



$i_o(\infty) = 0 = K_1$

$\tau = ?$



$\tau = R_{eq} C$ $\quad R_{eq} = \dfrac{R_2 R_3}{R_2 + R_3} + R_4 = 4k\Omega$

$\tau = 0.8s$

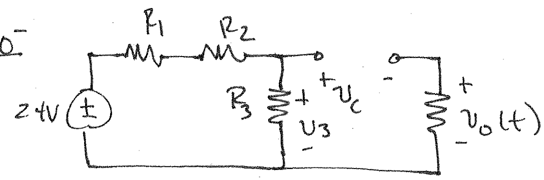
$$\boxed{i_o(t) = e^{-1.25t} \text{ mA}}$$

7.31 Find $v_o(t)$ for $t > 0$ in the network in Fig. P7.30 using the step-by-step technique.

SOLUTION:

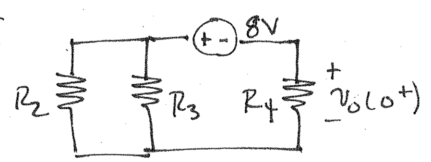
$v_o(t) = K_1 + K_2 e^{-t/\tau}$ $\qquad R_1 = R_2 = R_3 = 4k\Omega \qquad R_4 = 2k\Omega$

$t = 0^-$



$$v_c = v_3 = \frac{24 R_3}{R_1 + R_2 + R_3} = 8V$$

$t = 0^+$



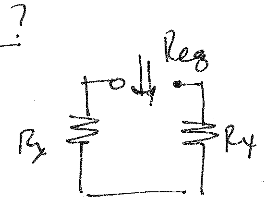
$R_x = R_2 // R_3 = 2k\Omega$

$$v_o(0^+) = -\frac{8 R_4}{R_4 + R_x} = -4V = K_1 + K_2$$

$t = \infty$

$v_o(\infty) = 0 = K_1$

$\tau = ?$



$\tau = R_{eq} C \qquad R_{eq} = R_x + R_4 = 4k\Omega$

$\tau = 0.8s$

$$\boxed{v_o(t) = -4e^{-1.25t} \text{ V}}$$

7.32 Use the step-by-step technique to find $i_o(t)$ for $t > 0$ in the network in Fig. P7.32.

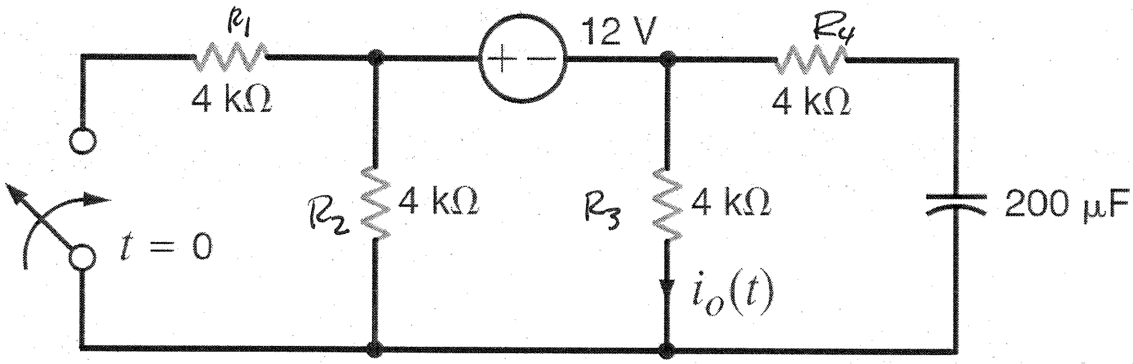
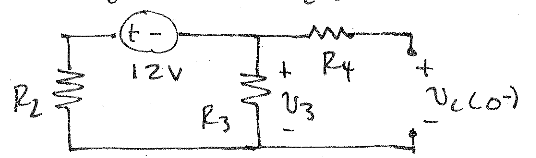


Figure P7.32

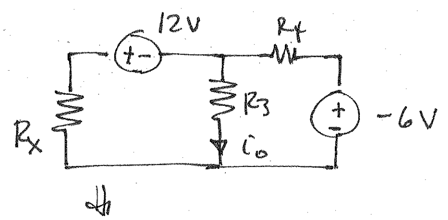
SOLUTION: $i_o(t) = K_1 + K_2 e^{-t/\tau}$

$t = 0^-$

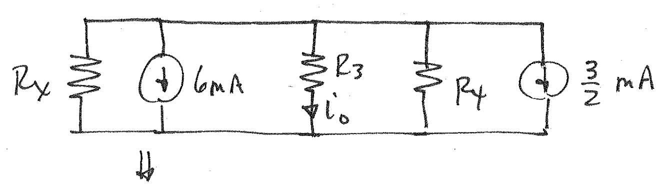


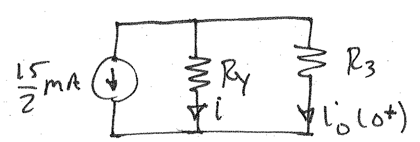
$v_c(0^-) = v_3 = -\frac{12 R_3}{R_2 + R_3} = -6V$

$t = 0^+$



$R_x = R_1 // R_2 = 2k\Omega$

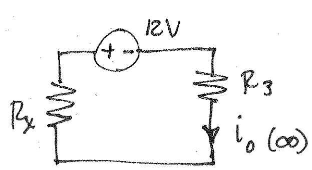




$R_y = R_x // R_4 = 4/3\ k\Omega$

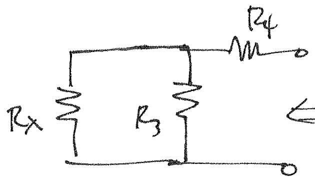
$i_o(0^+) = \frac{-15}{2000} \frac{R_y}{R_y + R_3} = -1.875\ mA = K_1 + K_2$

$t = \infty$



$i_o(\infty) = \frac{-12}{R_x + R_3} = -2mA = K_1$

$\underline{\tau = ?}$



$\Leftarrow R_{eq} = R_4 + \dfrac{R_x R_3}{R_x + R_3} = 5.33\,k\Omega$

$\tau = R_{eq} C = 1.07\,s$

$$\boxed{i_o(t) = -2 + 0.125\,e^{-0.9375t} \quad mA}$$

7.33 Find $v_o(t)$ for $t > 0$ in the network in Fig. P7.33 using the step-by-step method. **CS**

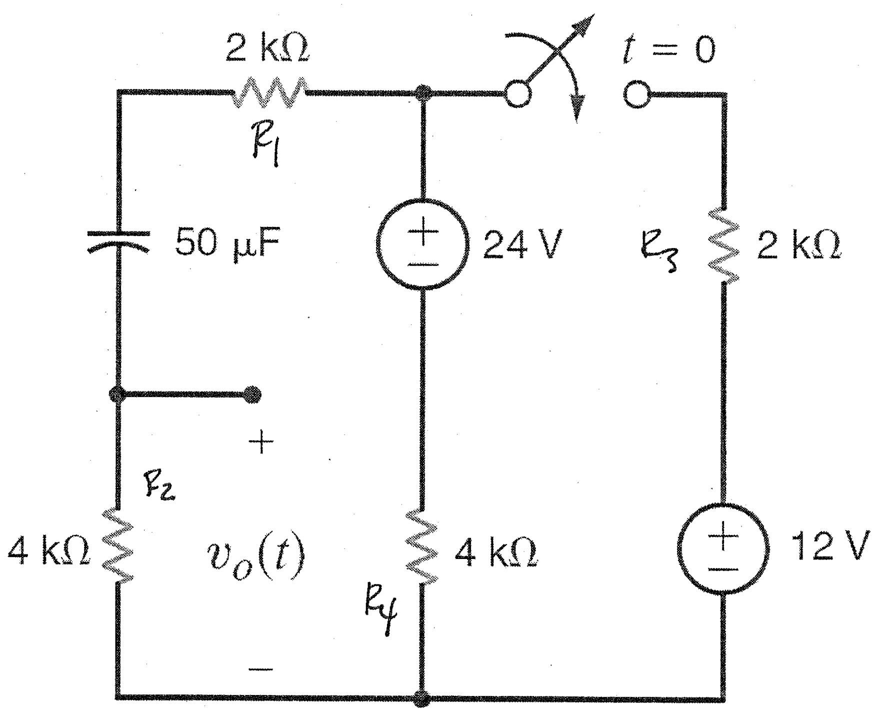
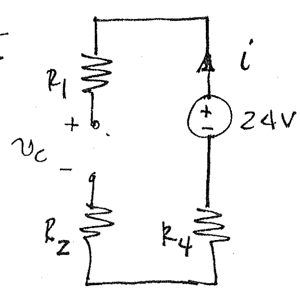


Figure P7.33

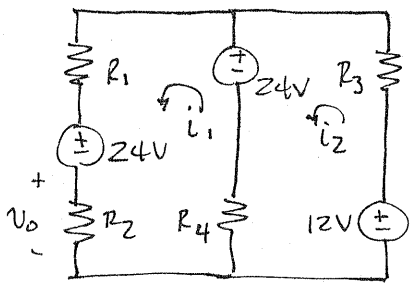
SOLUTION: $v_o(t) = K_1 + K_2 e^{-t/\tau}$

$\underline{t = 0^-}$



$i = 0$ so $v_c(0^-) = 24V$

$\underline{t=0^+}$



$$24 = i_1(R_1+R_2+R_4) - i_2 R_4 + 24$$

or, $$i_1(R_1+R_2+R_4) = i_2 R_4$$

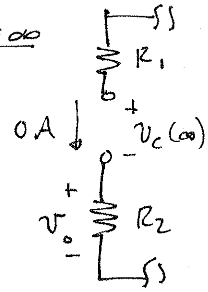
$$12 = i_2(R_3+R_4) + 24 - i_1 R_4$$

or $$i_1 R_4 - i_2(R_3+R_4) = 12$$

$$i_1 = -\frac{12}{11}\ mA \qquad v_o(0^+) = i_1 R_2$$

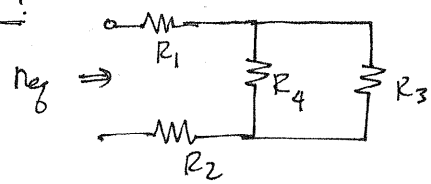
$$v_o(0^+) = \frac{48}{11}\ V = k_1 + k_2$$

$\underline{t=\infty}$



$$v_o(\infty) = 0 = k_1$$

$\underline{\tau = ?}$



$$R_{eq} = R_1 + R_2 + \frac{R_3 R_4}{R_3+R_4} = 7.33\,k\Omega$$

$$\tau = 367\ ms$$

$$\boxed{v_o(t) = -4.36\, e^{-2.73t}\ V}$$

7.34 Find $v_o(t)$ for $t > 0$ in the circuit in Fig. P7.34 using the step-by-step method. **PSV**

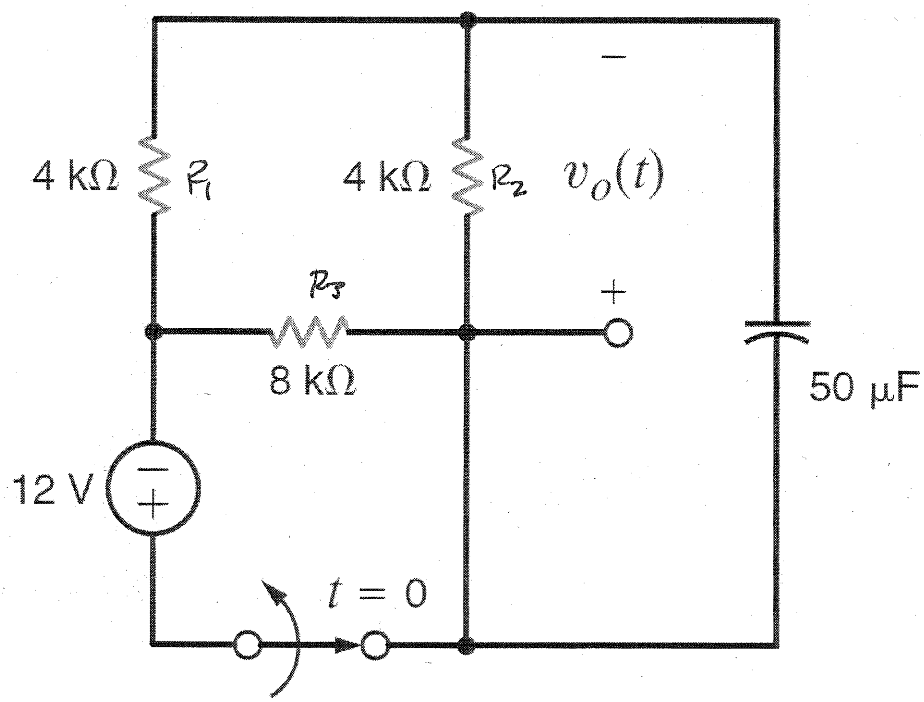
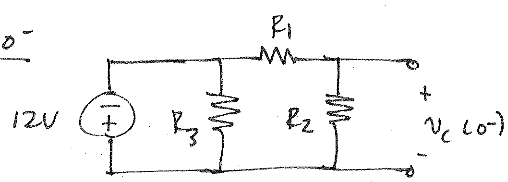


Figure P7.34

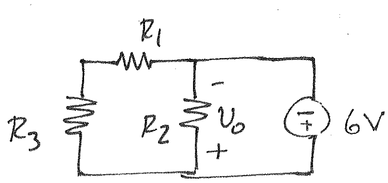
SOLUTION: $v_o(t) = K_1 + K_2 e^{-t/\tau}$

$t = 0^-$



$v_c(0^-) = -12\left(\frac{R_2}{R_1+R_2}\right) = -6V$

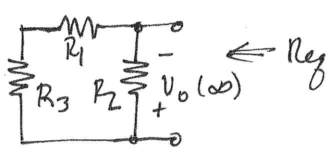
$t = 0^+$



$v_o(0+) = 6V = k_1 + k_2$

$t = \infty$

$v_o(\infty) = 0 = K_1$ $R_{eq} = R_2\ (R_1+R_3)/(R_1+R_2+R_3) = 3\ k\Omega$



$\tau = R_{eq} C = 150ms$

$$\boxed{v_o(t) = 6e^{-6.67t}\ V}$$

7.35 Use the step-by-step method to find $i_o(t)$ for $t > 0$ in the network in Fig. P7.35. CS

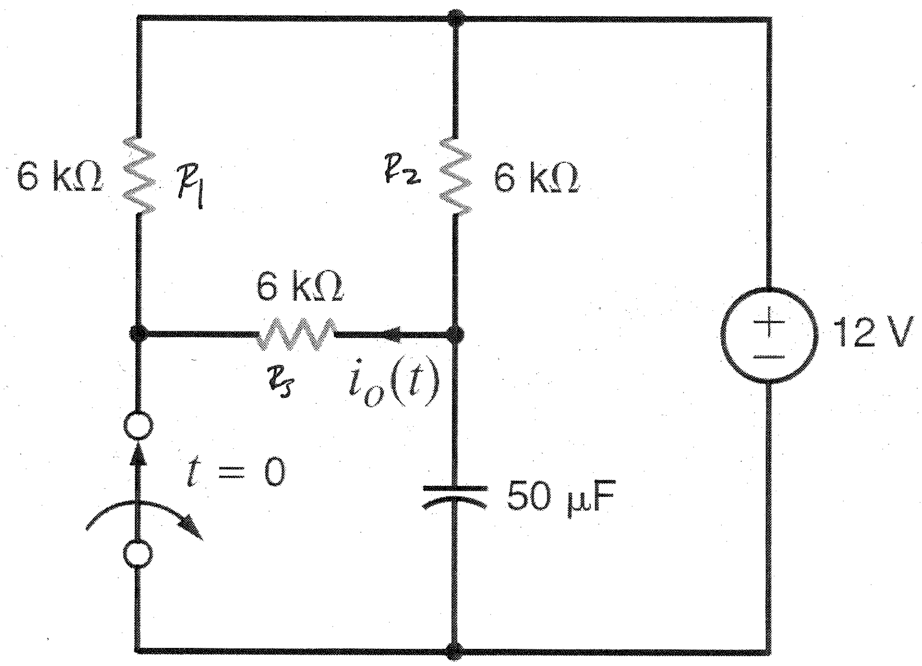
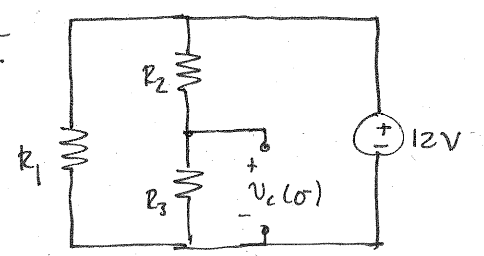


Figure P7.35

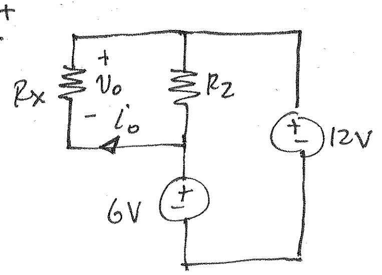
SOLUTION: $i_o(t) = K_1 + K_2 e^{-t/\tau}$

$\underline{t=0^-}$



$$v_c(0^-) = \frac{12 R_3}{R_3 + R_2} = 6V$$

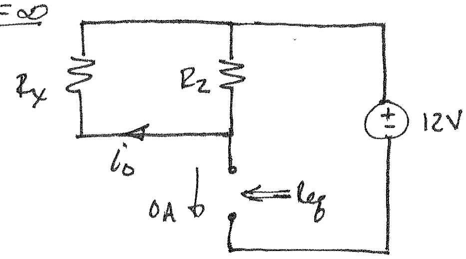
$\underline{t=0^+}$



$R_x = R_1 + R_3 = 12\,k\Omega$ $\quad v_o = 12 - 6 = 6V$

$$i_o = -\frac{v_o}{R_x} = -0.5\,mA = K_1 + K_2$$

$\underline{t=\infty}$



$i_o = 0 = K_1$

$\underline{\tau = ?}$ $R_{eq} = \dfrac{R_x R_2}{R_x + R_2} = 4k\Omega$ $\tau = R_{eq} C = 0.25$

$$\boxed{i_o(t) = -0.5\, e^{-5t} \text{ mA}}$$

7.36 Use the step-by-step technique to find $v_o(t)$ for $t > 0$ in the circuit in Fig. P7.36.

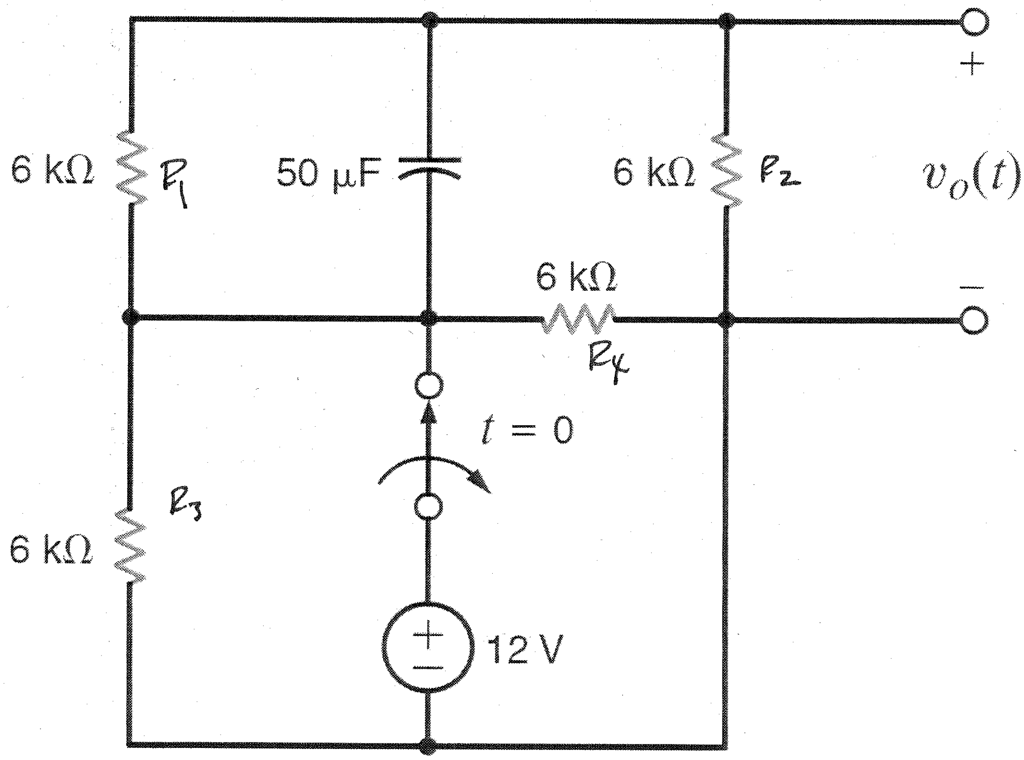
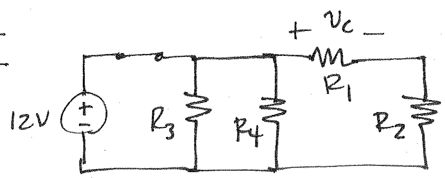


Figure P7.36

SOLUTION: $v_o(t) = K_1 + K_2 e^{-t/\tau}$

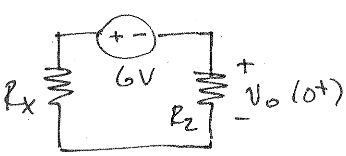
$t = 0^-$



$v_c(0^-) = \frac{12R_1}{R_1+R_2} = 6V$

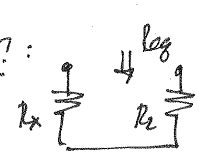
$R_x = R_3 // R_4 = 3k\Omega$

$t = 0^+$



$t = \infty$: $v_o(\infty) = 0 = K_1$

$\tau = ?$:



$R_{eq} = R_x + R_2 = 9k\Omega$

$\tau = R_{eq}C = 0.45s$

$v_o(0^+) = \frac{-6R_2}{R_x+R_2} = -4V = K_1 + K_2$

$$\boxed{v_o(t) = -4e^{-2.22t} \; V}$$

7.37 Find $i_o(t)$ for $t > 0$ in the network in Fig. P7.37 using the step-by-step method. CS

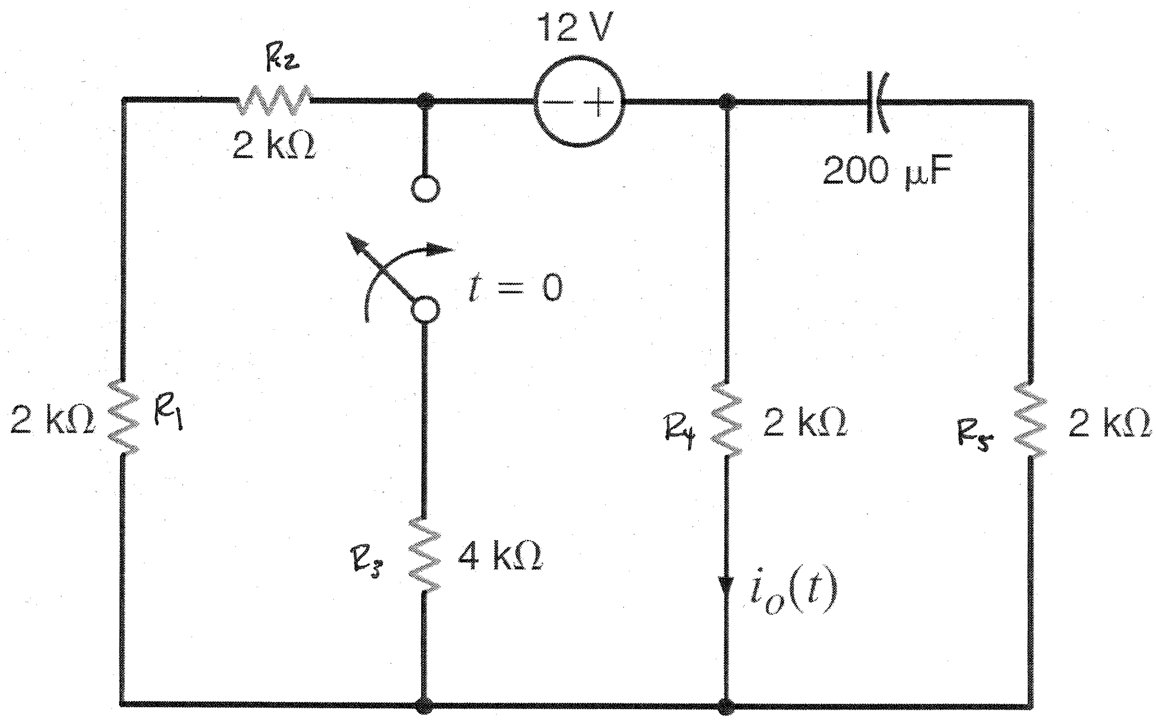
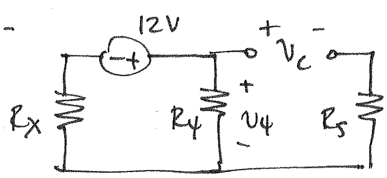


Figure P7.37

SOLUTION: $i_o(t) = K_1 + K_2 e^{-t/\tau}$

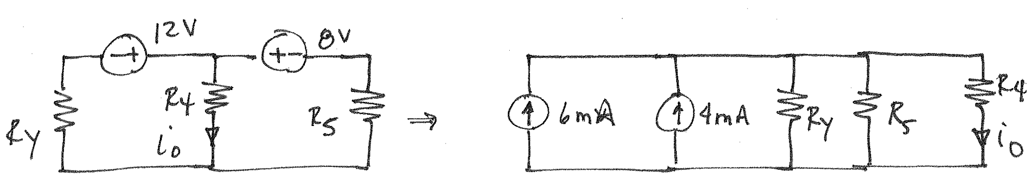
$t = 0^-$



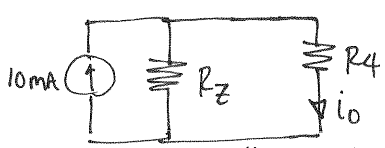
$v_C(0^-) = v_4 = \frac{12R_4}{R_4+R_x}$ $\quad R_x = R_1 + R_2 = 4k\Omega$

$v_C(0^-) = 8V$

$t = 0^+$



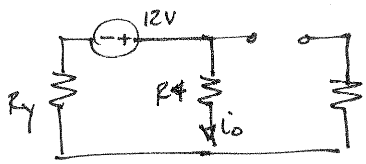
$R_y = R_x // R_3 = 2k\Omega$



$R_z = R_y // R_5 = 1k\Omega$

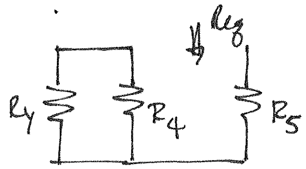
$i_o(0^+) = \frac{10^{-2} R_z}{R_z + R_4} = 3.33\ mA = K_1 + K_2$

$\underline{t = \infty}$



$$i_o(\infty) = \frac{+12}{R_Y + R_4} = +3mA = K_1$$

$\underline{\tau = ?}$



$$R_{eq} = R_5 + (R_4 // R_Y) = 3k\Omega$$

$$\tau = R_{eq} C = 0.6 s$$

$$\boxed{i_o(t) = 3 + 0.33 e^{-1.67t} \text{ mA}}$$

7.38 Use the step-by-step technique to find $i_o(t)$ for $t > 0$ in the network in Fig. P7.38.

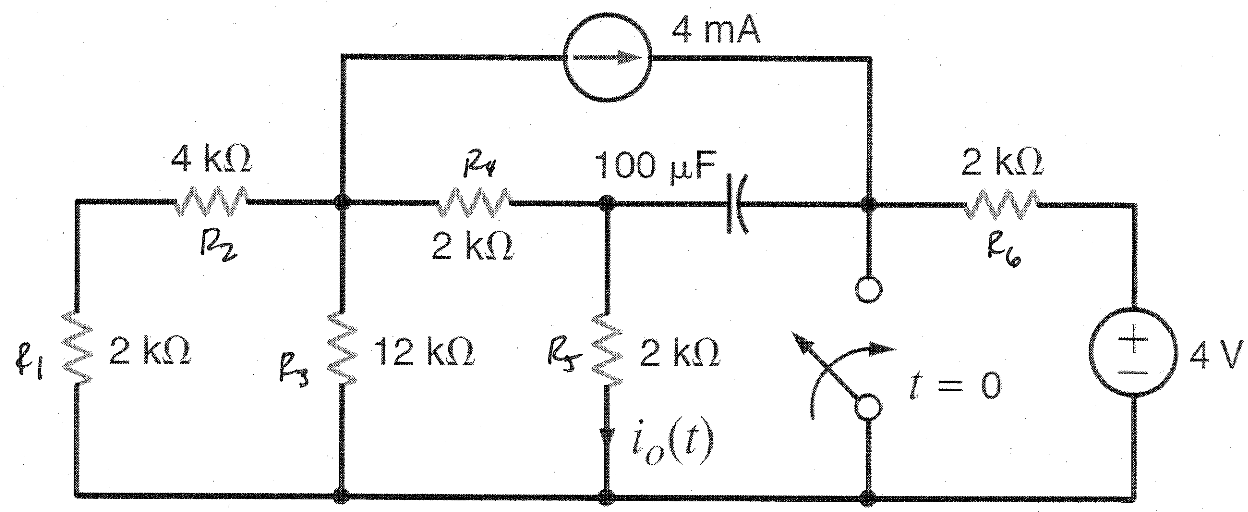
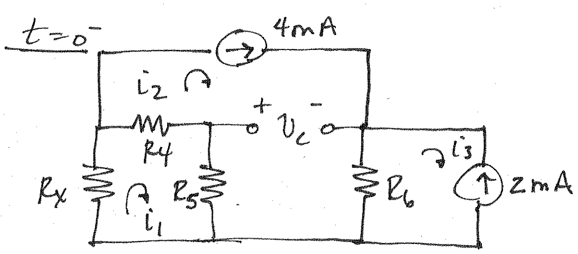


Figure P7.38

SOLUTION: $i_o(t) = K_1 + K_2 e^{-t/\tau}$

$t = 0^-$



$R_x = R_3 // (R_1 + R_2) = 4k\Omega$

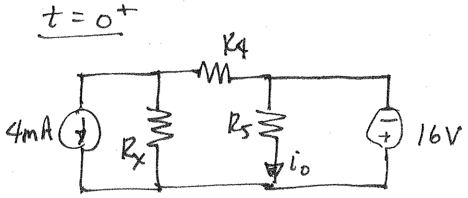
$i_2 = 4mA \qquad i_3 = -2mA$

$i_1(R_x + R_4 + R_5) - i_2(R_4 + R_5) = 0$

$i_1 = 2mA$

$v_C(0^-) = (i_1 - i_2)R_5 + (i_3 - i_2)R_6 = -16\ V$

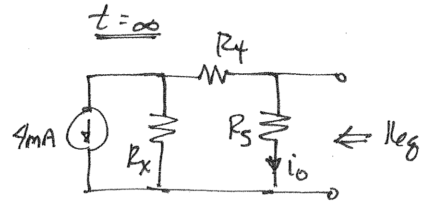
$t = 0^+$



$i_o(0^+) = -\frac{16}{R_5} = -8mA$

$K_1 + K_2 = -8mA$

$t = \infty$



$i_o(\infty) = \frac{-4\times10^{-3} R_x}{R_x + R_4 + R_5}$

$i_o(\infty) = -2mA = K_1$

$\tau = ?$

$R_{eq} = R_5 // (R_4 + R_x)$
$= 1.5k\Omega$

$\tau = R_{eq}C = 0.15s$

$\boxed{i_o(t) = -2 - 6e^{-6.67t}\ mA}$

7.39 Use the step-by-step method to find $i_o(t)$ for $t > 0$ in the circuit in Fig. P7.39.

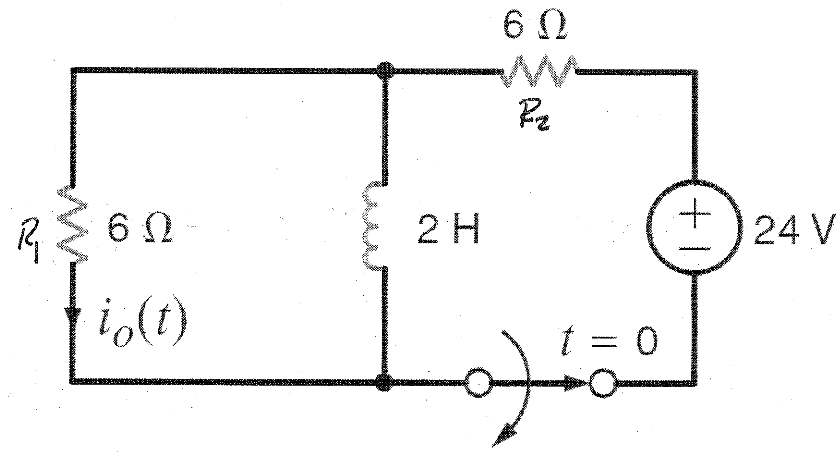
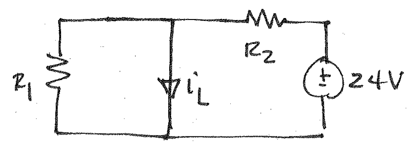


Figure P7.39

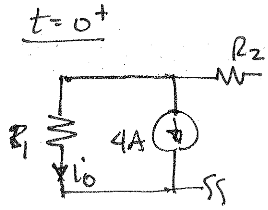
SOLUTION: $i_o(t) = K_1 + K_2 e^{-t/\tau}$

$t=0^-$



$i_L(0^-) = 24/R_2 = 4A$

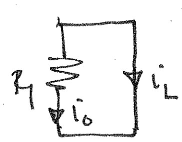
$t=0^+$



$i_o(0^+) = -4A$

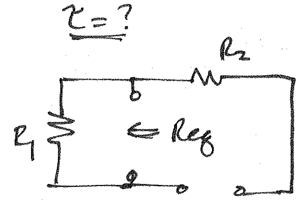
$K_1 + K_2 = -4A$

$t=\infty$



$i_o(\infty) = 0 = K_1$

$\tau = ?$



$R_{eq} = R_1 = 6\Omega$

$\tau = L/R_{eq} = \frac{1}{3}$ s

$$\boxed{i_o(t) = -4e^{-3t} \text{ A}}$$

7.40 Find $i_o(t)$ for $t > 0$ in the network in Fig. P7.40 using the step-by-step method.

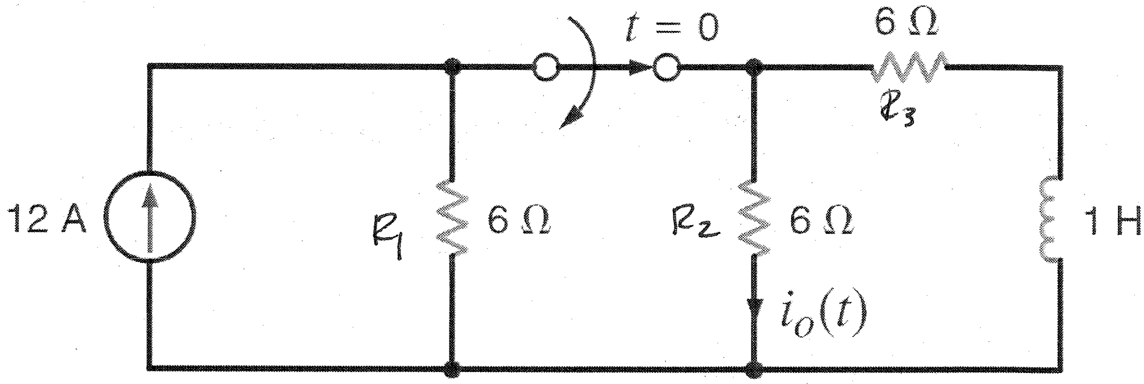
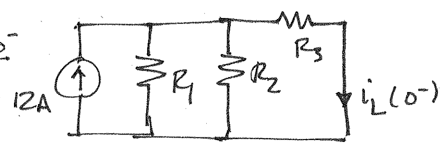


Figure P7.40

SOLUTION: $i_o(t) = K_1 + K_2 e^{-t/\tau}$

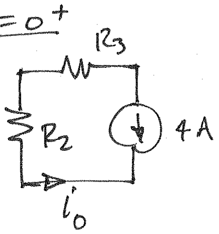
$t = 0^-$



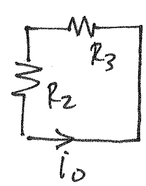
$R_x = R_1 // R_2 = 3\Omega$

$i_L(0^-) = \frac{12 R_x}{R_x + R_3} = 4A$

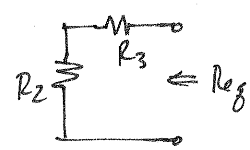
$t = 0^+$



$t = \infty$



$\tau = ?$



$R_{eq} = R_2 + R_3 = 12\Omega$

$\tau = L/R_{eq} = \frac{1}{12}$ s

$i_o = -4A = K_1 + K_2$

$i_o(\infty) = 0 = K_1$

$$\boxed{i_o(t) = -4e^{-12t} \text{ A}}$$

7.41 Find $i_o(t)$ for $t > 0$ in the network in Fig. P7.41 using the step-by-step method. CS

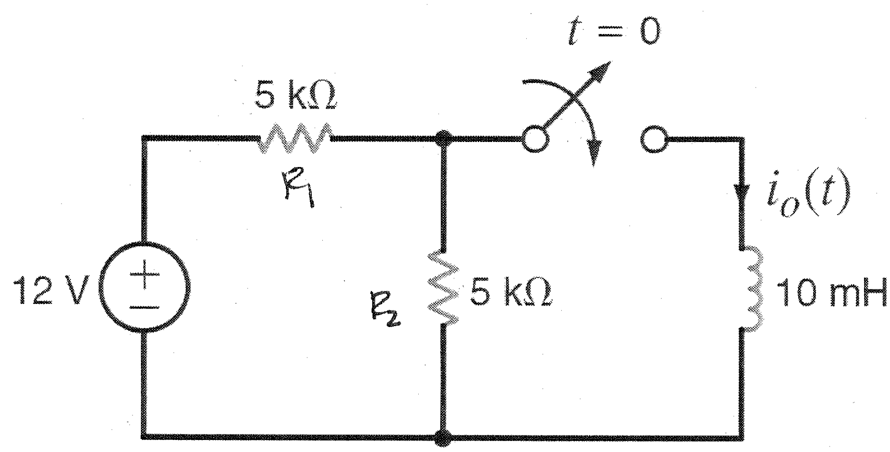
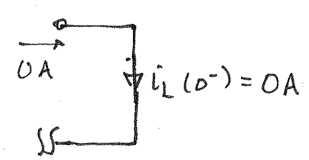


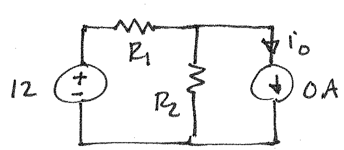
Figure P7.41

SOLUTION: $i_o(t) = K_1 + K_2 e^{-t/\tau}$

$t=0^-$

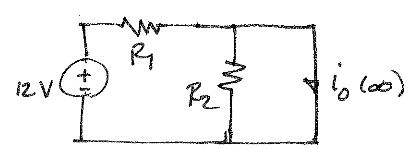


$t=0^+$



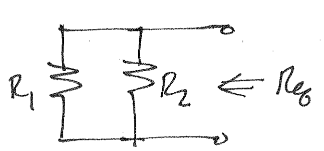
$i_o(0^+) = 0 = K_1 + K_2$

$t=\infty$



$i_o(\infty) = \frac{12}{R_1} = 2.4\,mA = K_1$

$\tau = ?$



$R_{eq} = R_1 // R_2 = 2.5\,k\Omega$

$\tau = \frac{L}{R_{eq}} = 4\mu s$

$$\boxed{i_o(t) = 2.4 - 2.4e^{-2.5\times10^{5}t}\ mA}$$

7.42 Find $v_o(t)$ for $t > 0$ in the network in Fig. P7.42 using the step-by-step method.

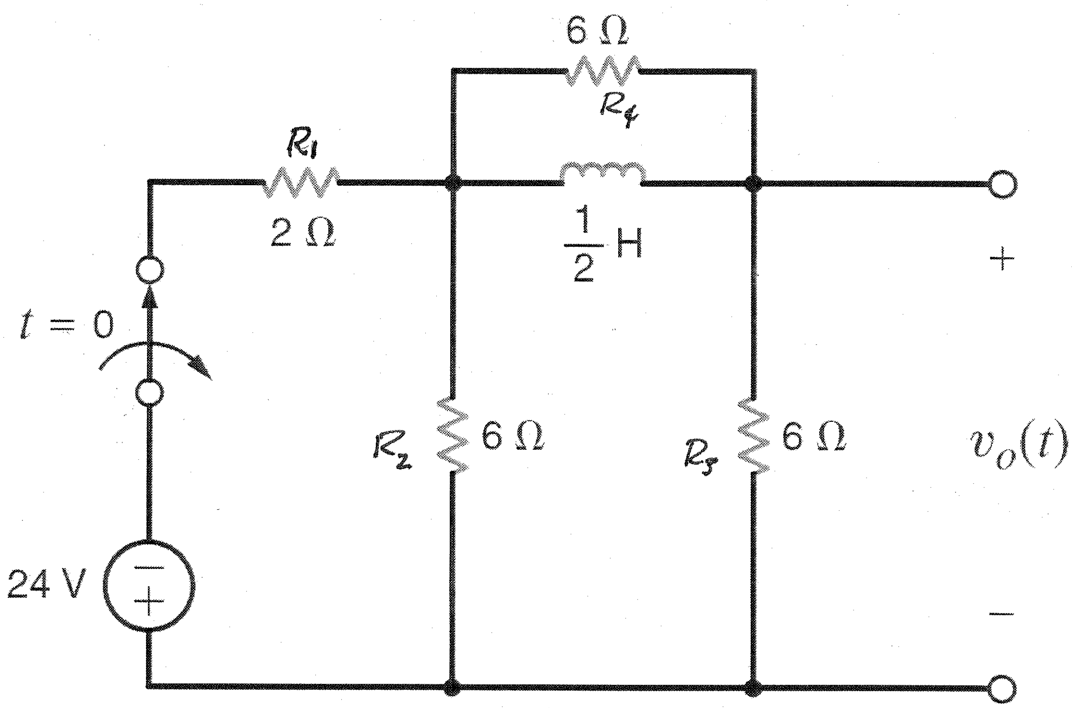
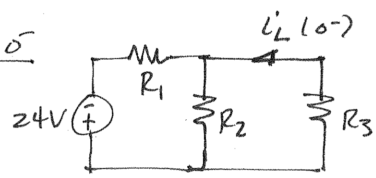


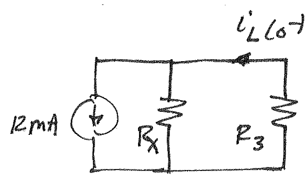
Figure P7.42

SOLUTION: $v_o(t) = K_1 + K_2 e^{-t/\tau}$

$t = 0^-$



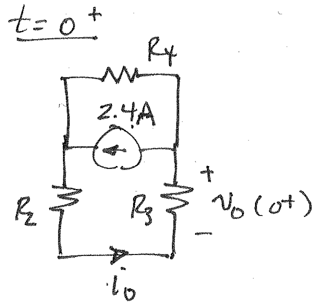
$\Rightarrow$



$R_x = R_1 // R_2 = 1.5k\Omega$

$i_L(0^-) = \dfrac{12\times10^{-3} R_x}{R_x + R_3} = 2.4A$

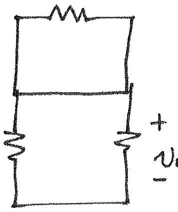
$t = 0^+$



$i_o = \dfrac{2.4\,R_4}{R_4 + R_3 + R_2} = 0.8\,mA$

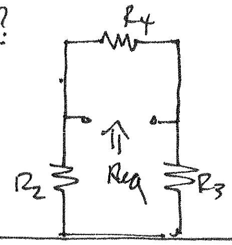
$v_o(0^+) = -R_3\, i_o(0^+) = -4.8V = K_1 + K_2$

$\underline{t=\infty}$



$v_o(\infty) = 0 = K_1$

$\underline{\tau = ?}$



$R_{eq} = R_4 \,//\, (R_2 + R_3) = 4\,\Omega$

$\tau = L/R_{eq} = \frac{1}{8}$ s

$$\boxed{v_o(t) = -4.8e^{-8t} \text{ V}}$$

7.43 Use the step-by-step method to find $v_o(t)$ for $t > 0$ in the network in Fig. P7.43. **PSV**

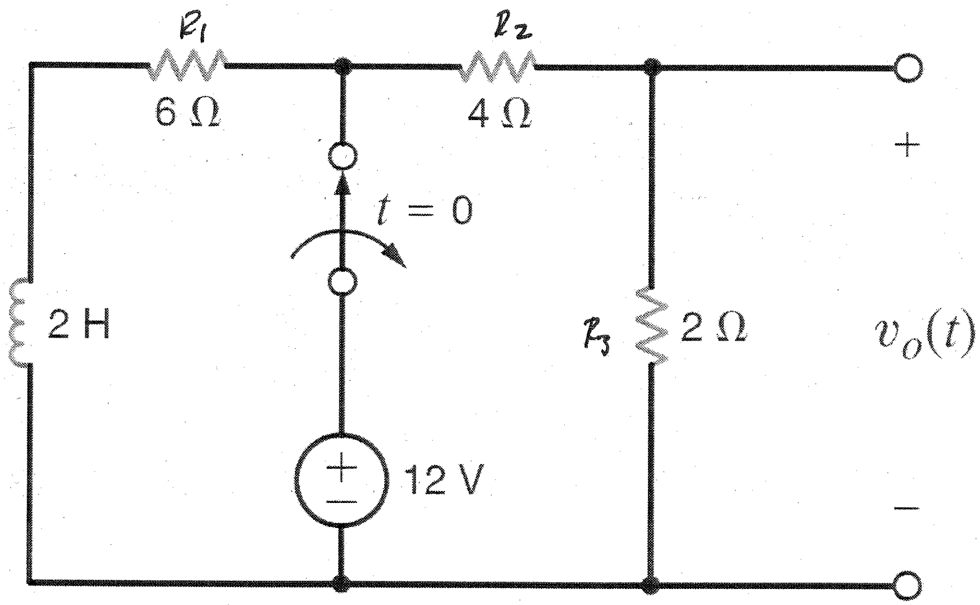
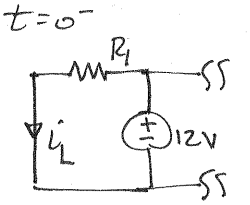


Figure P7.43

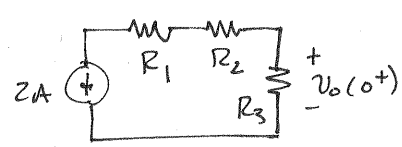
SOLUTION: $v_o(t) = K_1 + K_2 e^{-t/\tau}$

$t = 0^-$



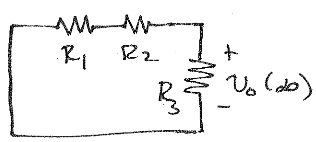
$i_L(0^-) = \frac{12}{R_1} = 2A$

$t = 0^+$



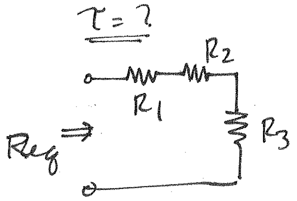
$v_o(0^+) = -2 R_3 = -4V$
$= K_1 + K_2$

$t = \infty$



$v_o(\infty) = 0 = K_1$

$\tau = ?$



$R_{eq} = R_1 + R_2 + R_3 = 12\Omega$

$\tau = L/R_{eq} = \frac{1}{6}\,s$

$\boxed{v_o = -4e^{-6t}\ V}$

7.44 Find $i_o(t)$ for $t > 0$ in the network in Fig. P7.44 using the step-by-step method.

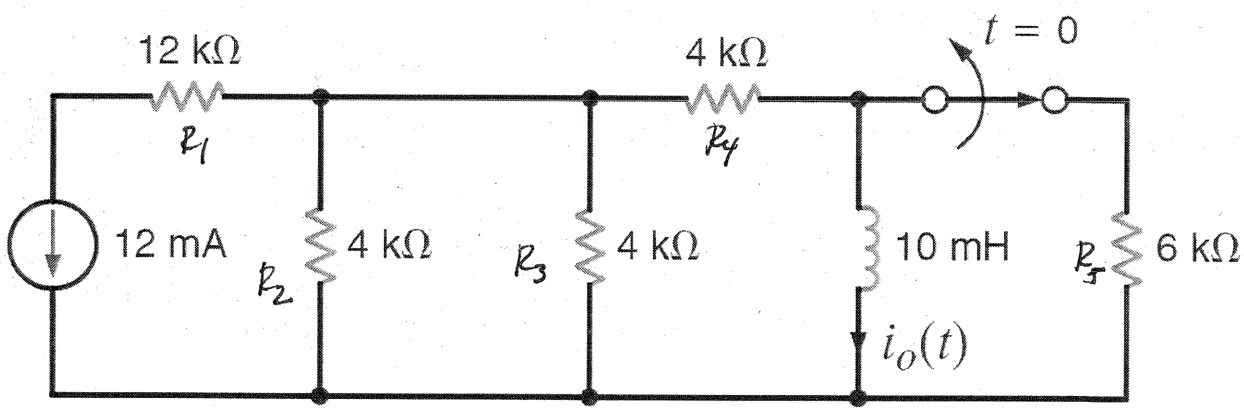
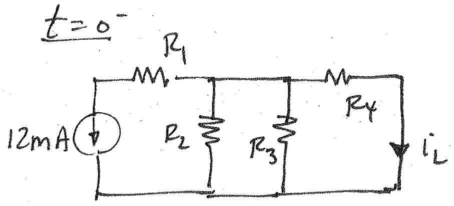


Figure P7.44

SOLUTION: $i_o(t) = K_1 + K_2 e^{-t/\tau}$

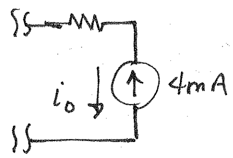
$\underline{t=0^-}$



$R_2 = R_3 = R_4 = 4k\Omega$

$i_L(0^-) = -\dfrac{12\times10^{-3}}{3} = -4mA$

$\underline{t=0^+}$



$i_o(0^+) = -4mA$
$= K_1 + K_2$

$\underline{t=\infty}$ Same as $t=0^-$!

$i_o(\infty) = -4mA = K_1$

So, $K_2 = 0$

$\boxed{i_o = -4mA}$

7.45 Use the step-by-step technique to find $v_o(t)$ for $t > 0$ in the circuit in Fig. P7.45.

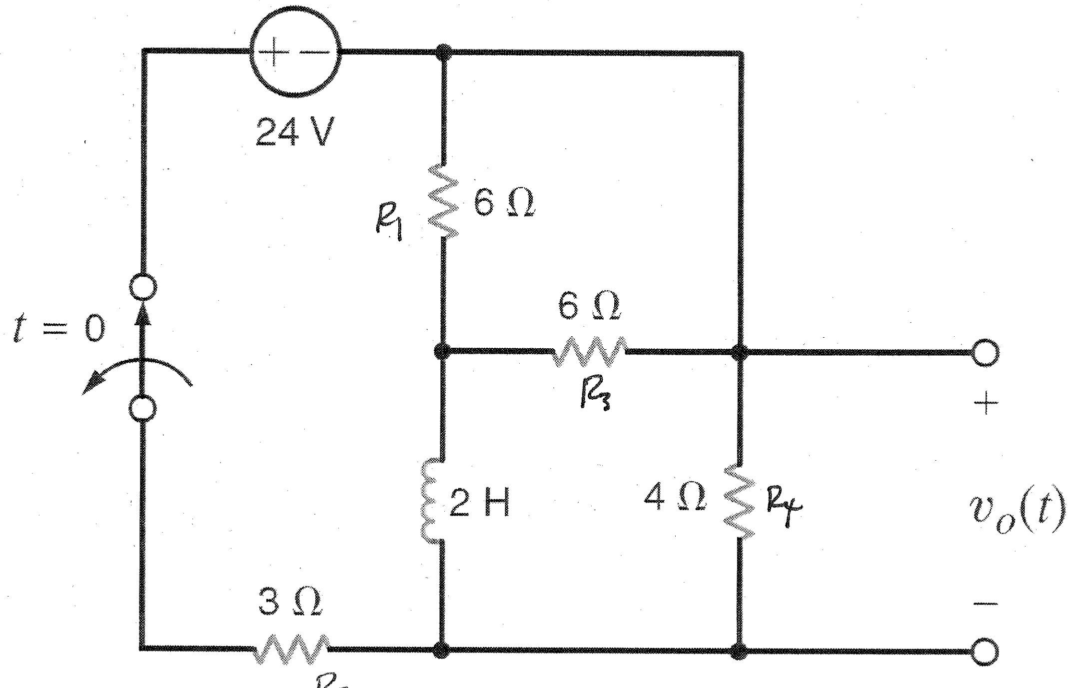
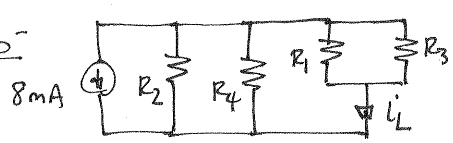


Figure P7.45

SOLUTION: $v_o(t) = K_1 + K_2 e^{-t/\tau}$

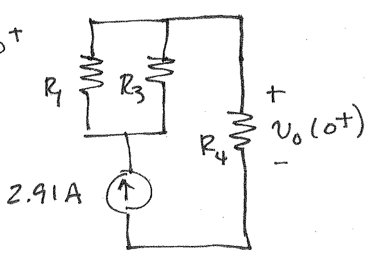
$t = 0^-$



$R_x = R_2 // R_4 = \frac{12}{7}\,\Omega$ $\quad R_y = R_1 // R_3 = 3\,\Omega$

$i_L = \dfrac{-8\times10^{-3} R_x}{R_x + R_y} = -2.91\ A$

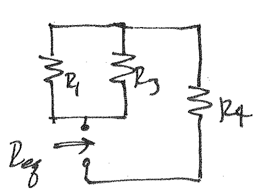
$t = 0^+$



$v_o(0^+) = 2.91 R_4 = 11.64V$
$= K_1 + K_2$

$t = \infty$ $\quad v_o = 0 = K_1$

$\tau = ?$



$R_{eq} = R_4 + (R_1 // R_3) = 7\,\Omega$

$\tau = L/R_{eq} = \frac{2}{7}\ s$

$$\boxed{v_o(t) = 11.64 e^{-3.5t}\ V}$$

7.46 Use the step-by-step method to find $v_o(t)$ for $t > 0$ in the network in Fig. P7.46.

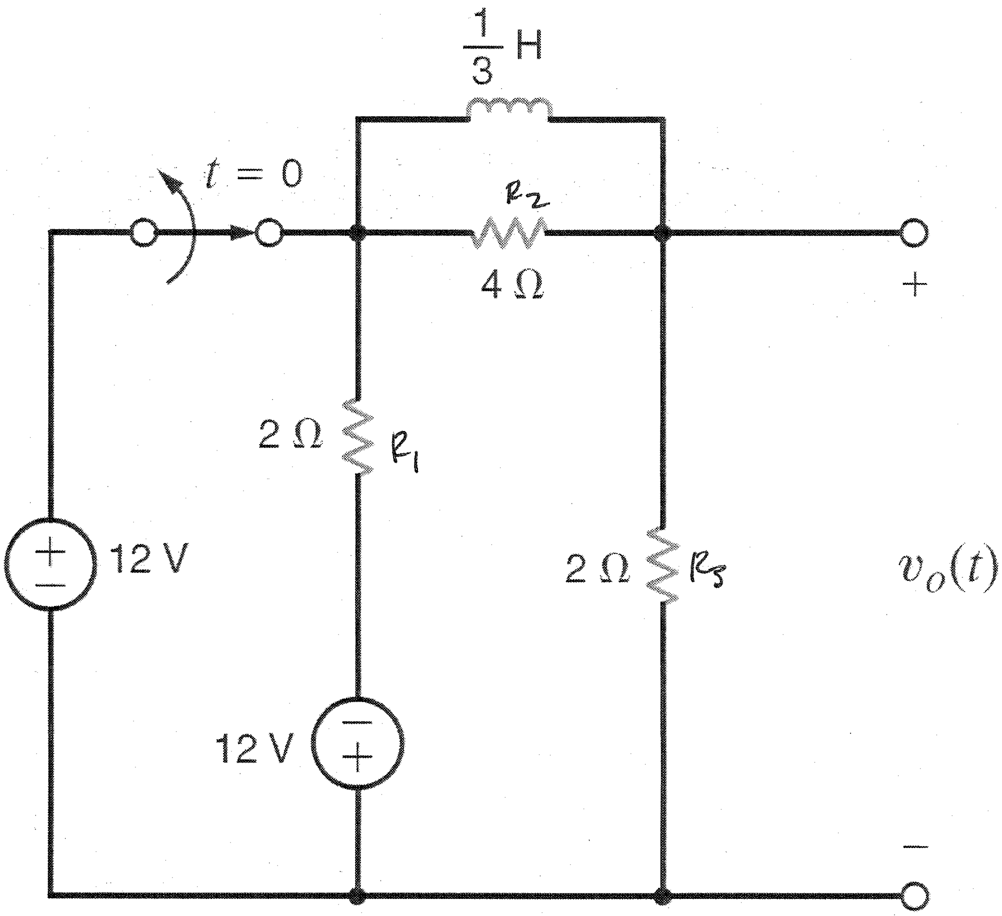
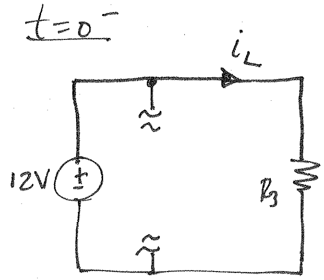


Figure P7.46

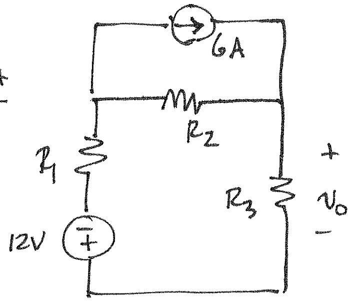
SOLUTION: $v_o(t) = K_1 + K_2 e^{-t/\tau}$

$t = 0^-$

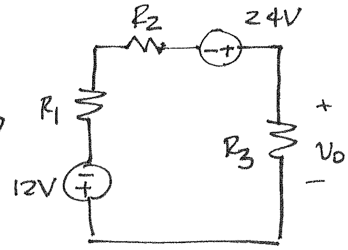


$i_L(0^-) = \frac{12}{R_3} = 6\,A$

$t = 0^+$



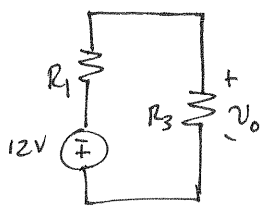
$\Rightarrow$



$$v_o = \frac{(24-12)\,R_3}{R_1+R_2+R_3}$$

$$v_o(0^+) = 3\,V = K_1 + K_2$$

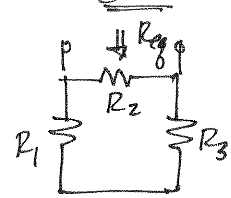
$t = \infty$



$$v_o(\infty) = \frac{-12\,R_3}{R_1+R_3}$$

$$v_o(\infty) = -6V = K_1$$

$\tau = ?$



$$R_{eq} = R_2 \,//\, (R_1+R_3)$$

$$R_{eq} = 2\Omega$$

$$\tau = \frac{L}{R_{eq}} = \frac{1}{6}\ s$$

$$\boxed{v_o(t) = -6 + 9e^{-6t}\ V}$$

7.47 Find $v_o(t)$ for $t > 0$ in the circuit in Fig. P7.47 using the step-by-step method.

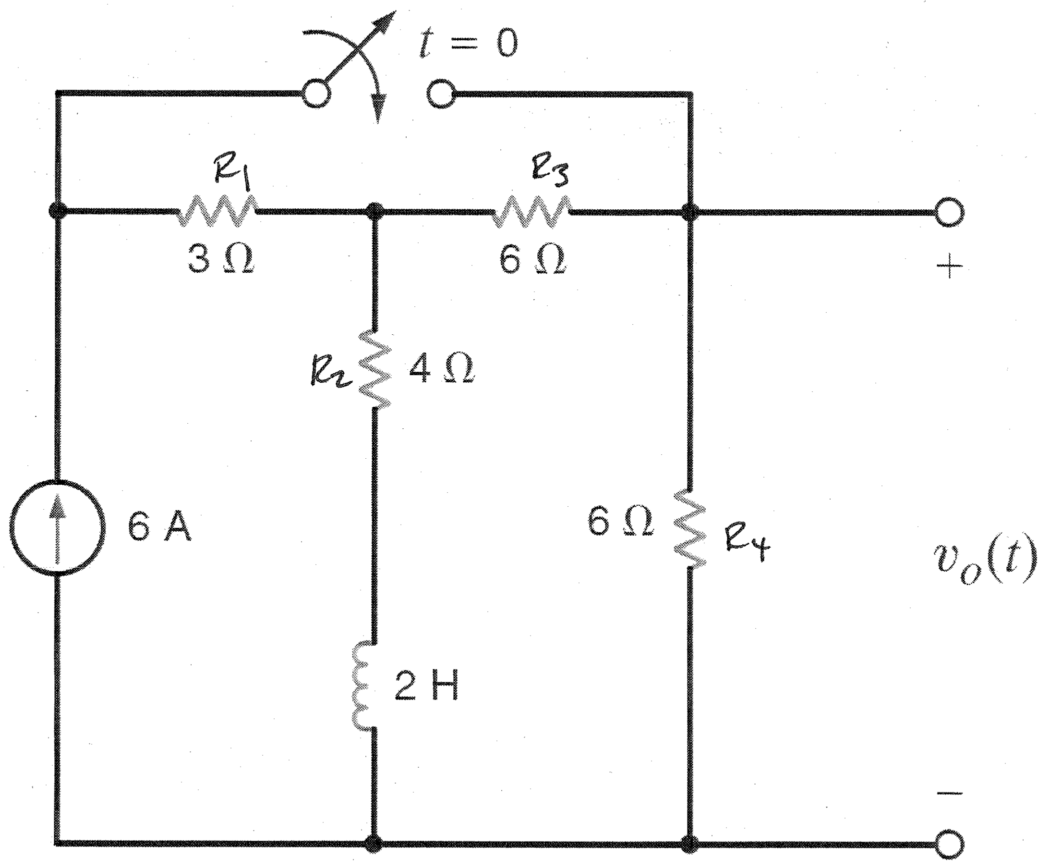
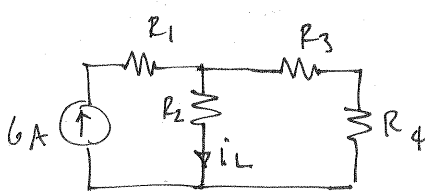


Figure P7.47

SOLUTION: $v_o(t) = K_1 + K_2 e^{-t/\tau}$

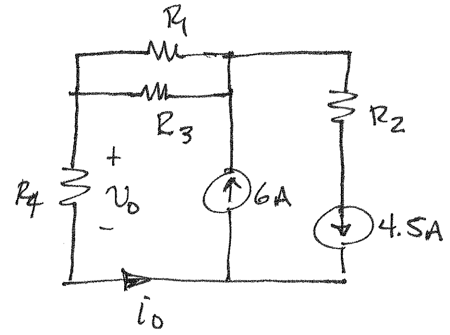
$t = 0^-$



$R_x = R_3 + R_4 = 12\Omega$

$i_L = \frac{6\, R_x}{R_x + R_2} = 4.5A$

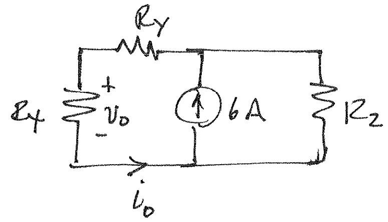
$\underline{t=0^+}$



$i_o = 6 - 4.5 = 1.5A$

$v_o = i_o R_4 = 9V = K_1 + K_2$

$\underline{t=\infty}$

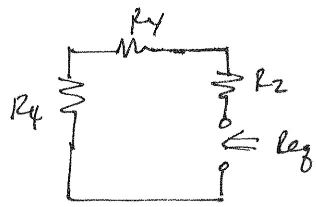


$R_y = R_1 // R_3 = 2\Omega$

$i_o = \dfrac{6 R_2}{R_2 + R_y + R_4}$ $\quad i_o = 2A$

$v_o(\infty) = i_o R_4 = 12V = K_1$

$\underline{\tau = ?}$



$R_{eq} = R_2 + R_y + R_4 = 12\Omega$

$\tau = L/R_{eq} = 1/6 \ s$

$$\boxed{v_o(t) = 12 - 3e^{-6t} \ V}$$

7.48 Find $v_o(t)$ for $t > 0$ in the network in Fig. P7.48 using the step-by-step technique.

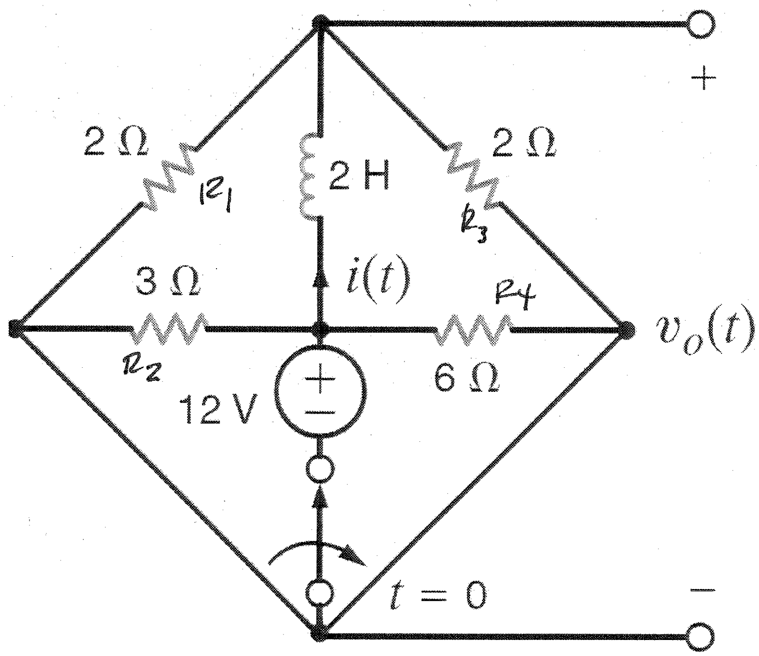
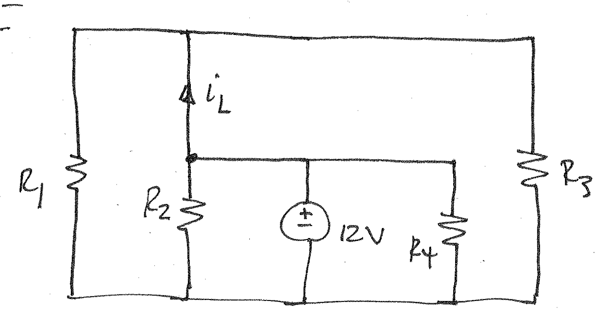
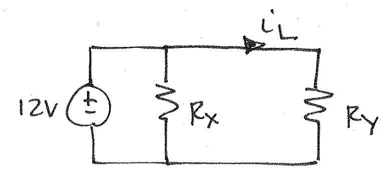


Figure P7.48

SOLUTION: $v_o(t) = K_1 + K_2 e^{-t/\tau}$

$t = 0^-$



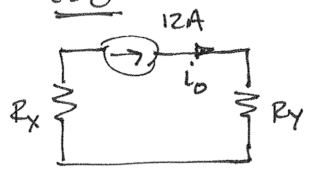


$R_x = R_2 // R_4 = 2\Omega$

$R_y = R_1 // R_3 = 1\Omega$

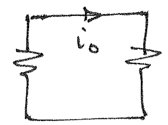
$i_L = \frac{12}{R_y} = 12A$

$\underline{t = 0^+}$



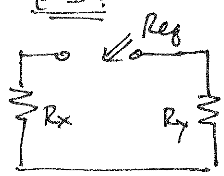
$i_o = 12A = K_1 + K_2$

$\underline{t = \infty}$



$i_o = 0 = K_1$

$\underline{\tau = ?}$



$R_{eq} = R_x + R_y$

$R_{eq} = 3\,\Omega$

$\tau = \dfrac{L}{R_{eq}} = \dfrac{2}{3}\,s$

$$\boxed{i_o = 12e^{-1.5t}\ A}$$

7.49 Use the step-by-step method to find $v_o(t)$ for $t > 0$ in the circuit in Fig. P7.49.

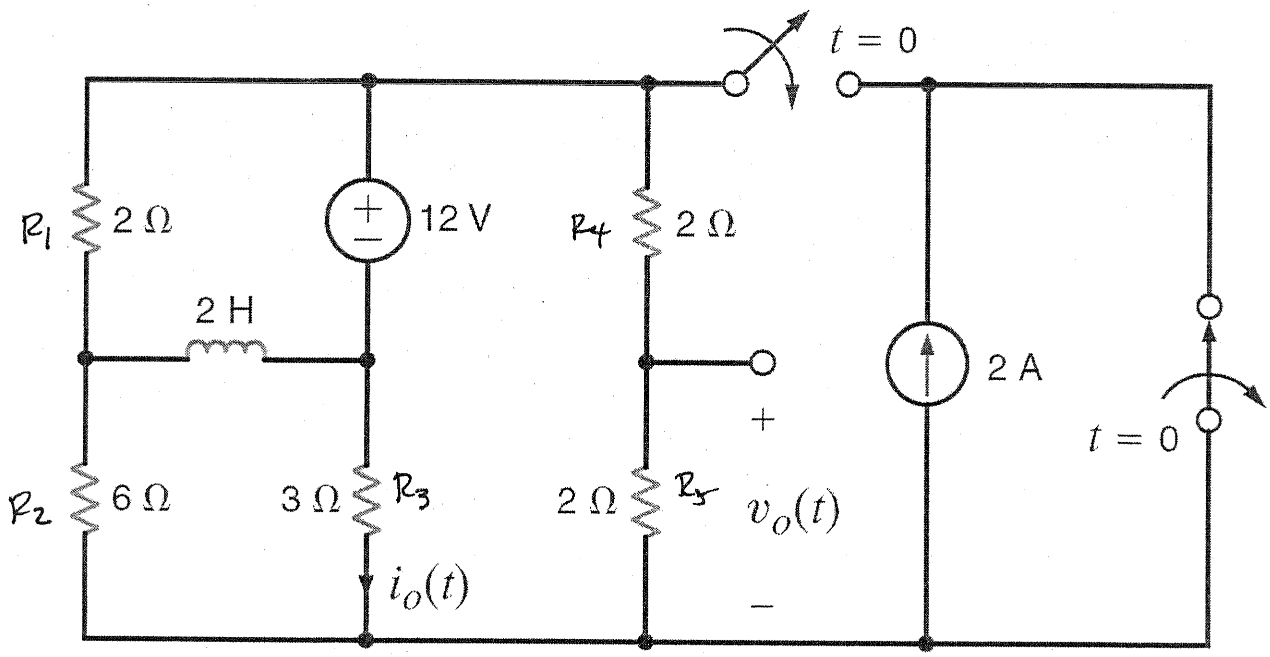
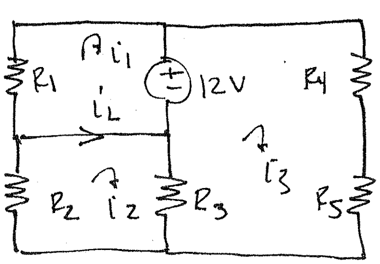


Figure P7.49

SOLUTION: $v_o(t) = K_1 + K_2 e^{-t/\tau}$

$t = 0^-$



mesh analysis:

$i_1 R_1 + 12 = 0 \Rightarrow i_1 = -6A$

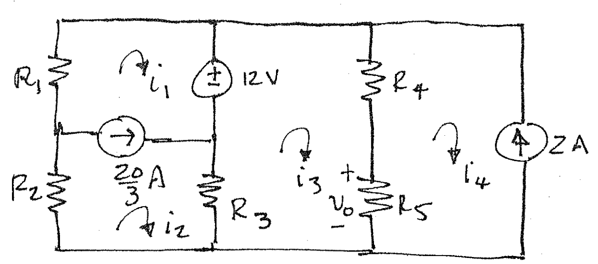
$i_2(R_2+R_3) - i_3 R_3 = 0 \Rightarrow i_3 = 3i_2$

$12 = i_3(R_3+R_4+R_5) - i_2 R_3 \Rightarrow 7i_3 - 3i_2 = 12$

yields $i_3 = 2A$ & $i_2 = \frac{2}{3}$ A

$i_L(0^-) = i_2 - i_1 = \frac{20}{3}$ A

$t = 0^+$



$i_2 - i_1 = 20/3$

$i_4 = -2$

$12 = i_3 (R_3 + R_4 + R_5) - i_2 R_3 - i_4 (R_4 + R_5)$

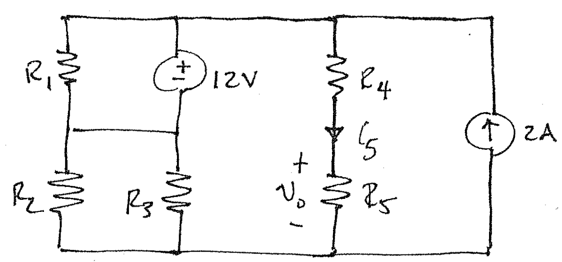
$0 = i_1 R_1 + i_2 R_2 + i_3 (R_4 + R_5) - i_4 (R_4 + R_5)$

$i_3 = 0.706 A$

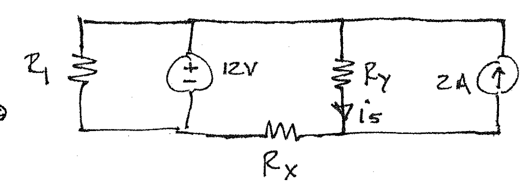
$v_o = R_5 (i_3 - i_4) = 5.41 V$

$5.41 = K_1 + K_2$

$t = \infty$



$\Rightarrow$



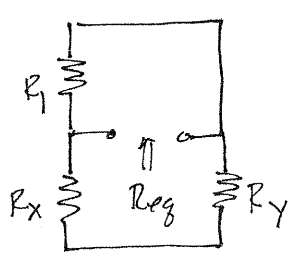
$R_X = R_2 // R_3 = 2\Omega \qquad R_Y = R_4 + R_5 = 4\Omega$

$v_o(\infty) = i_5 R_5 = 2 i_5$

Find i_5 by superposition: $i_5 = \frac{12}{R_X + R_Y} + \frac{2 R_X}{R_X + R_Y} = \frac{8}{3} A$

$v_o = \frac{16}{3} = 5.33 V = K_1$

$\tau = ?$



$R_{eq} = R_1 // (R_X + R_Y) = 1.5\Omega$

$\tau = L / R_{eq} = \frac{4}{3} s$

$$\boxed{v_o = 5.33 + 0.08 e^{-0.75t} \ V}$$

7.50 Find $i(t)$ for $t > 0$ in the circuit of Fig. P7.50 using the step-by-step method.

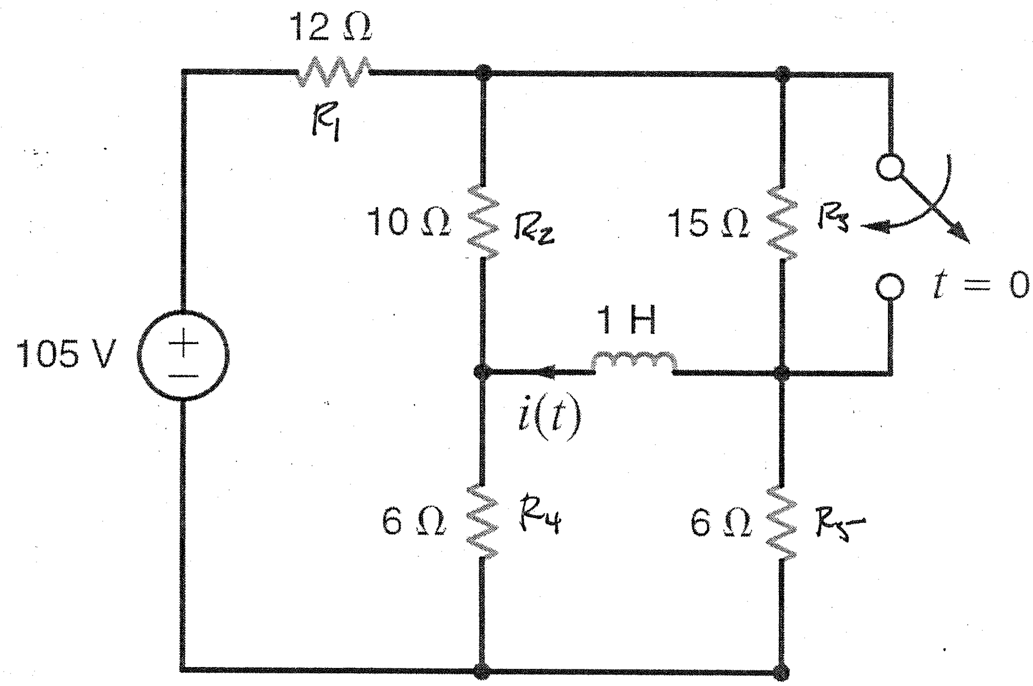
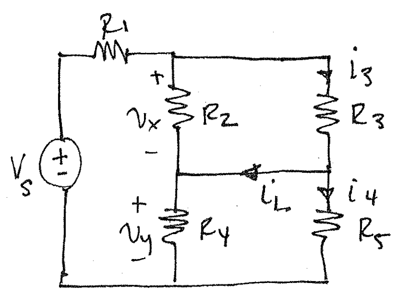


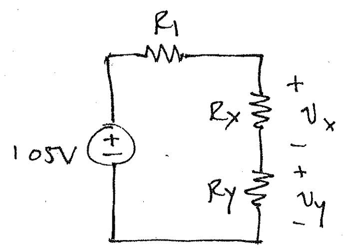
Figure P7.50

SOLUTION: $i(t) = K_1 + K_2 e^{-t/\tau}$

$t = 0^-$



$\Rightarrow$



$R_x = R_2 // R_3 = 6\,\Omega$

$R_y = R_4 // R_5 = 3\,\Omega$

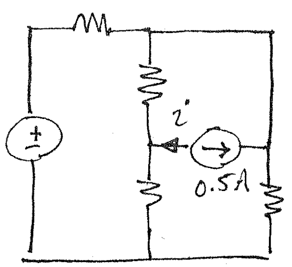
$$v_x = \frac{105\,R_x}{R_x + R_y + R_1} = 30\text{V}$$

$$v_y = \frac{105\,R_y}{R_1 + R_x + R_y} = 15\text{V}$$

$$i_3 = \frac{v_x}{R_3} = 2\text{A} \qquad i_4 = \frac{v_y}{R_5} = 2.5\text{A}$$

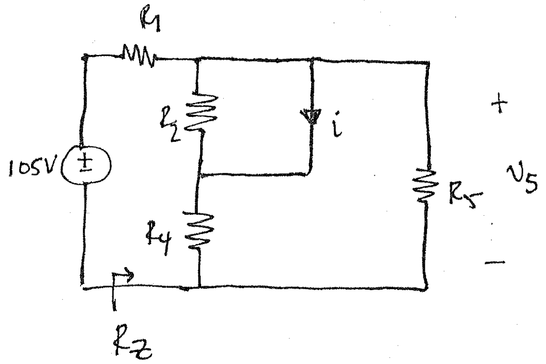
$$i_L = i_3 - i_4 = -0.5\text{A}$$

$\underline{t = 0^+}$



$i = -0.5A = K_1 + K_2$

$\underline{t = \infty}$

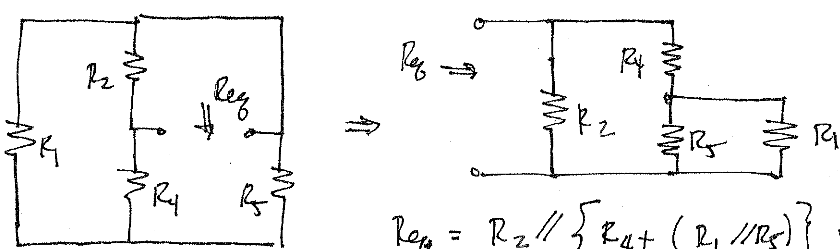


$R_Z = R_4 // R_5 = 3\Omega$

$v_5 = \dfrac{105 R_Z}{R_Z + R_1} = 21V$

$i(\infty) = \dfrac{v_5}{R_4} = 3.5A = K_1$

$\underline{\tau = ?}$



$R_{eq} = R_2 // \{R_4 + (R_1 // R_5)\} = 5\Omega$

$\tau = \dfrac{L}{R_{eq}} = \dfrac{1}{5} s$

$$\boxed{i(t) = 3.5 - 4e^{-5t} \text{ A}}$$

7.51 Find $v_C(t)$ for $t > 0$ in the circuit of Fig. P7.51 using the step-by-step method.

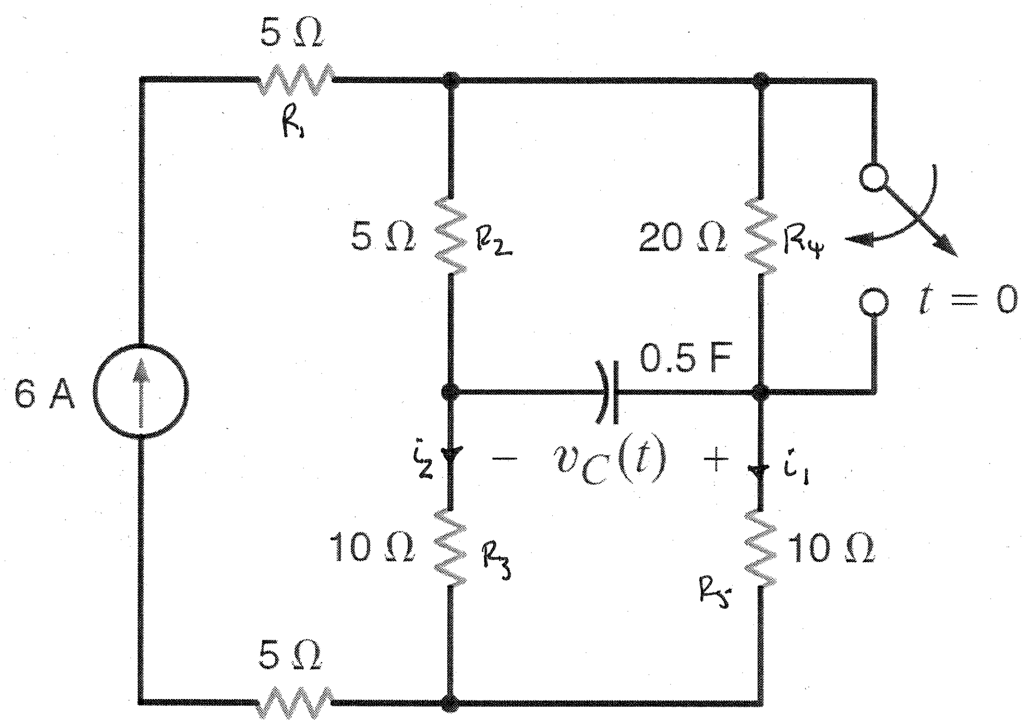


Figure P7.51

SOLUTION: $v_c(t) = K_1 + K_2 e^{-t/\tau}$

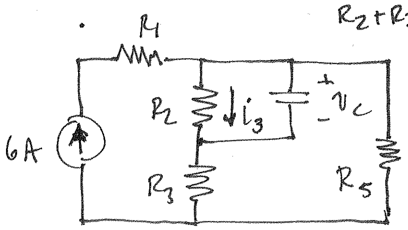
$\underline{t=0^-}$: $R_A = R_4 + R_5 = 30\,\Omega$ $R_B = R_2 + R_3 = 15\,\Omega$

$i_1 = \dfrac{6R_B}{R_B + R_A} = 2A$ $i_2 = \dfrac{6R_A}{R_A + R_B} = 4A$ $v_c(0^-) = i_1 R_5 - i_2 R_3$

$v_c(0^-) = -20V$

$\underline{t=0^+}$: $v_c(0^+) = v_c(0^-) = -20 = K_1 + K_2$

$\underline{t \to \infty}$: $i_3 = \dfrac{6R_5}{R_2 + R_3 + R_5} = 2.4A$ $v_c(\infty) = R_2 i_3 = 12V = K_1$



$\tau = C\left[R_2 // (R_3 + R_5)\right] = 2s$

$$\boxed{v_c(t) = 12 - 32e^{-t/2}\ V}$$

7.52 Find $i(t)$ for $t > 0$ in the circuit of Fig. P7.52 using the step-by-step method.

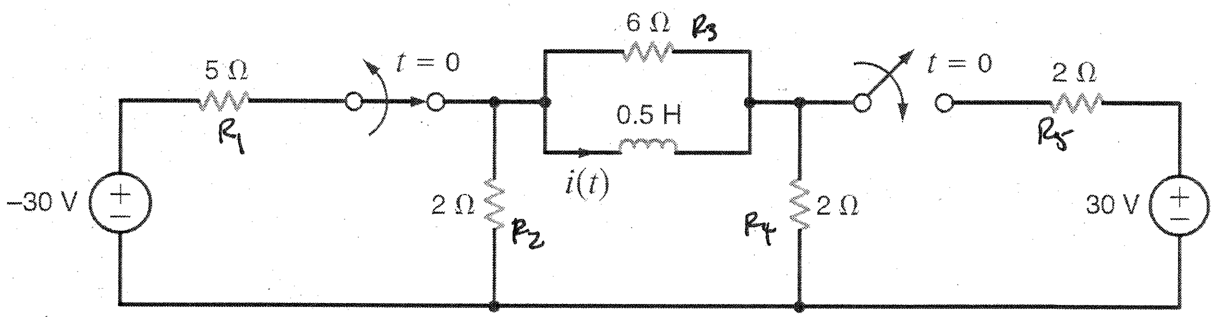
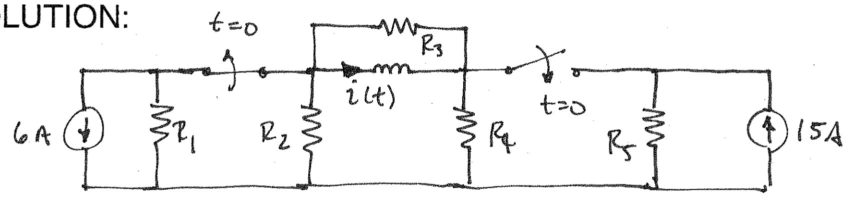


Figure P7.52

SOLUTION:

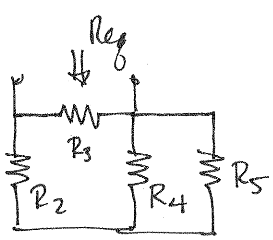


$t=0^-$: $\quad i(0^-) = \dfrac{-6\,(1/R_4)}{\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_4}} = -2.5\text{ A}$

$t=0^+$: $\quad i(0^+) = i(0^-) = -2.5\text{ A} = K_1 + K_2$

$K_2 = 2.5\text{ A}$

$t\to\infty$: $\quad i(\infty) = \dfrac{-15\,(1/R_2)}{\frac{1}{R_2}+\frac{1}{R_4}+1/R_5} = -5\text{A} = K_1$



$\tau = \dfrac{L}{R_{eq}}$

$t = \frac{1}{4}\text{ s}$

$R_{eq} = R_3 \,//\, [R_2 + R_A]$

$R_{eq} = 2\Omega$

$R_A = R_4 // R_5$

$R_A = 1\Omega$

$$\boxed{i(t) = -5 + 2.5e^{-4t}\text{ A}}$$

7.53 Find $i_o(t)$ for $t > 0$ in the circuit in Fig. P7.53 using the step-by-step method.

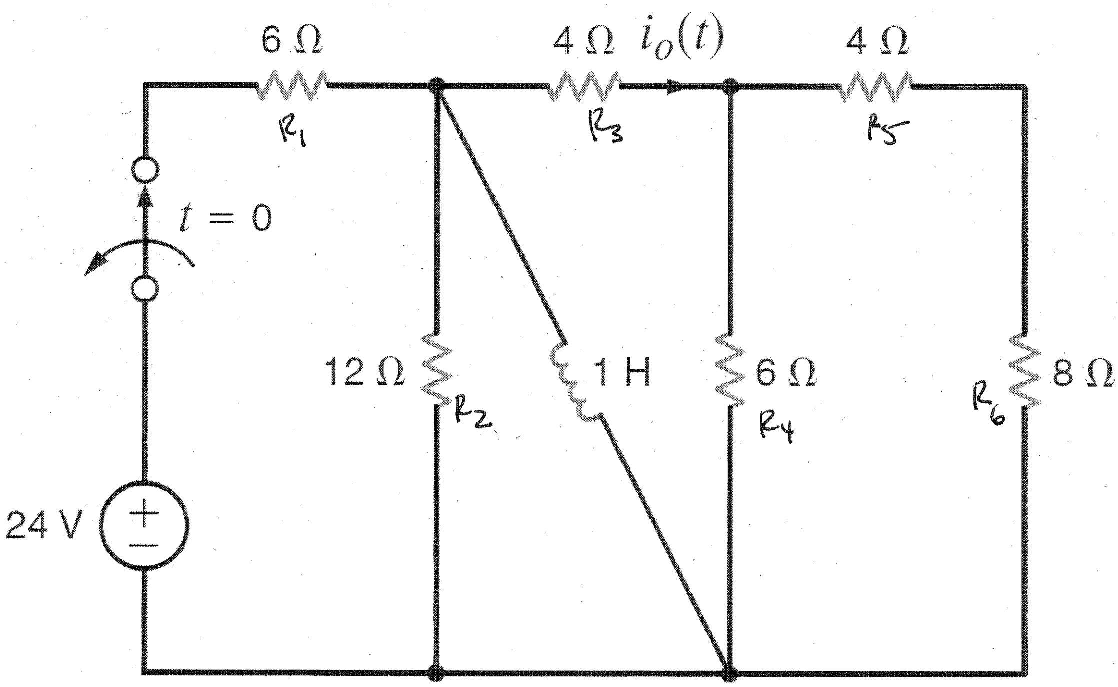
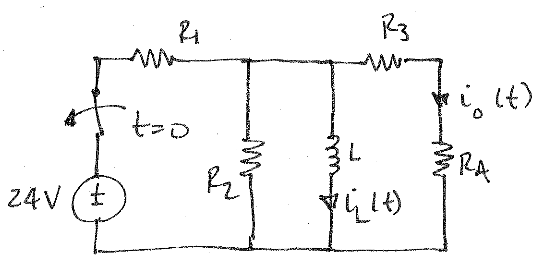


Figure P7.53

SOLUTION:



$R_A = R_4 \,//\, (R_5 + R_6) = 4\Omega$

$\underline{t=0^-}$: $i_L(0^-) = \dfrac{24}{R_1} = 4\,A$

$\underline{t=0^+}$: $i_o = \dfrac{-i_L(0^-)\,R_2}{R_2 + R_3 + R_A} = -2.4A = K_1 + K_2$

$\underline{t\to\infty}$: $i_o = 0 = K_1$

$\tau = L/R_{eq}$ $\quad R_{eq} = R_2 \,//\, (R_3 + R_A) = 4.8\,\Omega$ $\quad \tau = 0.208\,s$

$$\boxed{i_o(t) = -2.4e^{-4.8t}\ A}$$

7.54 Find $v_o(t)$ for $t > 0$ in the network in Fig. P7.54 using the step-by-step method.

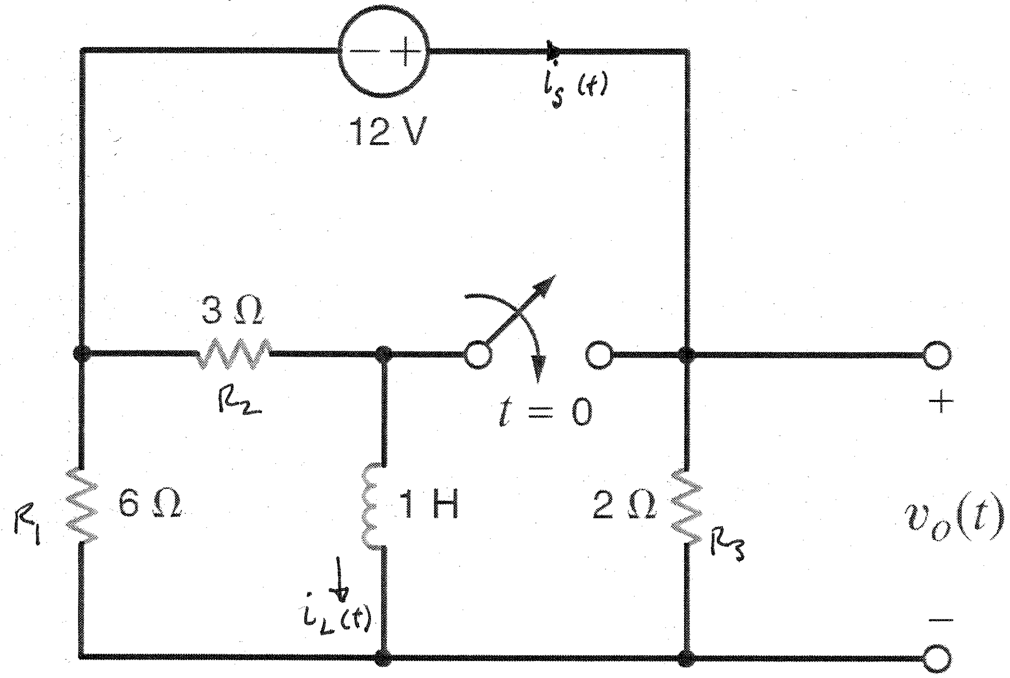
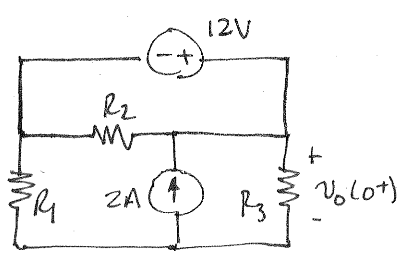


Figure P7.54

SOLUTION:

$\underline{t=0^-}$: $\quad i_s = \dfrac{12}{R_3 + (R_1 // R_2)} = 3\text{A} \qquad i_L = \dfrac{-i_s R_1}{R_1 + R_2} = -2\text{A}$

$\underline{t=0^+}$: $\quad$ By superposition: $\quad v_o(0^+) = \dfrac{12 R_3}{R_1 + R_3} + \dfrac{2 R_1 R_3}{R_1 + R_3} = 6\text{V} = K_1 + K_2$



$\underline{t \to \infty}$: $\quad v_o(\infty) = 0 = K_1$

$R_{eq} = R_1 // R_3 = 1.5\,\Omega \qquad \tau = \dfrac{L}{R} = \dfrac{2}{3}\,s$

$$\boxed{v_o(t) = 6e^{-1.5t}\ \text{V}}$$

7.55 Find $i_o(t)$ for $t > 0$ in the network in Fig. P7.55 using the step-by-step method. **PSV**

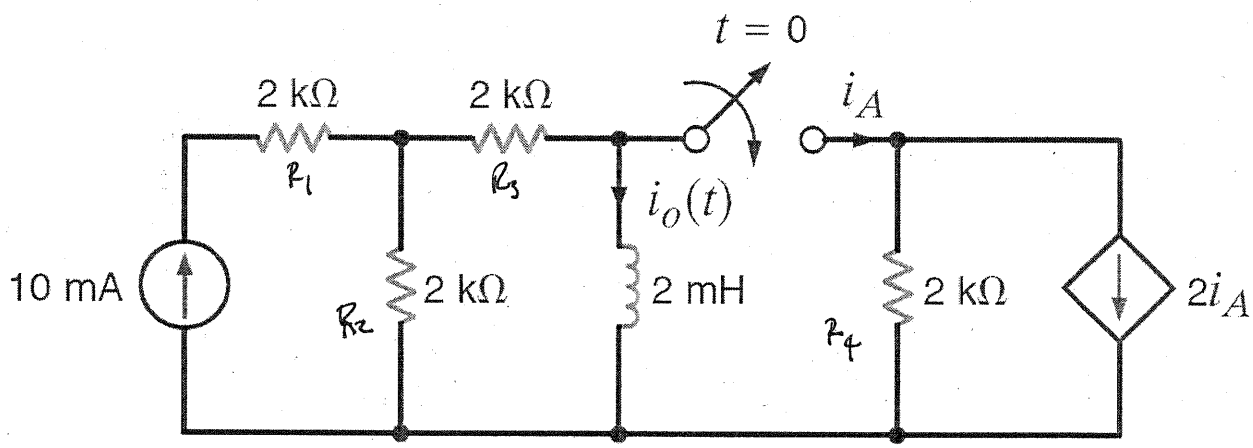
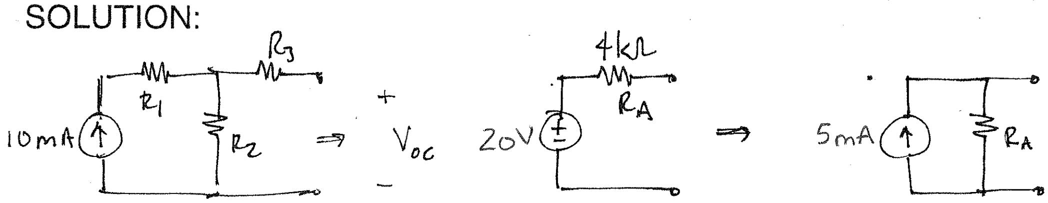
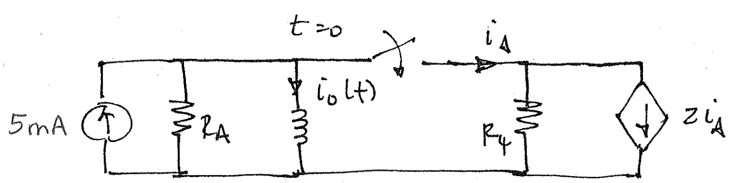


Figure P7.55

SOLUTION:

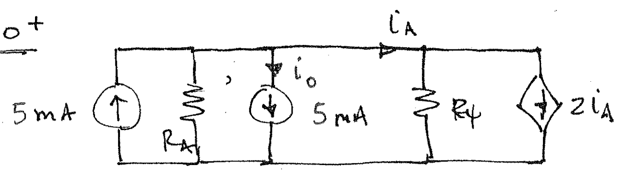




$\underline{t = 0^-}$ $\quad i_A = 0$

$i_o = 5 \times 10^{-3} = 5\ mA$

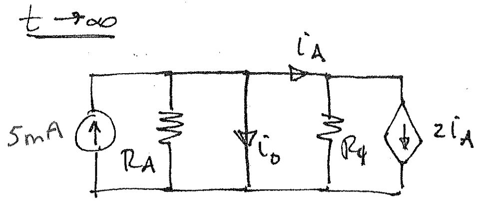
$\underline{t = 0^+}$



$i_o(0^+) = i_o(0^-) = 5\ mA = K_1 + K_2$

$\underline{t \to \infty}$

$i_A = 0 \qquad i_o = 0.5mA = K_1 \Rightarrow K_2 = 0$



$\boxed{i_o(t) = 5\ mA}$

7.56 Find $i_L(t)$ for $t > 0$ in the circuit of Fig. P7.56 using the step-by-step method.

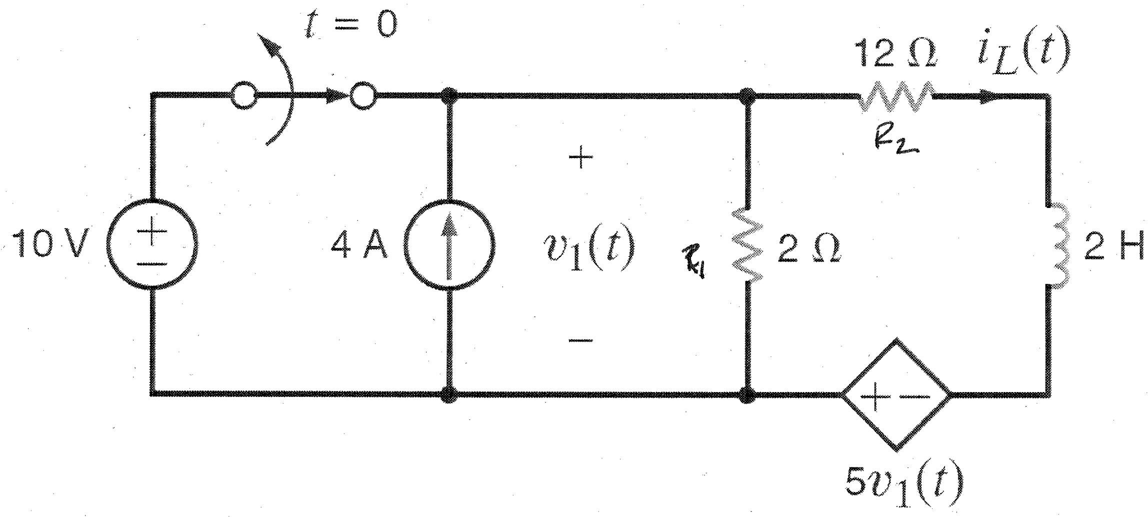


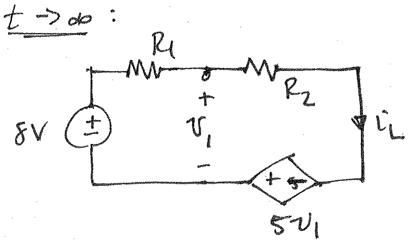
Figure P7.56

SOLUTION:

$\underline{t=0^-}$: $v_1 = 10\text{V}$ $\quad v_1 = R_2 i_L - 5v_1 \Rightarrow i_L = \frac{6v_1}{R_2} = 5\text{A}$

$\underline{t=0^+}$ $\quad i_L(0^+) = i_L(0^-) = 5A = K_1 + K_2$

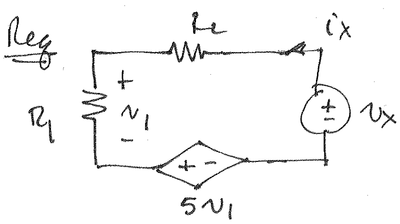
$\underline{t \to \infty}$:



$i_L = 6v_1/R_2 \Rightarrow v_1 = R_2 i_L/6$

$8 = i_L(R_1+R_2) - 5v_1 \Rightarrow i_L(\infty) = 2A = K_1$

$\underline{R_{eq}}$



$R_{eq} = v_x/i_x \qquad v_1 = i_x R_1$

$v_x = i_x(R_1+R_2) + 5v_1 = i_x(6R_1+R_2)$

$R_{eq} = 24\Omega \qquad \tau = L/R_{eq} = \frac{1}{12}\text{ s}$

$$\boxed{i_L(t) = 2 + 3e^{-12t}\ \text{A}}$$

7.57 Use the step-by-step technique to find $v_o(t)$ for $t > 0$ in the network in Fig. P7.57.

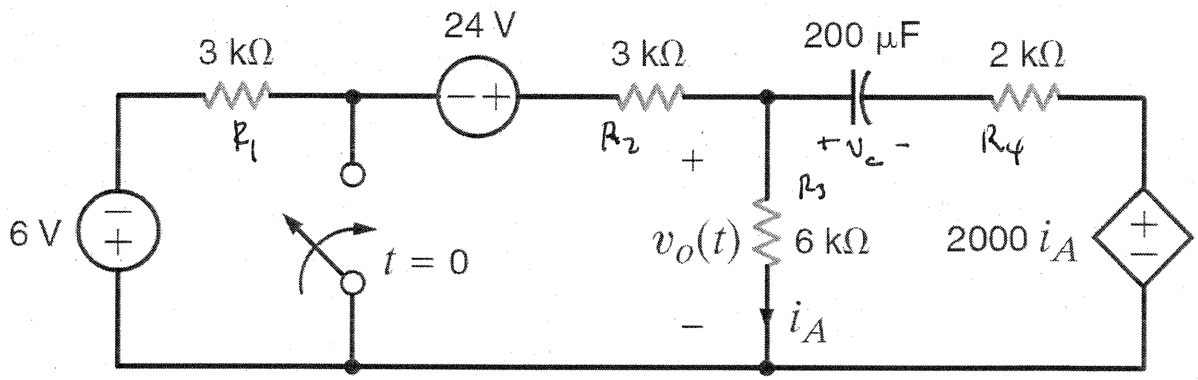


Figure P7.57

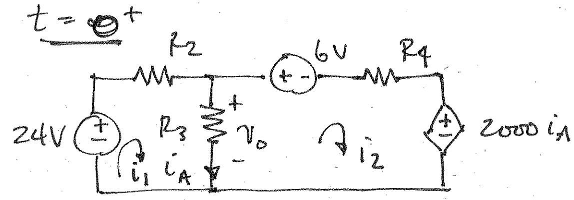
SOLUTION:

$t = 0^-$: $\quad 6 - 24 + i_A R_1 + i_A R_2 + i_A R_3 = 0 \qquad i_A = 1.5\text{mA}$

$v_c(0^-) = v_0(0^-) - 2000 i_A(0^-) \qquad v_0(0^-) = R_3 i_A(0^-) = 9\text{V}$

$v_c(0^-) = 6\text{V}$

$t = 0^+$



$24 = i_1(R_2 + R_3) - R_3 i_2$

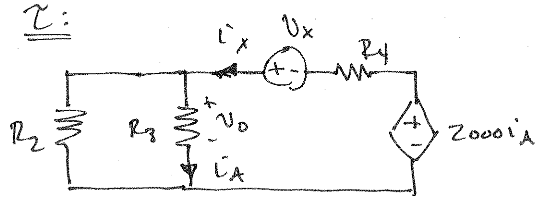
$-6 = i_2(R_3 + R_4) - R_3 i_1 + 2000 i_A$

$i_A = i_1 - i_2$

$v_0(0^+) = i_A R_3 = 13.2\text{V} = K_1 + K_2$

$t = \infty$: $\quad i_A = \dfrac{24}{R_2 + R_3} = 2.67\text{A} \qquad v_0 = i_A R_3 = 16\text{V} = K_1$

τ:



$R_{eq} = v_x / i_x$

$v_x + 2000 i_A = i_x (R_A + R_4)$; $R_A = R_2 // R_3 = 2\text{k}\Omega$

$i_A = \dfrac{i_x R_2}{R_2 + R_3} = i_x/3 \Rightarrow R_{eq} = 3.33\text{k}\Omega$

$\tau = C R_{eq} = 0.667\text{ s}$

$$\boxed{v_0(t) = 16 - 2.8 e^{-1.5t} \text{ V}}$$

7.58 Use the step-by-step method to find $v_o(t)$ for $t > 0$ in the network in Fig. P7.58. CS

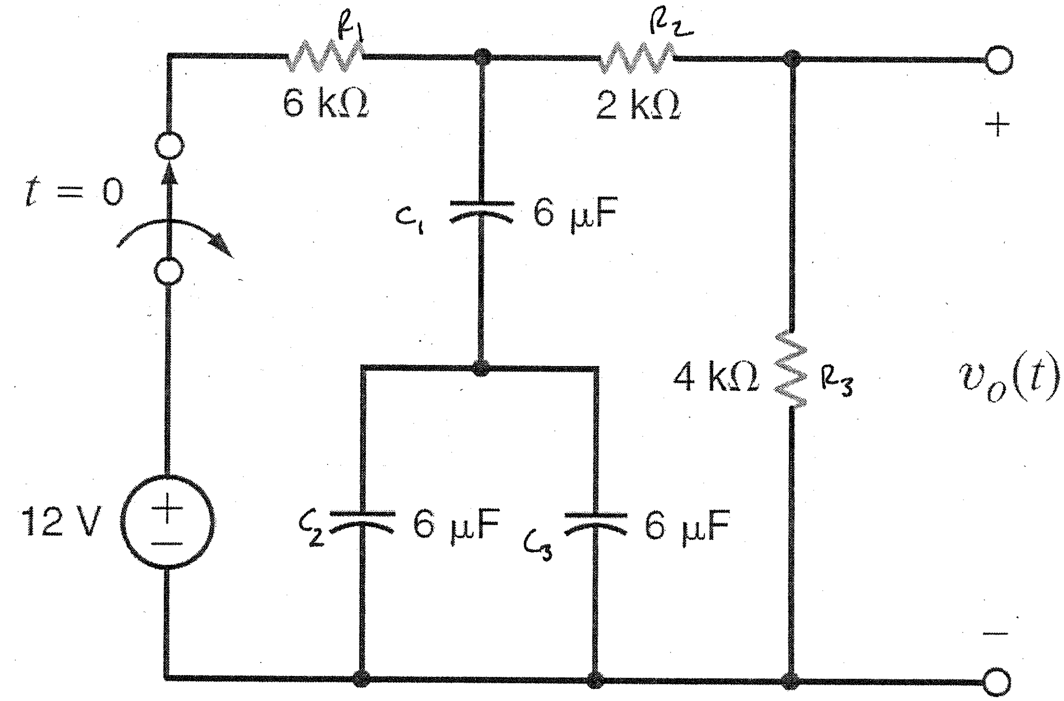
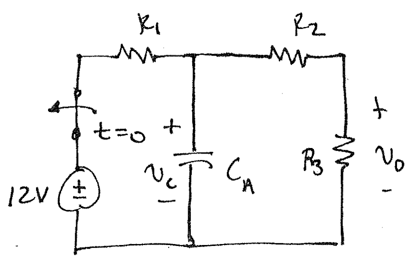


Figure P7.58

SOLUTION:



$$C_A = \frac{C_1(C_2+C_3)}{C_1+C_2+C_3} = 4\mu F$$

$t=0^-$: $v_c = \dfrac{12(R_2+R_3)}{R_1+R_2+R_3} = 6\,V$

$t=0^+$: $v_c = 6\,V$ $\quad v_o = \dfrac{v_c R_3}{R_2+R_3} = 4\,V = K_1 + K_2$

$t=\infty$: $v_o = 0 = K_1$

τ: $\quad R_{eq} = R_2 + R_3 = 6k\Omega \qquad \tau = C_A R_{eq} = 24\,ms$

$$\boxed{v_o(t) = 4e^{-41.67t}\ V}$$

7.59 Find $i_o(t)$ for $t > 0$ in the circuit in Fig. P7.59 using the step-by-step method. CS

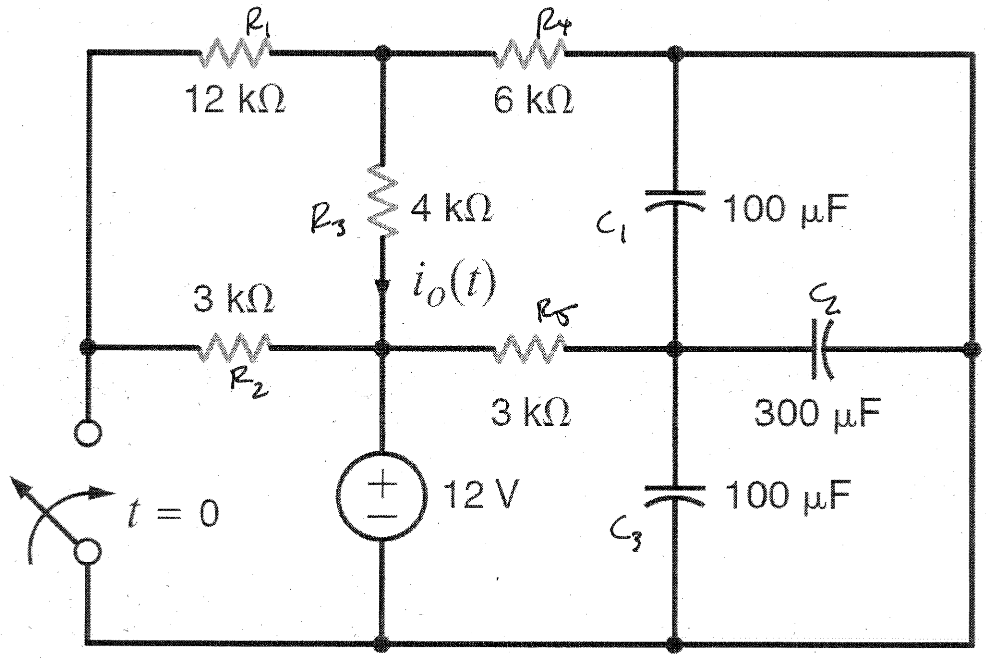
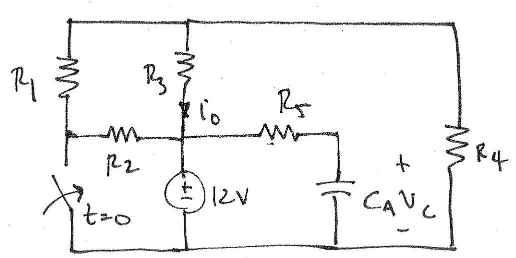


Figure P7.59

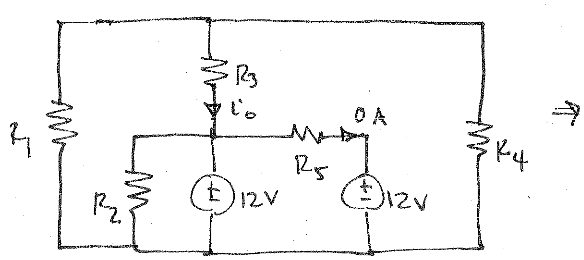
SOLUTION:



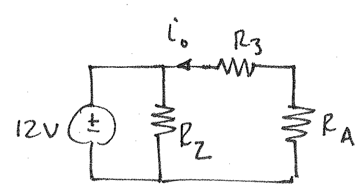
$C_A = C_1 + C_2 + C_3 = 500\mu F$

$t = 0^-$: $v_c = 12V$

$t = 0^+$: $v_c = 12V$



$\Rightarrow$



$R_A = R_1 // R_4 = 4k\Omega$

$i_o = \dfrac{-12}{R_3 + R_A} = -1.5mA = K_1 + K_2$

$\underline{\underline{t = \infty}}$ Same situation as $t = 0^+$, $i_o = -1.5mA = K_1 \Rightarrow K_2 = 0$

$i_o(t) = -1.5mA$

7.60 Find $v_o(t)$ for $t > 0$ in the network in Fig. P7.60 using the step-by-step method.

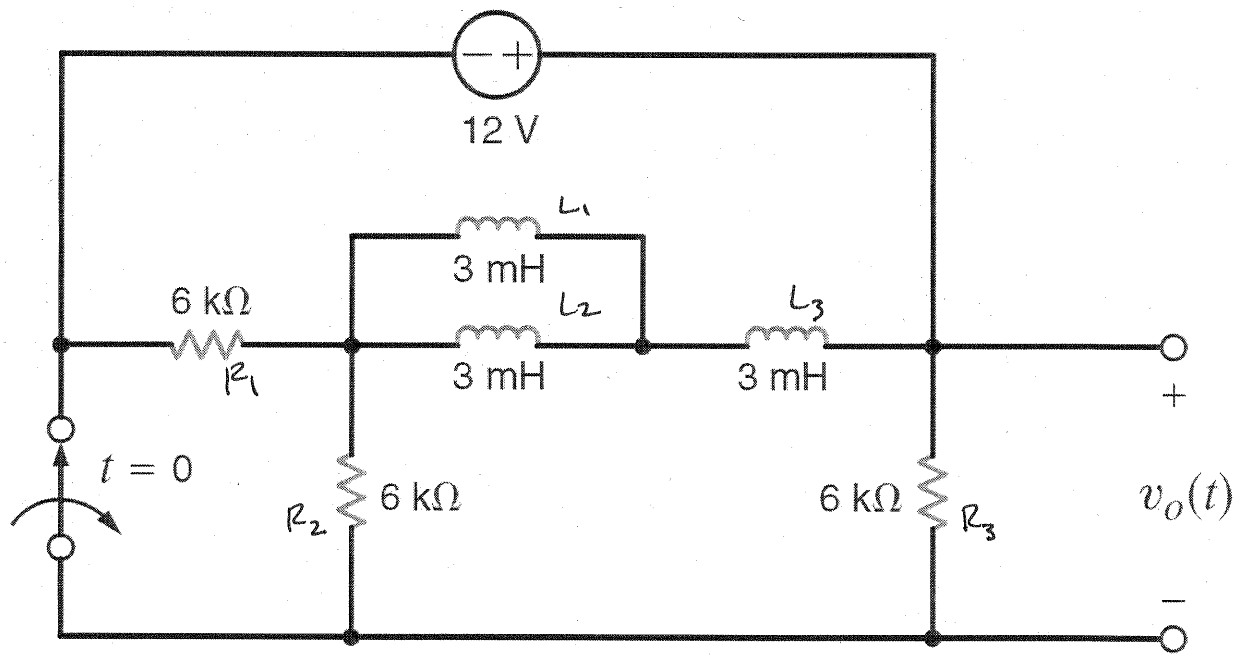
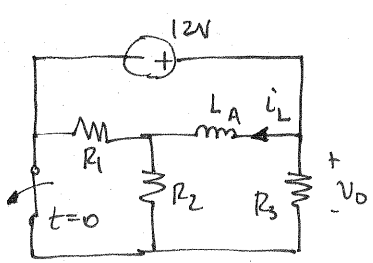


Figure P7.60

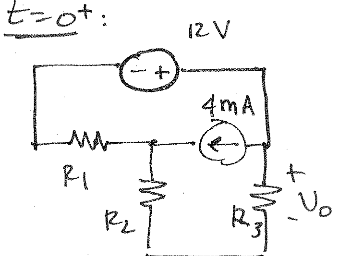
SOLUTION:



$$L_A = L_3 + \frac{L_1 L_2}{L_1 + L_2} = 4.5 mH$$

$\underline{t = 0^-}$ $\quad i_L = \frac{12}{R_1} + \frac{12}{R_2} = 4 mA$

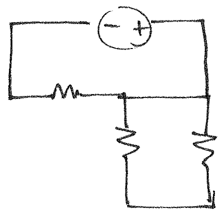
$\underline{t = 0^+}$:



Superposition: $V_o = \frac{12 R_3}{R_1 + R_2 + R_3} - \frac{4 \times 10^{-3} R_1}{R_1 + R_2 + R_3} R_3$

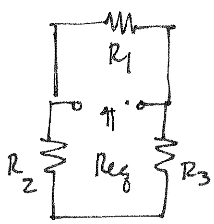
$$V_o(0+) = -4 V = K_1 + K_2$$

$\underline{t=\infty}$:



$v_o(\infty) = 0\,V = K_1$

$\underline{\tau = ?}$



$R_{eq} = \frac{R_1(R_2+R_3)}{R_1+R_2+R_3} = 4\,k\Omega$ $\quad \tau = L/R_{eq} = 1.125\,\mu s$

$$\boxed{v_o(t) = -4e^{-8.88\times10^{5}t}\ V}$$

7.61 Use the step-by-step method to find $i_o(t)$ for $t > 0$ in the network in Fig. P7.61.

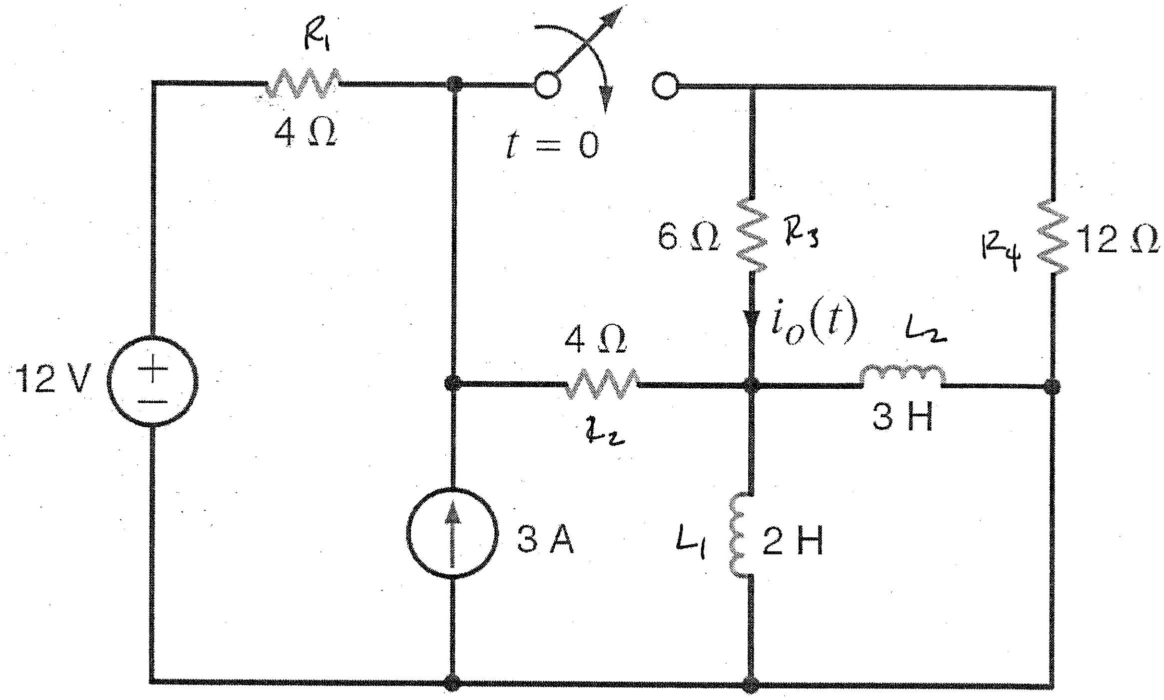
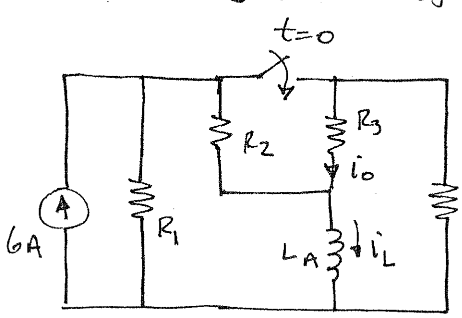


Figure P7.61

SOLUTION: Soure transformation

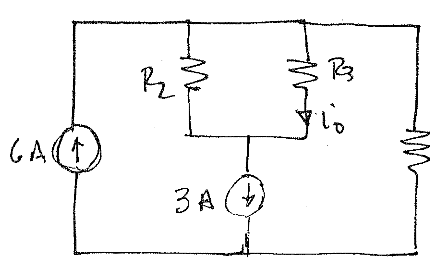


$\underline{t=0^-}$ $\quad i_L = \dfrac{6\,(1/R_2)}{\frac{1}{R_1}+\frac{1}{R_2}}$ $\qquad R_A = R_2 // R_3 = 2.4\Omega$

$i_L = 3A$ $\qquad L_A = \dfrac{L_1 L_2}{L_1+L_2} = 1.2\ H$

$\underline{t=0^+}$:

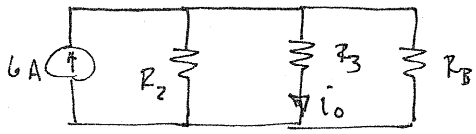
Superposition:



$i_o = \dfrac{3\,R_2}{R_2+R_3} =$ $\qquad 1.2\ A = K_1 + K_2$

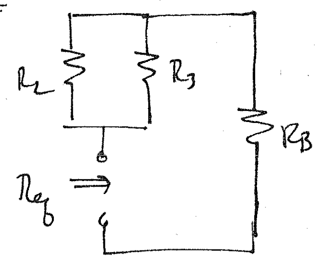
$R_B = R_1 // R_4 = 3\Omega$

$\underline{\underline{t=\infty}}$



$$i_o = \frac{6\left(1/R_3\right)}{\frac{1}{R_2}+\frac{1}{R_3}+\frac{1}{R_B}} = \frac{4}{3}\,A = K_1$$

$\underline{\underline{\tau}}$



$$R_{eq} = (R_2 \,//\, R_3) + R_B = 5.4\,\Omega$$

$$\tau = \frac{L_A}{R_{eq}} = 222\ \text{mS}$$

$$\boxed{i_o(t) = 1.33 - 0.133\,e^{-4.5t}\ A}$$

7.62 The current source in the network in Fig. P7.62a is defined in Fig. P7.62b. The initial voltage across the capacitor must be zero. (Why?) Determine the current $i_o(t)$ for $t > 0$.

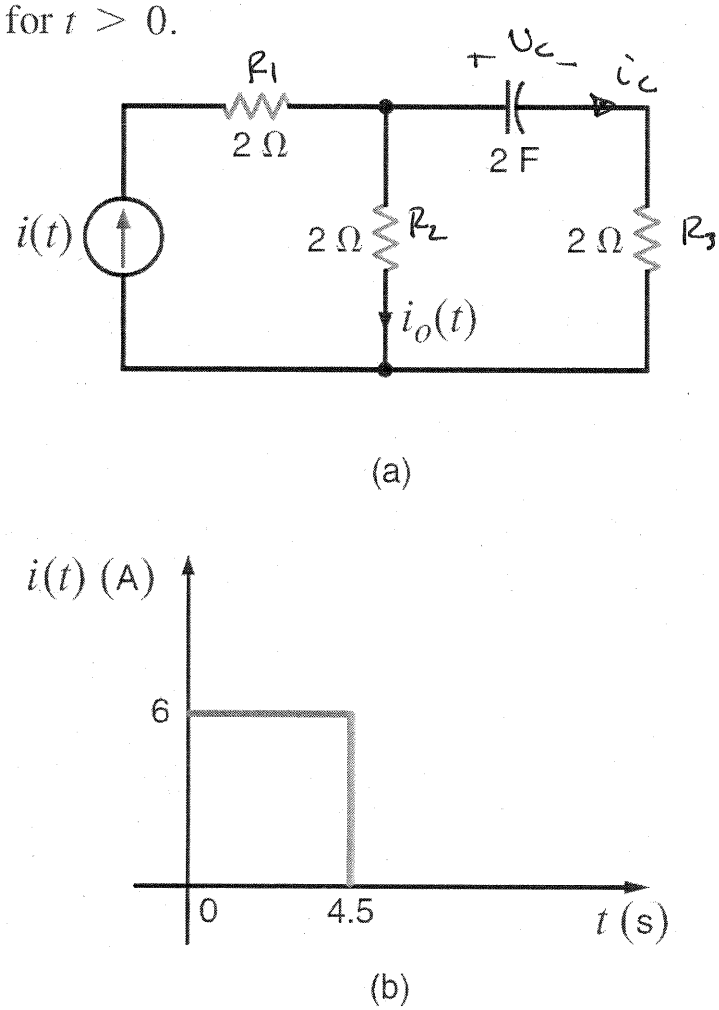


Figure P7.62

SOLUTION:

Since $i(t)$ is $\emptyset$ for $t<0$, no charge has accumulated on the capacitor and v_c must be $\emptyset$.

$v_c(t) = K_3 + K_4 e^{-t/\tau}$

$\underline{t=0^-}$: $v_c = 0$

$i_o(t) = K_1 + K_2 e^{-t/\tau}$

$\underline{t=0^+}$ $v_c = 0$, $i_o = \dfrac{i R_3}{R_2+R_3} = 3A = K_1 + K_2$

$t = \infty$ $\quad i_c = 0\,A \qquad i_o = 6 = K_1 \qquad v_c = i_o R_2 = 12V = K_3$

$K_2 = -3\,A \qquad K_4 = -12V$

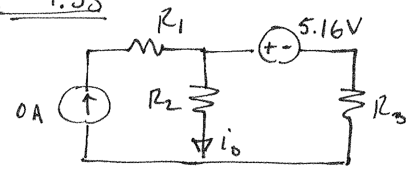
$\tau = C R_{eq} = C\,(R_2 + R_3) = 8s$

$$\left.\begin{aligned} i_o(t) &= 6 - 3e^{-t/8}\ A \\ v_c(t) &= 12 - 12e^{-t/8}\ V \end{aligned}\right\} \qquad 0 \le t \le 4.5s$$

$t = 4.5s$ $\quad v_c(4.5) = 5.16\,V \qquad i_o = K_5 + K_6 e^{-t'/\tau_2} \qquad t > 4.5s$

$t' = t - 4.5$

$t = 4.5s^+$



$i_o = \dfrac{5.16}{R_2 + R_3} = 1.29 = K_5 + K_6$

$t \to \infty$ $\quad i_o = 0 = K_5 \Rightarrow K_6 = 1.29A$

$\tau_2 = C R_{eq2} = C\,[R_2 + R_3] = 8s$

$i_o(t) = 1.29e^{-t'/8}\ A \qquad t > 4.5s$

final answer

$$\boxed{i_o(t) = \begin{cases} 6 - 3e^{-t/8}\ A & 0 \le t \le 4.5s \\ 1.29e^{-(t-4.5)/8}\ A & t > 4.5s \end{cases}}$$

7.63 Determine the equation for the voltage $v_o(t)$ for $t > 0$, in Fig. P7.63a when subjected to the input pulse shown in Fig. P7.63b.

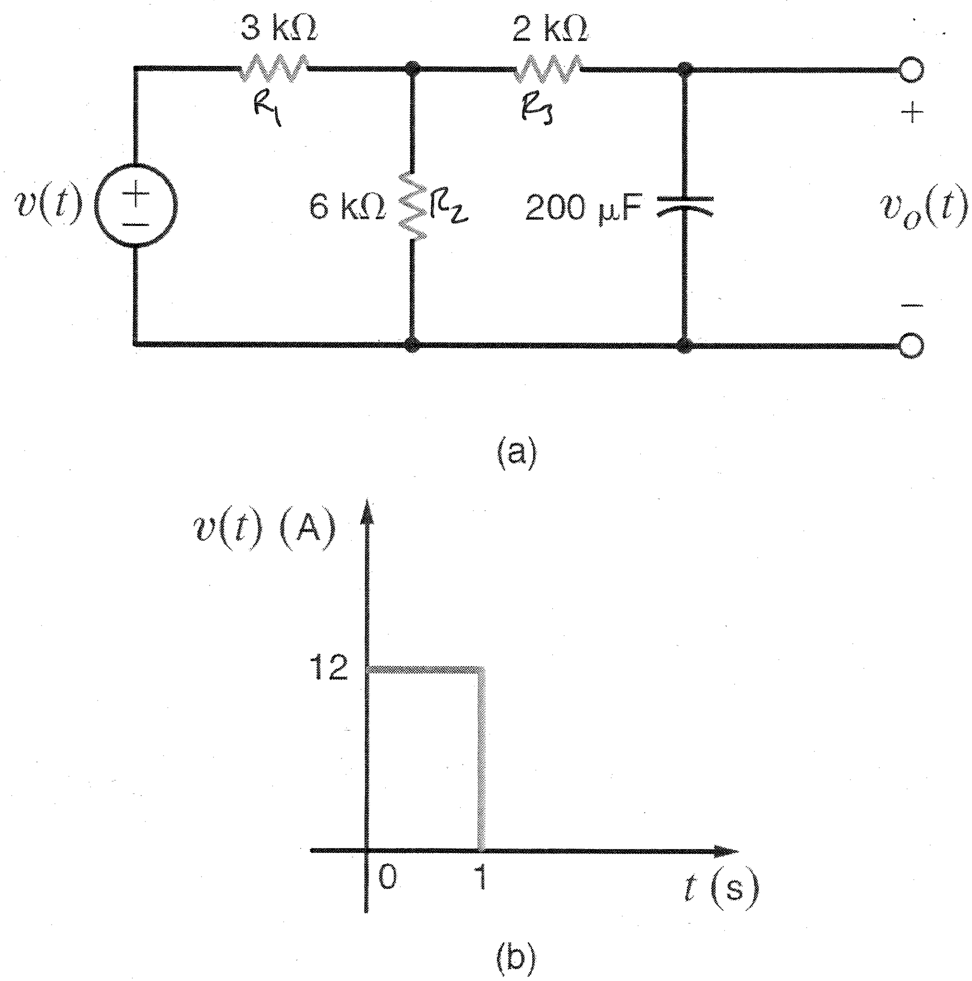
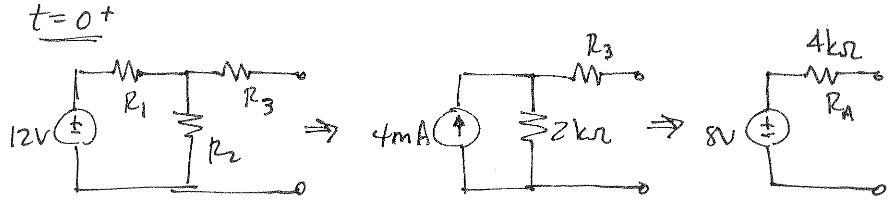


Figure P7.63

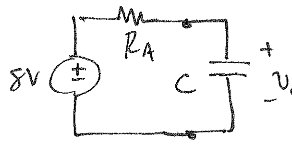
SOLUTION: $v_o = K_1 + K_2 e^{-t/\tau}$

$t = 0^-$ $\quad v_o = 0$

$t = 0^+$



$\underline{t=0^+}$



$v_o = v_o(0^-) = 0\,V = K_1 + K_2$

$\underline{t=\infty}$: $v_o = 8V = K_1$

$\underline{\tau}$ $\quad \tau = CR_A = 0.8s$

$v_o(t) = 8 - 8e^{-1.25t} \quad V \qquad 0 < t \leq 1$

for $t > 1s$, $\quad v_o = K_3 + K_4 e^{-t'/\tau} \qquad t' = t-1$

$\underline{\text{at } t = 1^-}$, $\quad v_o = 5.71\,V$

$\underline{\text{at } t = 1^+}$, $\quad v_o = 5.71V = K_3 + K_4$

$\underline{\text{at } t = \infty}$ $\quad v_o = 0 = K_3 \Rightarrow K_4 = 5.71\,V$

$$\boxed{v_o = \begin{cases} 8 - 8e^{-1.25t}\ V & 0 \leq t \leq 1 \\ 5.71\,e^{-1.25(t-1)}\ V & t > 1 \end{cases}}$$

7.64 Find the output voltage $v_o(t)$ in the network in Fig. P7.64 if the input voltage is $v_i(t) = 5\big(u(t) - u(t - 0.05)\big)$ V.

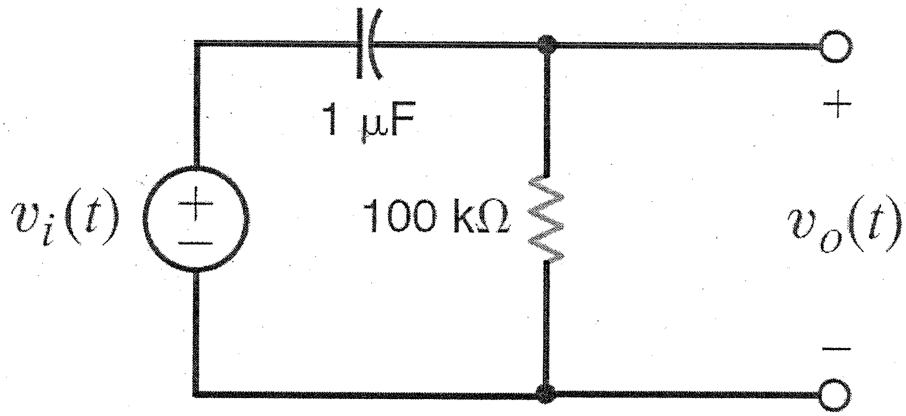
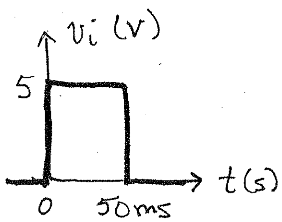


Figure P7.64

SOLUTION:



For $0 \le t \le 50ms$ $v_o = k_1 + k_2 e^{-t/\tau}$

For $t > 50 ms$ $v_o = k_3 + k_4 e^{-t/\tau}$

$\underline{t=0^-}$ $v_o = 0$ & $v_c = 0V$

$\underline{t=0^+}$ $v_c = 0$ & $v_o = v_i = 5 = k_1 + k_2$

$\underline{t \to \infty}$ $v_o = 0 = k_1 \Rightarrow k_2 = 5V$

$\tau = CR = 0.1s \Rightarrow v_o(t) = 5e^{-10t}$ $0 \le t \le 50ms$

$\underline{\text{at } t = 50ms^-}$ $v_o = 3.03V$ & $v_c = 1.97V$

$\underline{t = 50ms^+}$ $v_c = 1.97V$ & $v_o = v_i - v_c = -1.97V = k_3 + k_4$

$\underline{t \to \infty}$ $v_o = 0 = k_4 \Rightarrow v_o(t) = -1.97e^{-10(t-0.05)}$ V $t > 50ms$

$$\boxed{v_o = \begin{cases} 5e^{-10t} \text{ V} & 0 \le t \le 50ms \\ -1.97e^{-10(t-0.05)} \text{ V} & t > 50ms \end{cases}}$$

7.65 The voltage $v(t)$ shown in Fig. P7.65a is given by the graph shown in Fig. P7.65b. If $i_L(0) = 0$, answer the following questions: (a) how much energy is stored in the inductor at $t = 3$ s?, (b) how much power is supplied by the source at $t = 4$ s?, (c) what is $i(t = 6\text{ s})$?, and (d) how much power is absorbed by the inductor at $t = 3$ s?

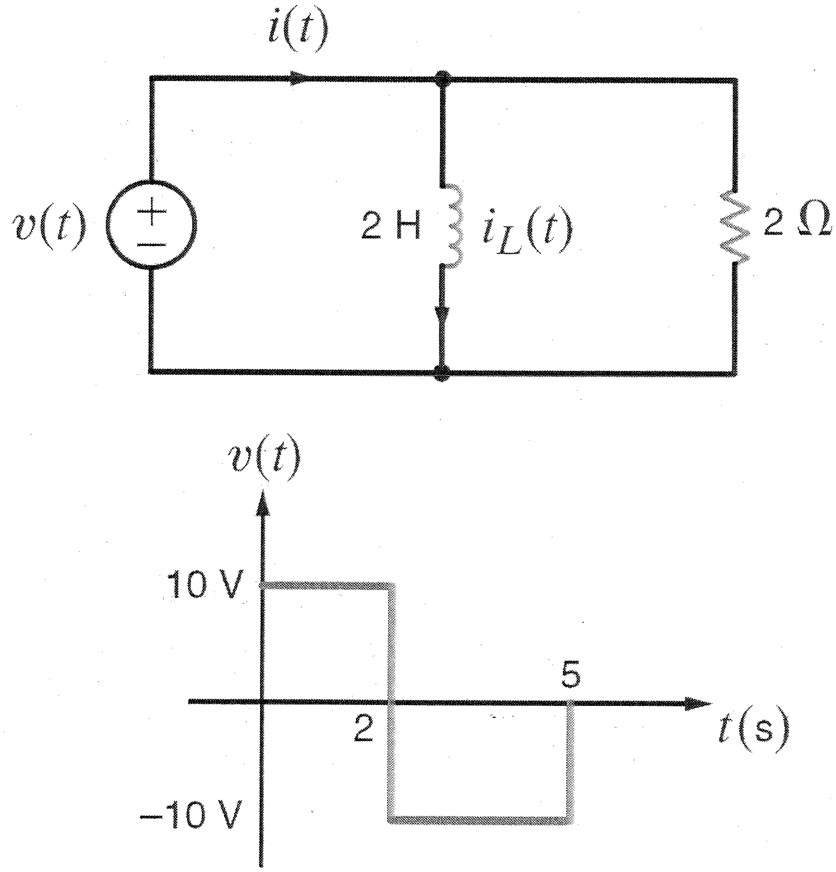


Figure P7.65

SOLUTION:

a) $W_L = \frac{1}{2} L i_L^2$ $\quad i_L(t) = \frac{1}{L}\int v_L\,dt = \frac{1}{L}\int v\,dt$

$i_L(3) = \frac{10}{2} t\Big|_0^2 - \frac{10}{2} t\Big|_2^3 = 5\text{A}$ $\quad \boxed{W_L(3) = 25\text{J}}$

b) $p_S(t) = v(t)\,i(t) = v(t)\left[i_L(t) + v(t)/R\right]$

$i_L(4) = \frac{1}{L}\int_0^4 v(t)\,dt = 5t\Big|_0^2 - 5t\Big|_2^4 = 0A$

$P_S(\cancel{t}4) = v^2(4)/R = 100/2$ $\boxed{P_S(4) = 50W}$

c) $i(6) = i_L(6) + \frac{v(6)}{R}$ $v(6) = 0$

$i_L(6) = \frac{1}{L}\int_0^6 v(\cancel{t})\,dt = 5t\Big|_0^2 - 5t\Big|_2^5 = -5A$

$\boxed{i(6) = -5A}$

d) $P_L = v(t)\,i_L(t)$ $i_L(3) = 5A$ $v(3) = -10V$

$\boxed{P_L(3) = -50W \text{ absorbed}}$

7.66 In the circuit in Fig. P7.66, $v_R(t) = 100e^{-400t}$ V for $t < 0$. Find $v_R(t)$ for $t > 0$.

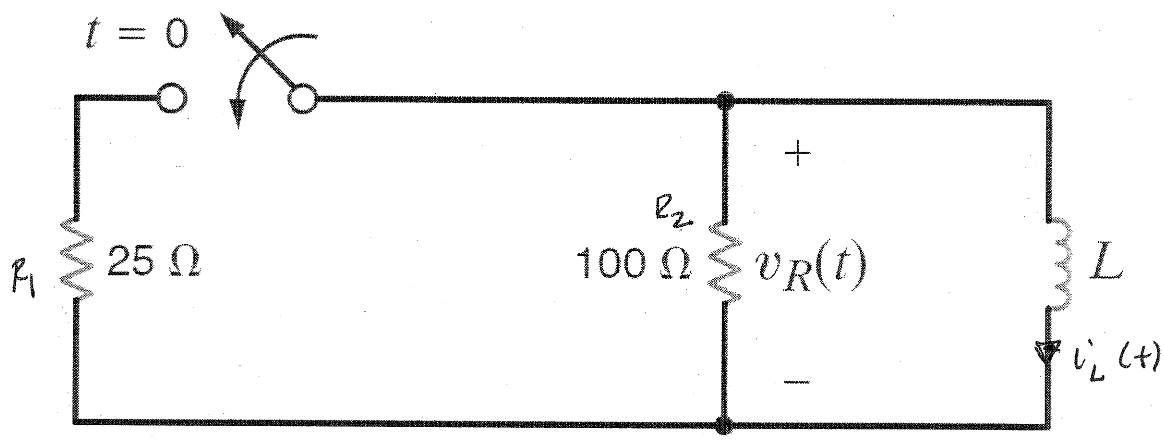


Figure P7.66

SOLUTION:

$t = 0^-$ $\quad v_R(0^-) = 100\,V \quad i_L(0^-) = -\frac{v_R(0^-)}{R_2} = -1A$

$\tau_1 = \frac{L}{R_2} = \frac{1}{400} \Rightarrow L = \frac{1}{4}\,H$

$t = 0^+$ $\quad i_L(0^+) = -1A \quad v_R(0^+) = -i_L(0^+)\frac{R_2 R_1}{R_1 + R_2} = 20V = K_1 + K_2$

$t = \infty$ $\quad v_R = 0 = K_1 \Rightarrow K_2 = 20\,V$

τ $\quad \tau_2 = \frac{L(R_1+R_2)}{R_1 R_2} = \frac{1}{80}\,s$

$$\boxed{v_R(t) = 20e^{-80t}\ V}$$

7.67 Given that $v_{C1}(0-) = -10$ V and $v_{C2}(0-) = 20$ V in the circuit in Fig. P7.67, find $i(0+)$.

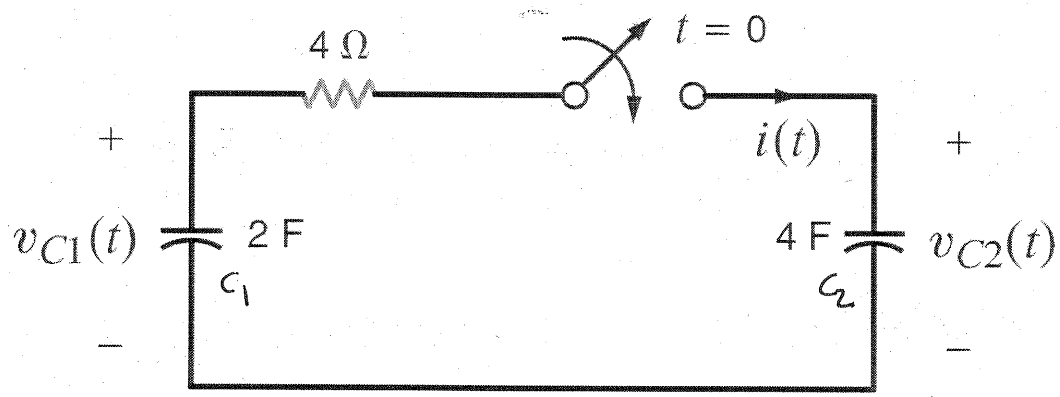
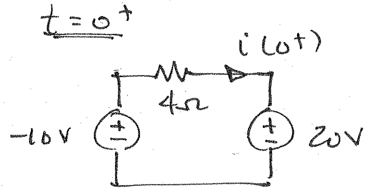


Figure P7.67

SOLUTION:

v_{c1} & v_{c2} cannot change instantaneously.

$\underline{t=0^+}$



$$i(0^+) = \frac{-10-20}{4} = -7.5A$$

$$\boxed{i(0^+) = -7.5A}$$

7.68 The switch in the circuit in Fig. P7.68 is closed at $t = 0$. If $i_1(0-) = 2$ A, determine $i_2(0+)$, $v_R(0+)$, and $i_1(t = \infty)$.

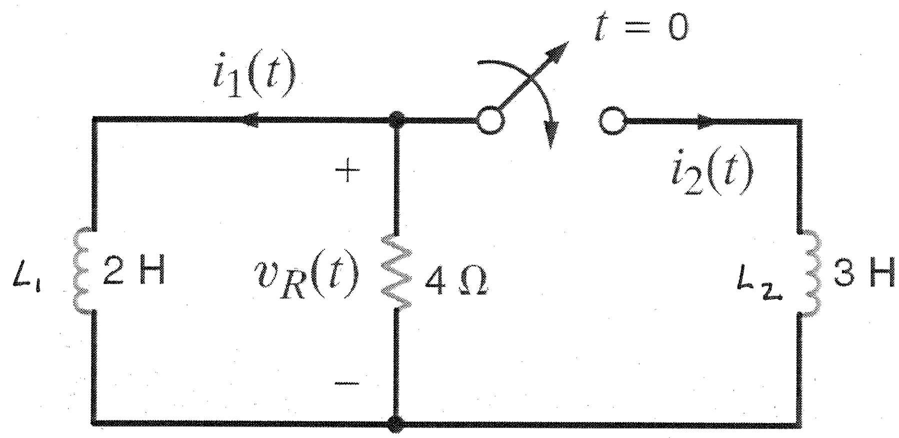


Figure P7.68

SOLUTION:

$\underline{t=0^-}$ $\quad i_1(0^-) = 2A \qquad i_2(0^-) = 0A$

$\underline{t=0^+}$ $\quad i_1(0^+) = i_1(0^-) = 2A \qquad i_2(0^+) = i_2(0^-) = 0A$

$v_R(0+) = -i_1(0+)(4) = -8V$

$\underline{t=\infty}$ all $v(t)$ & $i(t) \to 0$

$i_1(\infty) = 0$

$$\boxed{\begin{array}{l} v_R(0+) = -8V \\ i_2(0^+) = 0A \\ i_1(\infty) = 0A \end{array}}$$

7.69 In the network in Fig. P7.69 find $i(t)$ for $t > 0$. If $v_{C1}(0-) = -10$ V, calculate $v_{C2}(0-)$.

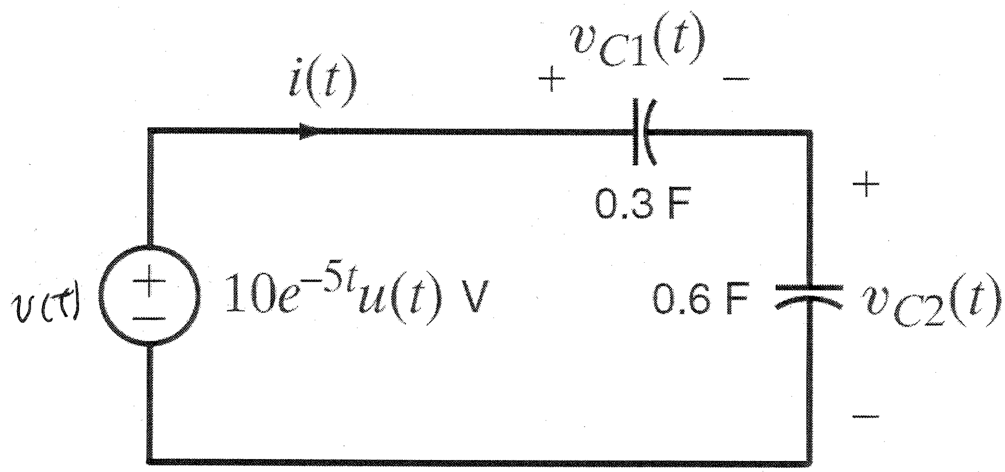
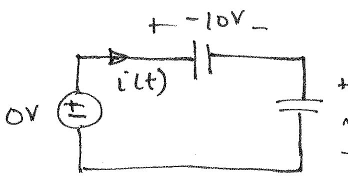


Figure P7.69

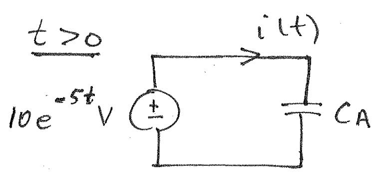
SOLUTION:

at $t = 0^-$



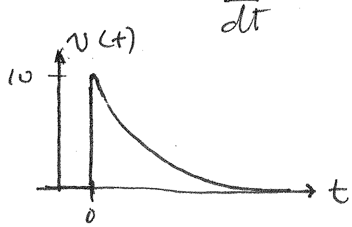
$v_{C2}(0^-) = -v_{C1}(0^-) = 10V$

$t > 0$



$C_A = \dfrac{C_1 C_2}{C_1 + C_2} = 0.2F$

$i(t) = C\dfrac{dv}{dt}$



$\dfrac{dv}{dt} = 10\,\delta(t) - 50e^{-5t} \quad t \geq 0$

$$\boxed{\begin{array}{l} i(t) = 2\delta(t) - 10e^{-5t}\text{ A} \quad t \geq 0 \\ v_2(0^-) = 10V \end{array}}$$

7.70 The switch in the circuit in Fig. P7.70 has been closed for a long time and is opened at $t = 0$. If $v_C(t) = 20 - 8e^{-0.05t}$ V, find R_1, R_2, and C.

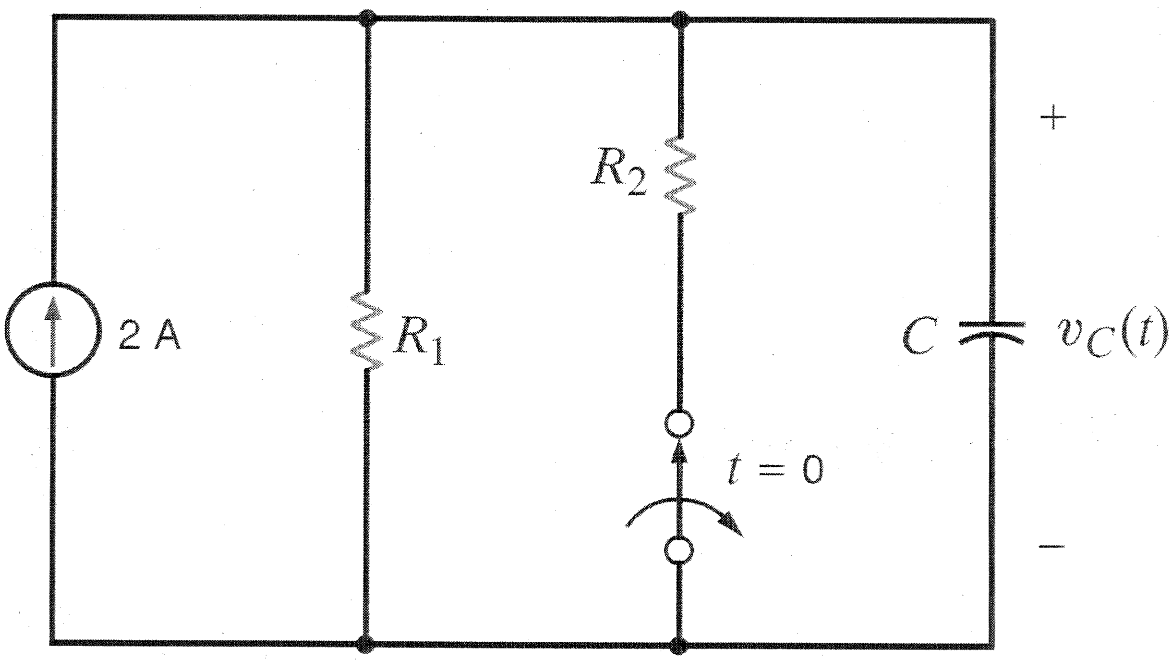


Figure P7.70

SOLUTION:

$v_C(t) = 20 - 8e^{-t/20}$ V $= K_1 + K_2 e^{-t/\tau}$

$K_1 = v_C(\infty) = 2R_1 = 20$ $\qquad R_1 = 10\,\Omega$

$K_1 + K_2 = v_C(0^+) = v_C(0^-) = \dfrac{2(R_1 R_2)}{R_1 + R_2} = 12 \Rightarrow$ $\qquad R_2 = 15\,\Omega$

$\tau = C R_1 = 20\,s$ $\qquad C = 2F$

$$\boxed{\begin{array}{l} C = 2F \\ R_1 = 10\,\Omega \\ R_2 = 15\,\Omega \end{array}}$$

7.71 Given that $i(t) = 13.33e^{-t} - 8.33e^{-0.5t}$ A for $t > 0$ in the network in Fig. P7.71, find the following: (a) $v_C(0)$, (b) $v_C(t = 1\text{ s})$, and (c) the capacitance C.

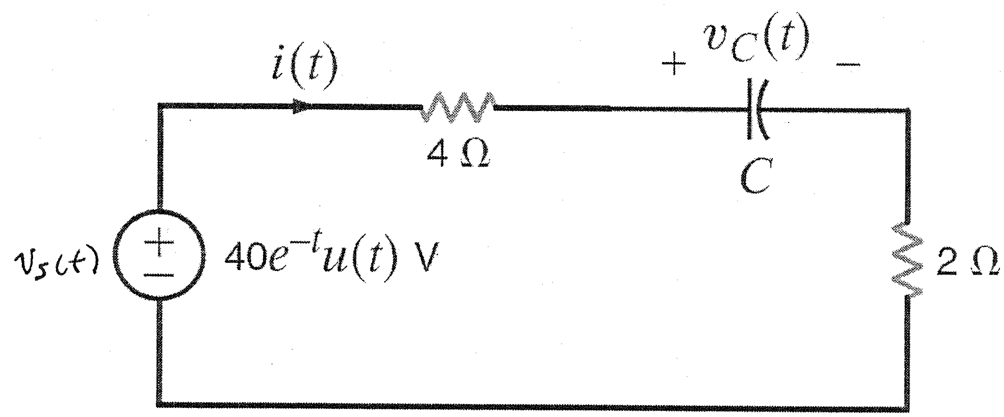


Figure P7.71

SOLUTION:

a) $v_c(0^-) = v_s(0^-) = 0\text{V} = v_c(0^+)$ $\boxed{v_c(0) = 0\text{V}}$

b) $v_c(t) = \frac{1}{C}\int i\,dt + K$

$$= \frac{1}{C}\left[16.66e^{-t/2} - 13.33e^{-t}\right] + K$$

$v_c(0) = 0 = \frac{1}{C}[3.33] + K \Rightarrow K = -3.33/C$

Need C.

c) $\tau = 2 = C[4+2] = 6C \Rightarrow \boxed{C = 1/3\text{ F}}$

Back to b)

$K = -10$ $v_c(t) = 50e^{-t/2} - 40e^{-t} - 10$

$\boxed{v_c(1) = 5.61\text{ V}}$

7.72 Given that $i(t) = 2.5 + 1.5e^{-4t}$ A for $t > 0$ in the circuit in Fig. P7.72, find R_1, R_2, and L.

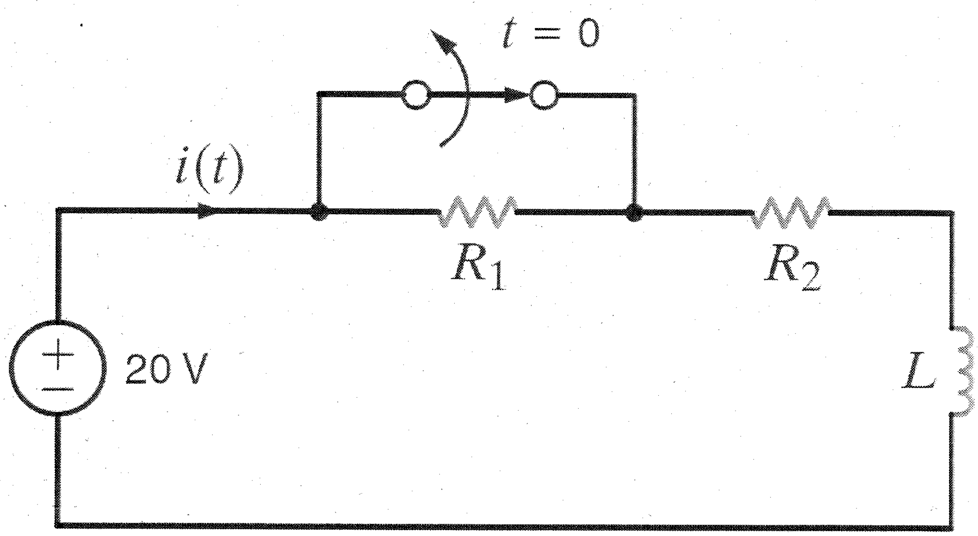


Figure P7.72

SOLUTION:

$i(t) = 2.5 + 1.5e^{-4t} = K_1 + K_2 e^{-t/\tau}$

$K_1 = 2.5 = i(\infty) = \frac{20}{R_1+R_2} \Rightarrow R_1 + R_2 = 8\,\Omega$

$K_1 + K_2 = 4 = i(0^+) = i_L(0^+) = i_L(0^-) = \frac{20}{R_2} \Rightarrow R_2 = 5\,\Omega$ $\Big\}$ $R_1 = 3\,\Omega$

$\tau = \frac{1}{4} = \frac{L}{R_1+R_2}$ $L = 2H$

$$\boxed{\begin{array}{l} L = 2H \\ R_1 = 3\,\Omega \\ R_2 = 5\,\Omega \end{array}}$$

7.73 The differential equation that describes the current $i_o(t)$ in a network is

$$\frac{d^2 i_o(t)}{dt^2} + 6\left[\frac{di_o(t)}{dt}\right] + 4i_o(t) = 0$$

Find **(a)** the characteristic equation of the network, **(b)** the network's natural frequencies, and **(c)** the expression for $i_o(t)$.

SOLUTION:

a) $s^2 + 6s + 4 = 0$

b) $s_{1,2} = \dfrac{-6 \pm \sqrt{36-16}}{2} = \left\{\begin{matrix} -0.764 \\ -5.24 \end{matrix}\right\} = s_{1,2}$

c) $2\zeta\omega_o = 6 \qquad \omega_o^2 = 4 \qquad \Rightarrow \zeta = 1.5$ over damped.

$$\boxed{i_o(t) = K_1 e^{-0.764t} + K_2 e^{-5.24t}}$$

7.74 The terminal current in a network is described by the equation

$$\frac{d^2 i_o(t)}{dt^2} + 8\left[\frac{di_o(t)}{dt}\right] + 16 i_o(t) = 0$$

Find **(a)** the characteristic equation of the network, **(b)** the network's natural frequencies, and **(c)** the equation for $i_o(t)$.

SOLUTION:

a) $s^2 + 8s + 16 = 0$

b) $s_{1,2} = \frac{-8 \pm \sqrt{64-64}}{2} = -4 \ r/s$

c) $2\zeta\omega_o = 8 \qquad \omega_o^2 = 16 \quad \Rightarrow \quad \zeta = 1 \quad \Rightarrow$ critically damped.

$$\boxed{i_o(t) = B_1 e^{-4t} + B_2 t e^{-4t}}$$

7.75 The voltage $v_1(t)$ in a network is defined by the equation

$$\frac{d^2v_1(t)}{dt^2} + 2\left[\frac{dv_1(t)}{dt}\right] + 5v_1(t) = 0$$

Find

(a) the characteristic equation of the network.

(b) the circuit's natural frequencies.

(c) the expression for $v_1(t)$. CS

SOLUTION:

a) $s^2 + 2s + 5 = 0$

b) $s_{1,2} = \dfrac{-2 \pm \sqrt{4-20}}{2} = -1 \pm j2$ r/s

c) underdamped!

$$v_1(t) = e^{-t}\left[A_1 \cos 2t + A_2 \sin 2t\right]$$

7.76 The output voltage of a circuit is described by the differential equation

$$\frac{d^2v_o(t)}{dt^2} + 8\left[\frac{dv_o(t)}{dt}\right] + 10v_o(t) = 0$$

Find **(a)** the characteristic equation of the circuit, **(b)** the network's natural frequencies, and **(c)** the equation for $v_o(t)$.

SOLUTION:

a) $s^2 + 8s + 10 = 0$

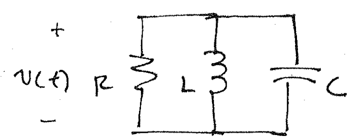
b) $s_{1,2} = \frac{-8 \pm \sqrt{64-40}}{2} = \begin{cases} -1.55 & r/s \\ -6.45 & r/s \end{cases}$

c) overdamped!

$$\boxed{v_o(t) = K_1 e^{-1.55t} + K_2 e^{-6.45t}}$$

7.77 The parameters for a parallel RLC circuit are $R = 1\ \Omega$, $L = 1/2$ H, and $C = 1/2$ F. Determine the type of damping exhibited by the circuit.

SOLUTION:



$$\frac{v(t)}{R} + \frac{1}{L}\int v\,dt + C\frac{dv}{dt} = 0$$

$$C\frac{d^2v}{dt^2} + \frac{1}{R}\frac{dv}{dt} + \frac{1}{L}v(t) = 0$$

$$\frac{d^2v}{dt^2} + \frac{1}{RC}\frac{dv}{dt} + \frac{1}{LC}v(t) = 0$$

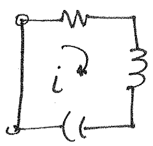
$$\omega_0^2 = \frac{1}{LC} = 4 \Rightarrow \omega_0 = 2 \text{ r/s}$$

$$2\zeta\omega_0 = 2 \Rightarrow \zeta = 1/2$$

$$\boxed{\text{underdamped}}$$

7.78 A series *RLC* circuit contains a resistor $R = 2\ \Omega$ and a capacitor $C = 1/2$ F. Select the value of the inductor so that the circuit is critically damped.

SOLUTION:



$$Ri + L\frac{di}{dt} + \frac{1}{C}\int i\,dt = 0$$

$$\frac{d^2i}{dt^2} + \frac{R}{L}\frac{di}{dt} + \frac{1}{LC}i(t) = 0$$

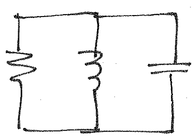
$$\frac{1}{LC} = \omega_0^2 \qquad 2\zeta\omega_0 = \frac{R}{L}$$

for $\zeta = 1$, $\quad 2\omega_0 = \frac{R}{L} \Rightarrow \frac{2}{\sqrt{LC}} = \frac{R}{L} \Rightarrow \sqrt{L} = \frac{R\sqrt{C}}{2}$

$$L = \frac{R^2C}{4}. \qquad \boxed{L = \frac{1}{2}H}$$

7.79 A parallel *RLC* circuit contains a resistor $R = 1\ \Omega$ and an inductor $L = 2$ H. Select the value of the capacitor so that the circuit is critically damped.

SOLUTION:



$$s^2 + \frac{s}{RC} + \frac{1}{LC} = 0$$

$$\omega_0^2 = \frac{1}{LC} \qquad 2\zeta\omega_0 = \frac{1}{RC} \qquad \zeta = 1$$

$$2\omega_0 = \frac{1}{RC} = \frac{2}{\sqrt{LC}} \Rightarrow \sqrt{C} = \frac{\sqrt{L}}{2R} \Rightarrow C = \frac{L}{4R^2} = \frac{1}{2}\,F$$

$$\boxed{C = \frac{1}{2}\,F}$$

7.80 For the underdamped circuit shown in Fig. P7.80, determine the voltage $v(t)$ if the initial conditions on the storage elements are $i_L(0) = 1$ A and $v_C(0) = 10$ V. CS

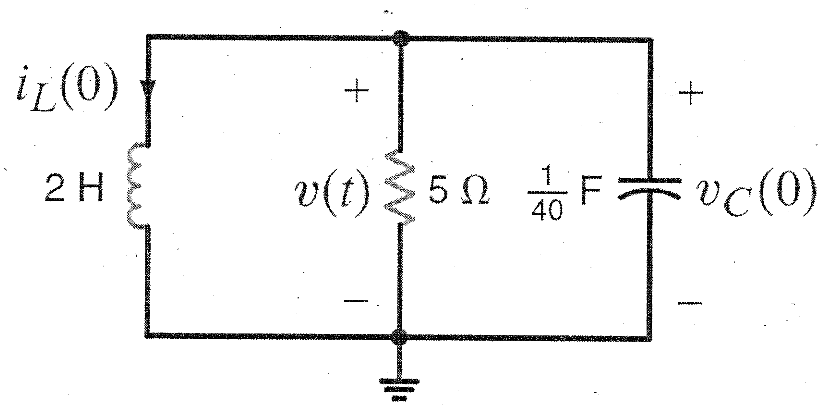


Figure P7.80

SOLUTION:

characteristic eq: $s^2 + \frac{1}{RC}s + \frac{1}{LC} = 0 \Rightarrow s^2 + 8s + 20 = 0$

natural freq: $s_{1,2} = \frac{-8 \pm \sqrt{64-80}}{2}$ $\quad s_{1,2} = -4 \pm j2$ r/s

$v(t) = e^{-4t}\left[A_1 \cos 2t + A_2 \sin 2t\right]$

$v_c(0) = v(0) = A_1 = 10$

$i_L(0) = 1 = -\frac{v(0)}{5} - C\frac{dv}{dt}\Big|_{t=0}$ $\quad v(0) = 10\,V$

$\frac{dv}{dt} = -4e^{-4t}\left[A_1 \cos 2t + A_2 \sin 2t\right] + e^{-4t}\left[-2A_1 \sin 2t + 2A_2 \cos 2t\right]$

$\frac{dv}{dt}\Big|_{t=0} = -40 + 2A_2 \rightarrow 1 = -2 + \frac{40}{40} - \frac{2A_2}{40} \Rightarrow A_2 = -40\,V$

$$\boxed{v(t) = e^{-4t}\left[10 \cos 2t - 40 \sin 2t\right]\ V}$$

7.81 In the critically damped circuit shown in Fig. P7.81, the initial conditions on the storage elements are $i_L(0) = 2$ A and $v_C(0) = 5$ V. Determine the voltage $v(t)$.

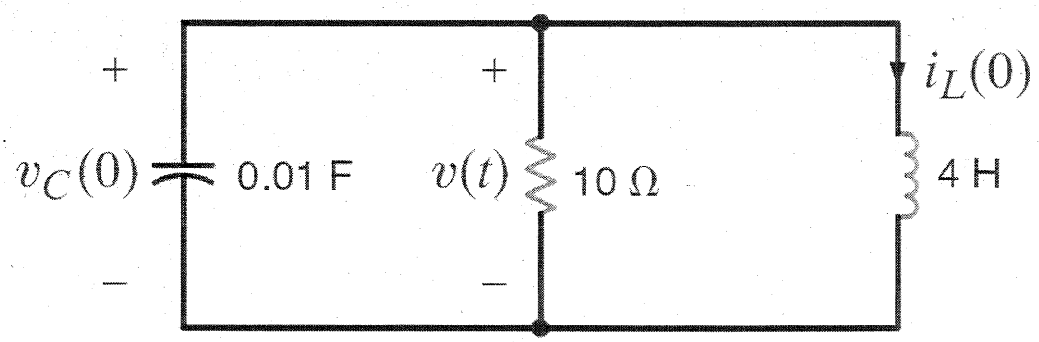


Figure P7.81

SOLUTION:

characteristic eq: $s^2 + \frac{s}{RC} + \frac{1}{LC} = 0 = s^2 + 10s + 25 = 0$

natural freq: $s_{1,2} = \frac{-10 \pm \sqrt{100 - 100}}{2} = -5$ critically damped!

$v(t) = B_1 e^{-5t} + B_2 t e^{-5t}$ V

$v_c(0) = v(0) = B_1 = 5$

$i_L(0) = 2 = -\frac{v(0)}{R} - C\frac{dv}{dt}\Big|_{t=0} = -\frac{1}{2} - C\frac{dv}{dt}\Big|_{t=0}$

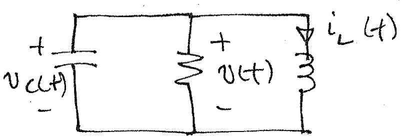
$\frac{dv}{dt}\Big|_{t=0} = \left[-5B_1 e^{-5t} + B_2 e^{-5t} - 5B_2 t e^{-5t}\right]_{t=0} = -5B_1 + B_2$

$2 = -\frac{1}{2} + \frac{25 - B_2}{100} \Rightarrow B_2 = -225$

$$\boxed{v(t) = 5e^{-5t} - 225te^{-5t} \text{ V}}$$

7.82 Given the circuit and the initial conditions from Problem 7.81, determine the current $i_L(t)$ that is flowing through the inductor.

SOLUTION:



$R = 10\Omega \quad L = 4H \quad C = 0.01F$

$v_C(0^-) = 5V \quad i_L(0^-) = 2A$

from 7.81, $\quad v(t) = 5e^{-5t} - 225te^{-5t}\ V$

$i_L = \frac{1}{L}\int v\,dt$

from integration tables: $\int te^{-5t}dt = -\frac{te^{-5t}}{5} - \frac{1}{25}e^{-5t}$

$i_L = \frac{1}{L}\int 5e^{-5t} - 225te^{-5t}dt = 2e^{-5t} + 11.25te^{-5t}\ A$

$$\boxed{i_L(t) = 2e^{-5t} + 11.25te^{-5t} A}$$

7.83 Find $v_C(t)$ for $t > 0$ in the circuit in Fig. P7.83. CS

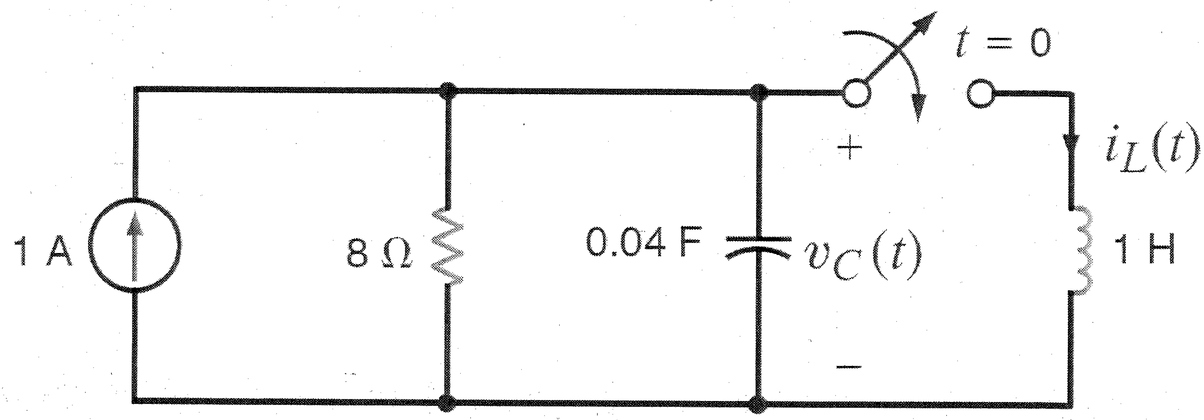


Figure P7.83

SOLUTION:

$\underline{t=0^-}$: $v_c(0^-) = 8V$ $\quad i_L(0^-) = 0$

$\underline{t>0}$: $\dfrac{d^2v_c}{dt^2} + \dfrac{1}{RC}\dfrac{dv_c}{dt} + \dfrac{v_c}{LC} = 0 \Rightarrow s^2 + 3.125s + 25 = 0$

natural frequencies: $s_{1,2} = -1.5625 \pm j4.75$ r/s $= \sigma \pm j\omega$

$v_c(t) = e^{-\sigma t}[A_1\cos\omega t + A_2\sin\omega t]$

$v_c(0^-) = A_1 = 8$

$-i_L(0^-) = 0 = \left.\dfrac{v_c}{R}\right|_{t=0} + C\left.\dfrac{dv_c}{dt}\right|_{t=0} = \dfrac{A_1}{R} + C[A_2\omega - \sigma A_1]$

yields $A_2 = -2.63$ V

$$\boxed{v_c(t) = e^{-\sigma t}[8\cos\omega t - 2.63\sin\omega t] \quad \sigma = 1.5625 \quad \omega = 4.75 \text{ r/s}}$$

7.84 Find $v_C(t)$ for $t > 0$ in the circuit in Fig. P7.84 if $v_C(0) = 0$.

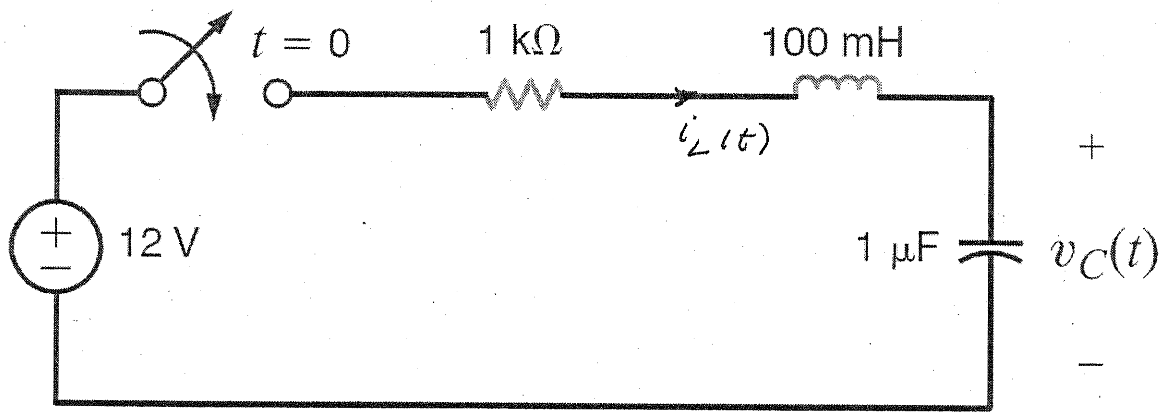


Figure P7.84

SOLUTION: $\underline{t=0^-}$: $v_C = 0$ $\quad i_L = 0$

$$\frac{d^2 i_L}{dt^2} + \frac{R}{L}\frac{di_L}{dt} + \frac{i_L}{LC} = 0 \Rightarrow s^2 + 10^4 s + 10^7 = 0$$

natural frequencies: $s_{1,2} = \begin{cases} -1127 = -\sigma_1 \\ -8873 = -\sigma_2 \end{cases}$

$$i_L(t) = K_1 e^{-1127t} + K_2 e^{-8873t} = K_1 e^{-\sigma_1 t} + K_2 e^{-\sigma_2 t}$$

$$i_L(0) = 0 = K_1 + K_2$$

$$12 = R i_L(0^+) + L \left.\frac{di_L}{dt}\right|_{t=0^+} + v_C(0^+) = RK_1 + RK_2 - L\sigma_1 K_1 - L\sigma_2 K_2 + v_C(0^+)$$

$v_C(0^+) = v_C(0^-) = 0\text{V} \Rightarrow K_1 = 15.5 \text{ mA} \quad K_2 = -15.5 \text{ mA}$

$$v_C(t) = \frac{1}{C}\int i_L(t)\,dt + K_3 \Rightarrow v_C(0^+) = 0 = -\frac{K_1}{C\sigma_1} - \frac{K_2}{C\sigma_2} + K_3$$

$K_3 = 12\text{V}$

$$\boxed{v_C(t) = 17.5e^{-\sigma_2 t} - 13.75e^{-\sigma_1 t} + 12 \text{ V} \quad \sigma_1 = 1127 \quad \sigma_2 = 8873}$$

7.85 Find $v_o(t)$ for $t > 0$ in the circuit in Fig. P7.85 and plot the response including the time interval just prior to closing the switch. **PSV**

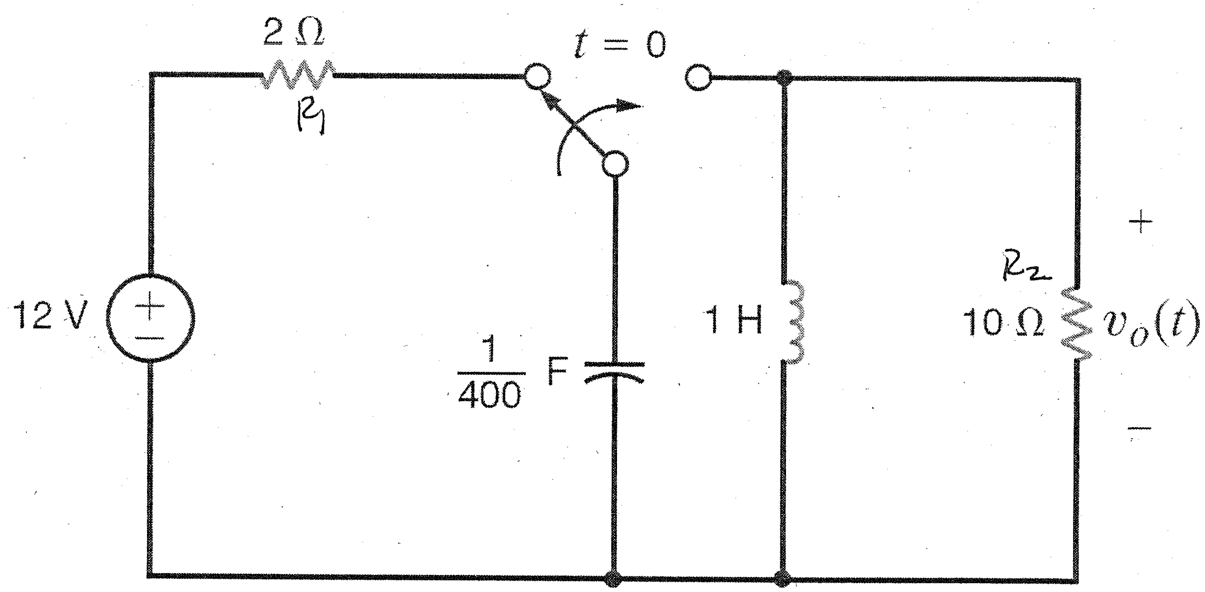


Figure P7.85

SOLUTION: $t=0^-$ $\quad v_C(0^-) = 12V \quad i_L(0^+) = 0 = i_L(0^-)$

$t>0$

$$\frac{d^2v_o}{dt^2} + \frac{dv_o}{dt}\left(\frac{1}{R_2C}\right) + \frac{v_o}{LC} = 0 \Rightarrow s^2 + 40s + 400 = 0$$

natural frequencies: $s_{1,2} = -20$ r/s

$$v_o(t) = B_1e^{-20t} + B_2te^{-20t}$$

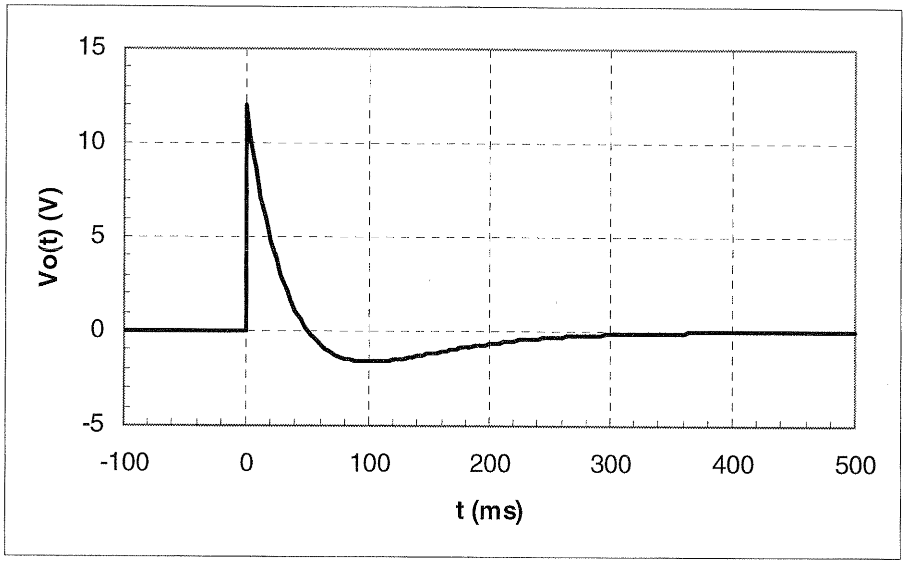
$$v_o(0^+) = v_C(0^+) = 12V = B_1$$

$$-i_L(0^+) = 0 = \frac{v_o(0^+)}{R_2} + C\left.\frac{dv_o}{dt}\right|_{t=0^+} = \frac{12}{R_2} + C\left[-20B_1 + B_2\right]$$

$$B_2 = -240V$$

$$\boxed{v_o(t) = 12e^{-20t} - 240te^{-20t}\ V}$$

PROBLEM 7.85



7.86 Find $v_o(t)$ for $t > 0$ in the circuit in Fig. P7.86 and plot the response including the time interval just prior to closing the switch.

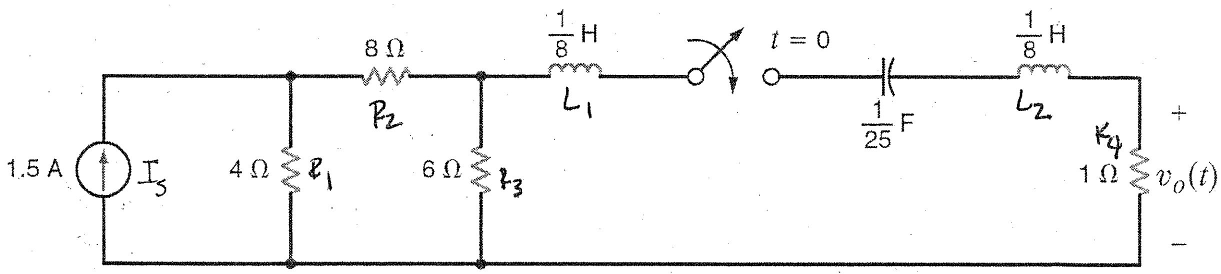
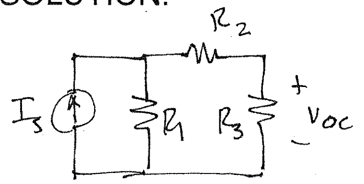
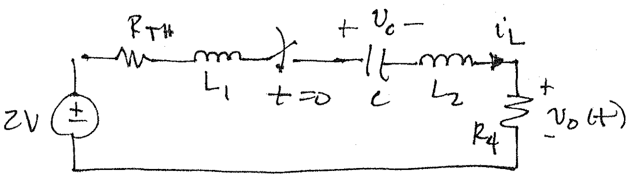


Figure P7.86

SOLUTION:



$$V_{oc} = \frac{I_s R_1}{R_1 + R_2 + R_3} R_3 = 2V \qquad R_{TH} = R_3 // (R_1 + R_2) = 4\Omega$$



$\underline{t=0^-}$: $i_{L1} = i_{L2} = i_L = 0A$

$v_c(0^-) = 0V$

$\underline{t>0}$:

$$\frac{d^2 i_L}{dt^2} + \frac{R}{L}\frac{di_L}{dt} + \frac{1}{LC} i_L = 0 \Rightarrow s^2 + \frac{R}{L}s + \frac{1}{LC} = 0$$

$R = R_{TH} + R_4$

$L = L_1 + L_2$

$s_{1,2} = -10 \ r/s \qquad i_L(t) = B_1 e^{-10t} + B_2 t e^{-10t}$

$i_L(0^+) = i_L(0^-) = 0 = B_1 \Rightarrow i_L(t) = B_2 t e^{-10t}$

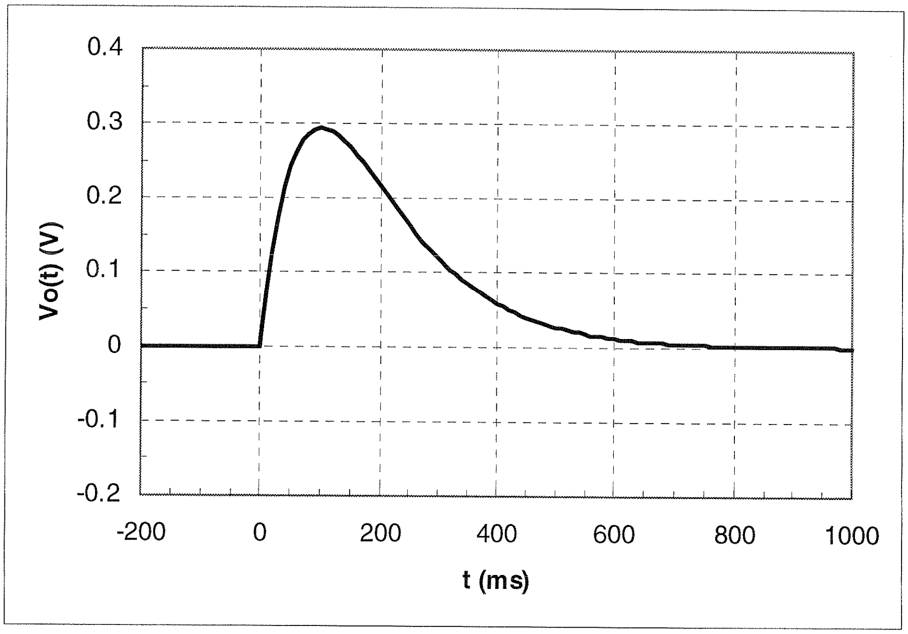
$$v_c(0^+) = 0 = 2 - R\, i_L(0^+) - L \left.\frac{di_L}{dt}\right|_{t=0} = 2 - 0 - L B_2$$

$$B_2 = \frac{2}{L} = 8$$

$$i_L(t) = 8te^{-10t} \ A$$

$v_o = R_4 i_L(t)$ $\qquad \boxed{v_o(t) = 8te^{-10t} \ V}$

PROBLEM 7.86



7.87 In the circuit shown in Fig. P7.87, switch action occurs at $t = 0$. Determine the voltage $v_o(t)$, $t > 0$.

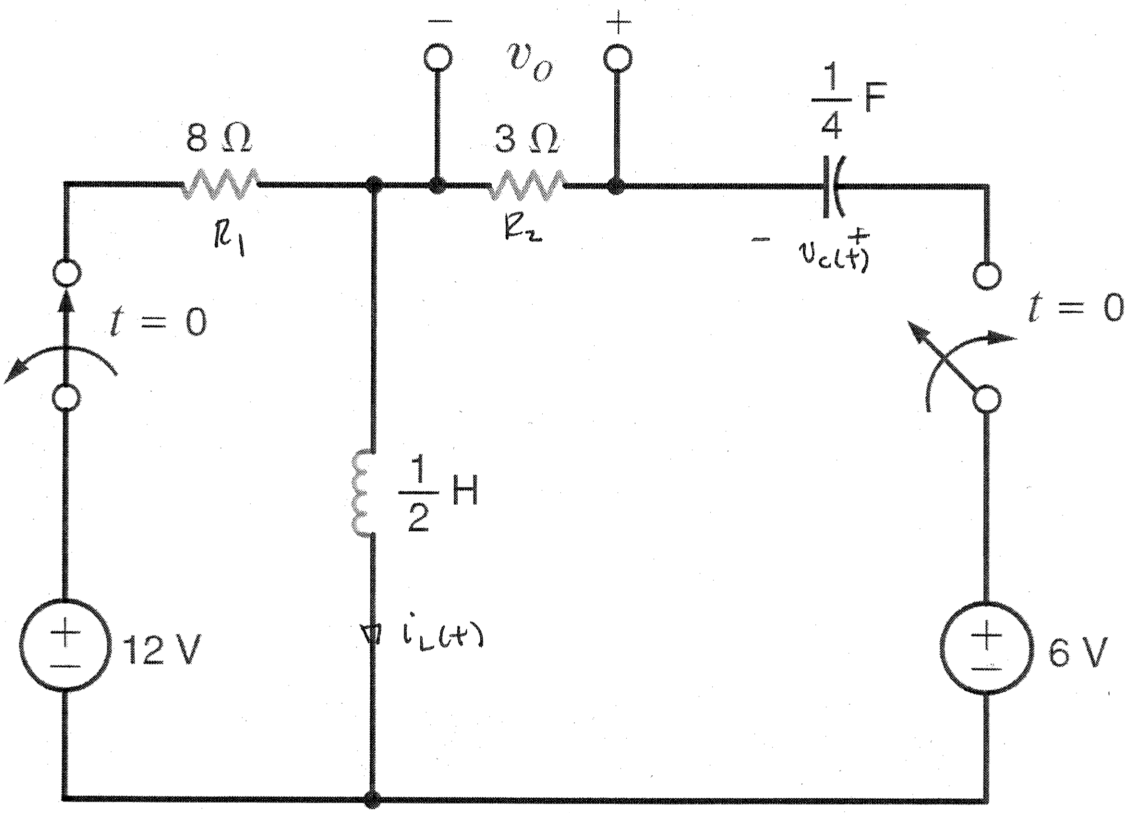


Figure P7.87

SOLUTION: $\underline{t=0^-}$: $\quad i_L(0^-) = 12/R_1 = 1.5A \qquad v_c(0^-) = 0V$

$\underline{t>0}$: $\quad 6 = \frac{1}{C}\int i_L dt + R_2 i_L(t) + L\frac{di_L}{dt} \Rightarrow \frac{d^2 i}{dt^2} + \frac{R_2}{L}\frac{di_L}{dt} + \frac{i_L(t)}{LC} = 0$

$$s^2 + 6s + 8 = 0 \quad \Rightarrow \quad s_{1,2} = \begin{Bmatrix} -2 = -\sigma_1 \\ -4 = -\sigma_2 \end{Bmatrix}$$

$$i_L(t) = K_1 e^{-\sigma_1 t} + K_2 e^{-\sigma_2 t} \qquad i_L(0^+) = 1.5 = K_1 + K_2$$

$$6 = v_c(0^+) + R_2 i_L(0^+) + L\frac{di_L}{dt}\Big|_{t=0} = 0 + R_2(1.5) - LK_1\sigma_1 - LK_2\sigma_2$$

yields,

$K_1 = 4.5V$ & $K_2 = -3V$ $\qquad v_o = R_2 i_L \qquad \boxed{v_o = 4.5e^{-2t} - 3e^{-4t} \text{ V}}$

7.88 Find $v_o(t)$ for $t > 0$ in the circuit in Fig. P7.88 and plot the response including the time interval just prior to moving the switch.

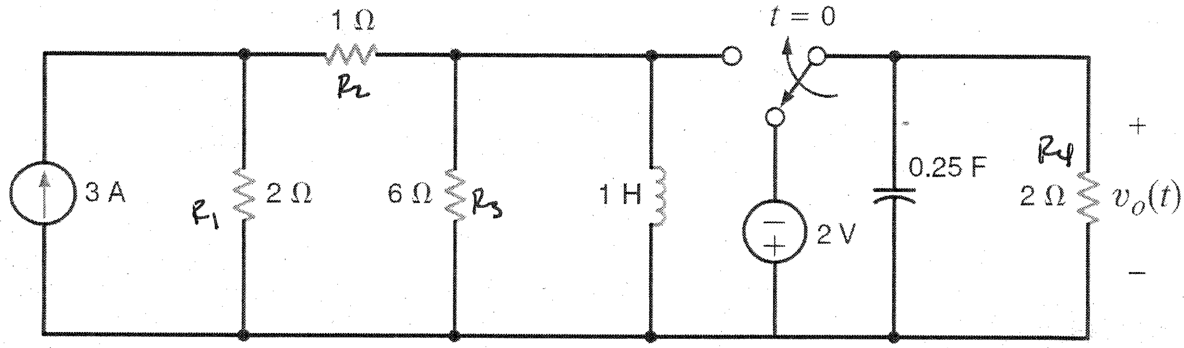
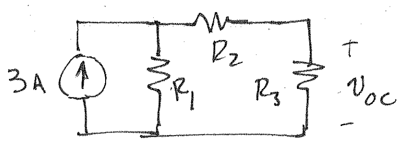


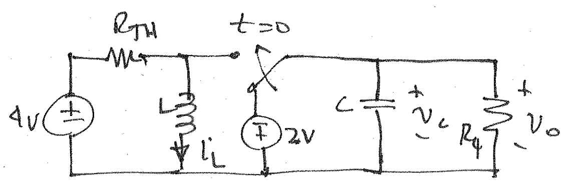
Figure P7.88

SOLUTION:



$$v_{oc} = 3\left[\frac{R_1 R_3}{R_1+R_2+R_3}\right] = 4V$$

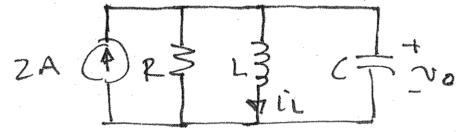
$$R_{TH} = R_3 // [R_1 + R_2] = 2\Omega$$



$\underline{t<0:}$ $i_L = 4/R_{TH} = 2A$

$v_c = -2V$

$\underline{t>0:}$



$$R = R_4 // R_{TH} = 1\Omega$$

$$\frac{d^2v_o}{dt^2} + \frac{dv_o}{dt}\left(\frac{1}{RC}\right) + \frac{v_o}{LC} = 0$$

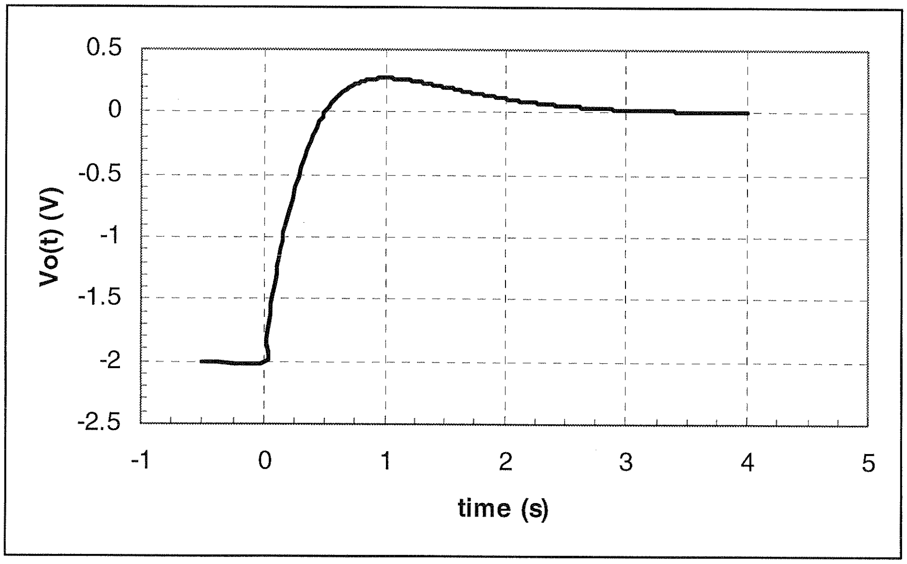
$$s^2 + 4s + 4 = 0 \Rightarrow s_{1,2} = -2 \text{ r/s} \qquad v_o(t) = B_1 e^{-2t} + B_2 t e^{-2t}$$

$$v_o(0^+) = v_c(0^-) = -2V = B_1$$

$$2 - i_L(0^+) = 0 = \frac{v_o(0^+)}{R} + C\left.\frac{dv_o}{dt}\right|_{t=0^+} = \frac{B_1}{R} - 2B_1C + CB_2 \Rightarrow B_2 = 4V$$

$$\boxed{v_o(t) = -2e^{-2t} + 4te^{-2t} \text{ V}}$$

PROBLEM 7.88



7.89 The switch in the circuit in Fig. P7.89 has been closed for a long time and is opened at $t = 0$. Find $i(t)$ for $t > 0$. **PSV**

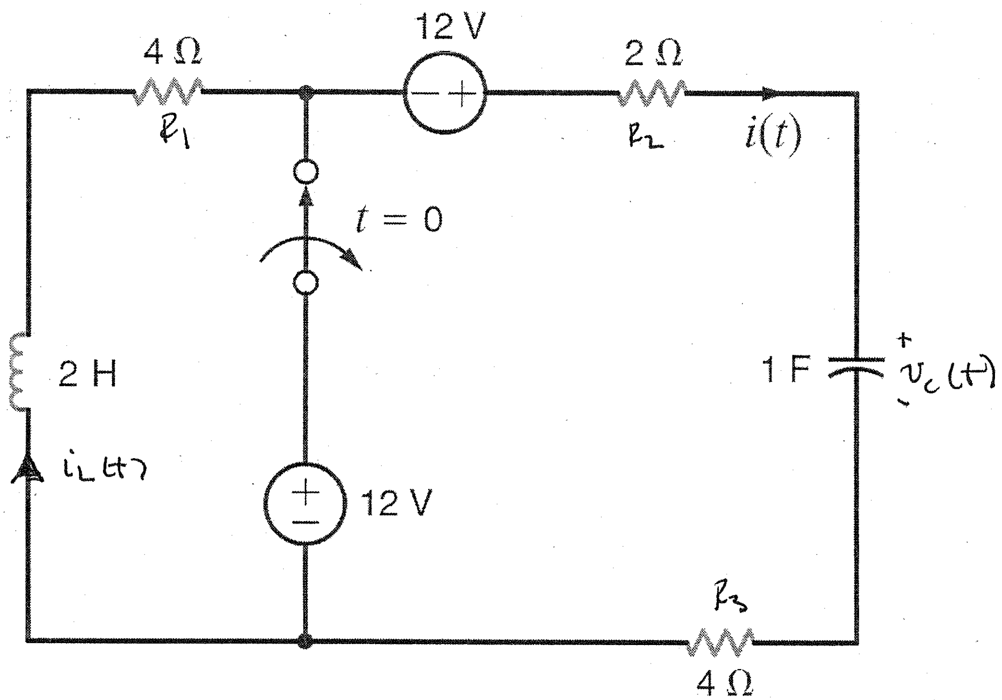


Figure P7.89

SOLUTION: $t=0^-$: $i_L(0^-) = -\frac{12}{R_1} = -3\,A$ $\quad v_c(0^-) = 24V$

$t>0$:

$$12 = (R_1+R_2+R_3)\,i(t) + \frac{1}{C}\int i(t)\,dt + L\,di(t)/dt = R\,i(t) + \frac{1}{C}\int i(t)dt + L\frac{di}{dt}$$

$$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0 \Rightarrow s^2 + 5s + 0.5 = 0 \qquad s_{1,2} = \begin{cases} -0.10 = -\sigma_1 \\ -4.89 = -\sigma_2 \end{cases}$$

$$i(t) = K_1 e^{-\sigma_1 t} + K_2 e^{-\sigma_2 t} \qquad i(0^+) = i_L(0^+) = -3 = K_1 + K_2$$

$$v_c(0^+) = 12 - R\,i(0^+) - L\left.\frac{di}{dt}\right|_{t=0^+} = 12 + 3R + L K_1\sigma_1 + L K_2 \sigma_2 = 24$$

yields

$K_1 = -1.19\,A$

$K_2 = -1.81\,A$

$$\boxed{i(t) = -1.19e^{-0.10t} - 1.81e^{-4.89t}\ A}$$

7.90 The switch in the circuit in Fig. P7.90 has been closed for a long time and is opened at $t = 0$. Solve for $i(t)$ for $t > 0$.

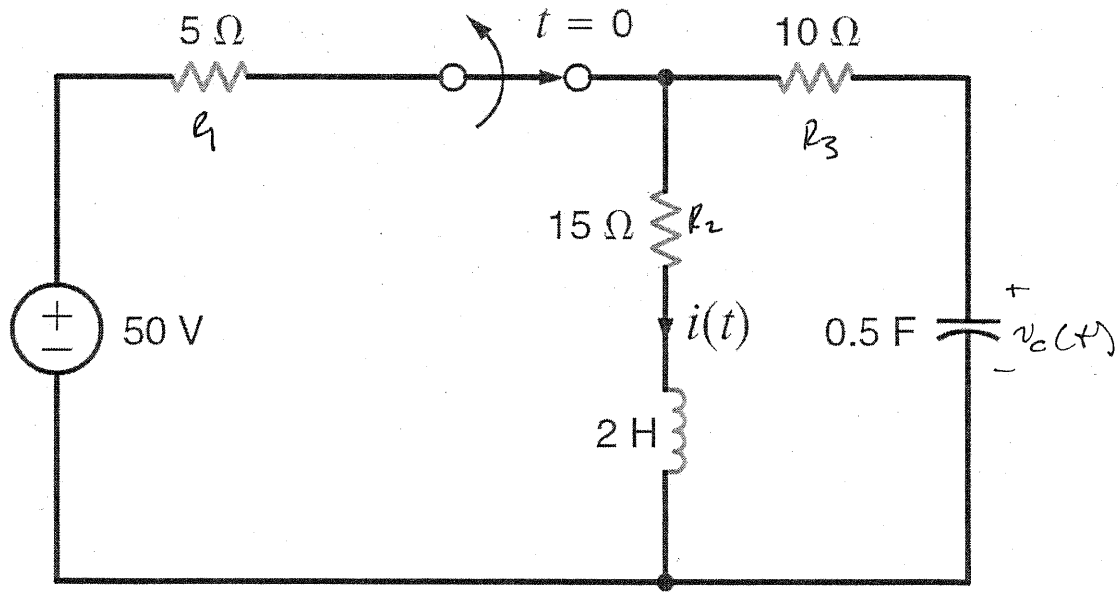


Figure P7.90

SOLUTION: $\underline{t = 0^-}$: $i(0^-) = \dfrac{50}{R_1 + R_2} = 2.5A$ $v_c(0^-) = i(0^-)\,R_2 = 37.5V$

$\underline{t > 0}$: $L\dfrac{di}{dt} + R\,i(t) + \dfrac{1}{C}\int i\,dt = 0$ $R = R_2 + R_3 = 25$

$v_c(t) = -\dfrac{1}{C}\int i\,dt$

$s^2 + \dfrac{R}{L}s + \dfrac{1}{LC} = 0 = s^2 + 12.5s + 1$

$s_{1,2} = \begin{Bmatrix} -0.08 = -\sigma_1 \\ -12.42 = -\sigma_2 \end{Bmatrix}$ $i(t) = K_1 e^{-\sigma_1 t} + K_2 e^{-\sigma_2 t}$

$i(0^+) = K_1 + K_2 = 2.5$ $v_c(0^+) = 37.5 = R\,i(0^+) + L\left.\dfrac{di}{dt}\right|_{t=0}$

$= R(2.5) - L\sigma_1 K_1 - L\sigma_2 K_2$

yields

$K_1 = 1.5$ $K_2 = 1$

$$\boxed{i(t) = e^{-\sigma_2 t} + 1.5\,e^{\sigma_1 t}\ A \qquad \sigma_1 = 0.08 \quad \sigma_2 = 12.42}$$

7.91 The switch in the circuit in Fig. P7.91 has been closed for a long time and is opened at $t = 0$. Solve for $i(t)$ for $t > 0$.

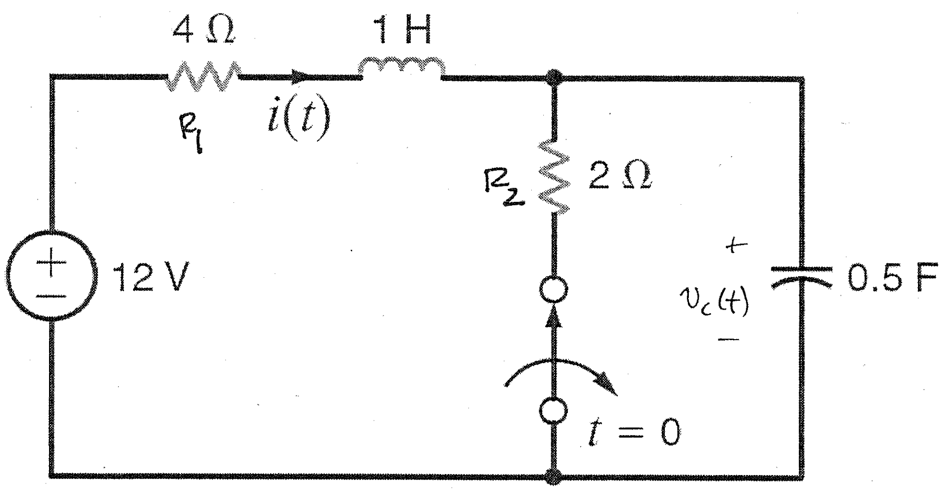


Figure P7.91

SOLUTION: $t=0^-$: $i(0^+) = \frac{12}{R_1+R_2} = 2A$ $\quad V_c(0^+) = i(0^+)R_2 = 4V$

$t=0^+$ $\quad 12 = L\,di/dt + R_1 i_1 + \frac{1}{C}\int i(t)dt \Rightarrow \frac{d^2i}{dt^2} + \frac{4di}{dt} + 2 = 0$

$s^2 + 4s + 1/2 = 0 \qquad S_{1,2} = \left\{ \begin{matrix} -0.586 = -\sigma_1 \\ -3.41 = -\sigma_2 \end{matrix} \right\}$

$i(t) = K_1 e^{-\sigma_1 t} + K_2 e^{-\sigma_2 t} \qquad i(0) = 2 = k_1 + k_2$

$v_L(0^+) = 12 - R_1 i(0^+) - L\frac{di}{dt}\Big|_{t=0} = 12 - 8 + \sigma_1 K_1 + \sigma_2 K_2 = 4$

$K_1 + K_2 = 2 \; \& \; \sigma_1 K_1 + \sigma_2 K_2 = 0 \Rightarrow k_1 = 2.414 \; \& \; K_2 = -0.414$

$$\boxed{i(t) = 2.414e^{-\sigma_1 t} - 0.414\,e^{-\sigma_2 t}\ A \qquad \sigma_1 = 0.586 \quad \sigma_2 = 3.41}$$

7.92 The switch in the circuit in Fig. P7.92 has been closed for a long time and is opened at $t = 0$. Find $i(t)$ for $t > 0$.

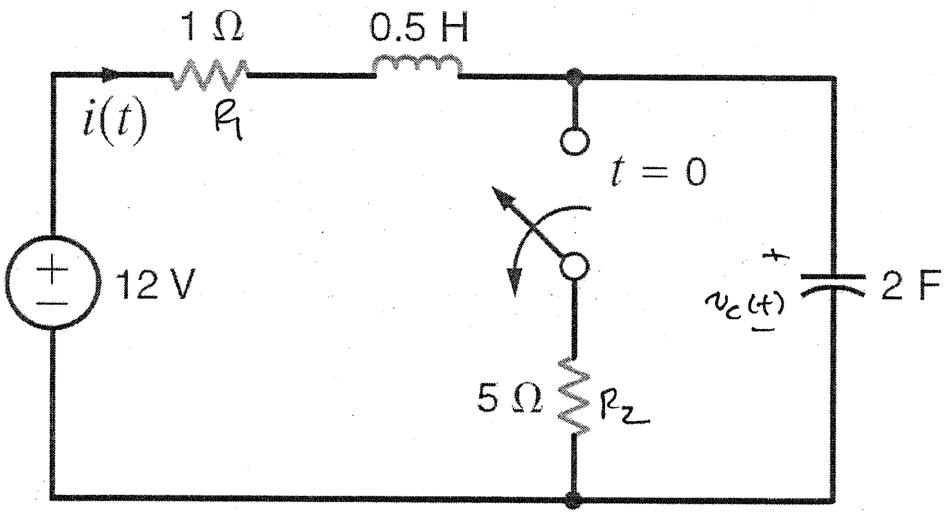


Figure P7.92

SOLUTION: $\underline{t=0^-}$: $i(0^-) = \frac{12}{R_1+R_2} = 2A$ $\quad v_c(0^-) = \frac{12 R_2}{R_1+R_2} = 10V$

$\underline{t=0^+}$: $12 = L\,di/dt + R_1 i + \frac{1}{C}\int i\,dt \Rightarrow s^2 + \frac{R_1}{L}s + \frac{1}{LC} = 0$

$s_{1,2} = -1\ r/s \qquad i(t) = B_1 e^{-t} + B_2 t e^{-t}$

$i(0^+) = 2A = B_1 \qquad 12 = L\frac{di}{dt}\Big|_{t=0} + R_1 i(0^+) + v_c(0^+)$

$12 = -LB_1 + LB_2 + R_1 B_1 + 10 \Rightarrow B_2 = 2$

$$\boxed{i(t) = 2e^{-t} + 2te^{-t}}$$

7.93 The switch in the circuit in Fig. P7.93 has been closed for a long time and is opened at $t = 0$. Find $i(t)$ for $t > 0$.

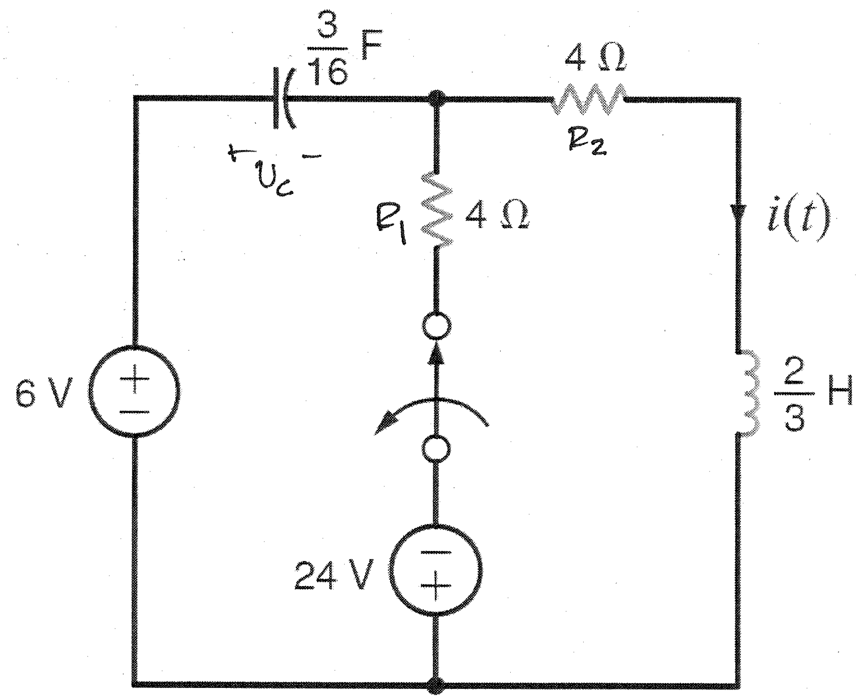


Figure P7.93

SOLUTION: $\underline{t=0^-}$: $i(0^-) = \frac{-24}{R_1+R_2} = -3A$ $\quad v_c(0^-) = 6 - i(0)R_2 = 18V$

$\underline{t=0^+}$ $\quad s^2 + \frac{R_2}{L}s + \frac{1}{LC} = 0 = s^2 + 6s + 8 \quad s_{1,2} = \begin{cases} -2 \\ -4 \end{cases}$

$i(t) = K_1 e^{-2t} + K_2 e^{-4t} \qquad i(0^+) = -3 = K_1 + K_2$

$v_c(0^+) = 6 - R_2 i(0^+) - L\frac{di}{dt}\Big|_{t=0^+} = 6 + 12 + 2L K_1 + 4L K_2 = 18$

yields, $K_1 = -6$ & $K_2 = 3$ $\Rightarrow$ $\boxed{i(t) = -6e^{-2t} + 3e^{-4t}\ A}$

7.94 The switch in the circuit in Fig. P7.94 has been closed for a long time and is opened at $t = 0$. Find $i(t)$ for $t > 0$.

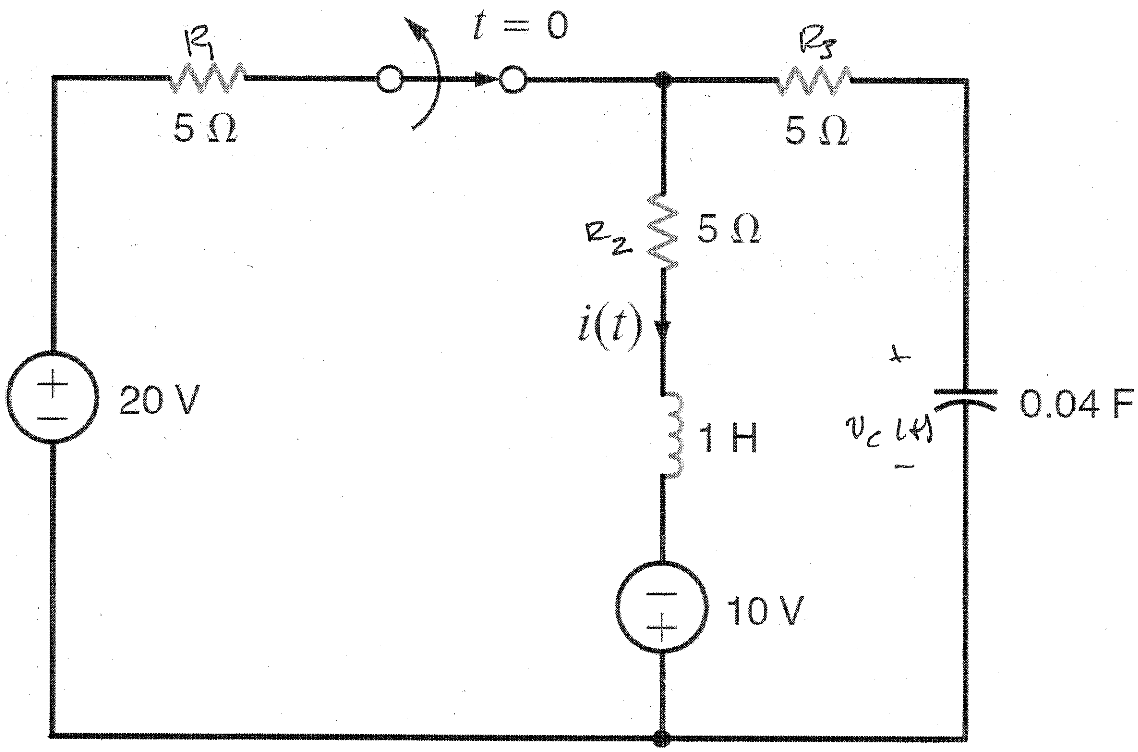


Figure P7.94

SOLUTION: $\underline{t=0^-}$: $i(0^-) = \frac{30}{R_1+R_2} = 3A$ $\quad v_c(0^-) = 20 - R_1 i(0^-) = 5V$

$\underline{t>0}$: $10 = L\frac{di}{dt} + Ri(t) + \frac{1}{C}\int i\,dt \quad R = R_2+R_3 = 10\Omega \quad v_c = -\frac{1}{C}\int i\,dt$

$s^2 + \frac{R}{L}s + \frac{1}{LC} = 0 = s^2 + 10s + 25 \Rightarrow s_{1,2} = -5\ r/s$

$i(t) = B_1 e^{-5t} + B_2 t e^{-5t} \qquad i(0^+) = 3 = B_1$

$-v_c(0^+) = -5 = 10 - Ri(0^+) - L\,di/dt|_{t=0^+} = 10 - 30 + L(5)B_1 - LB_2 \Rightarrow B_2 = 0$

$$\boxed{i(t) = 3e^{-5t}\ A}$$

7.95 The switch in the circuit in Fig. P7.95 has been closed for a long time and is opened at $t = 0$. Find $i(t)$ for $t > 0$.

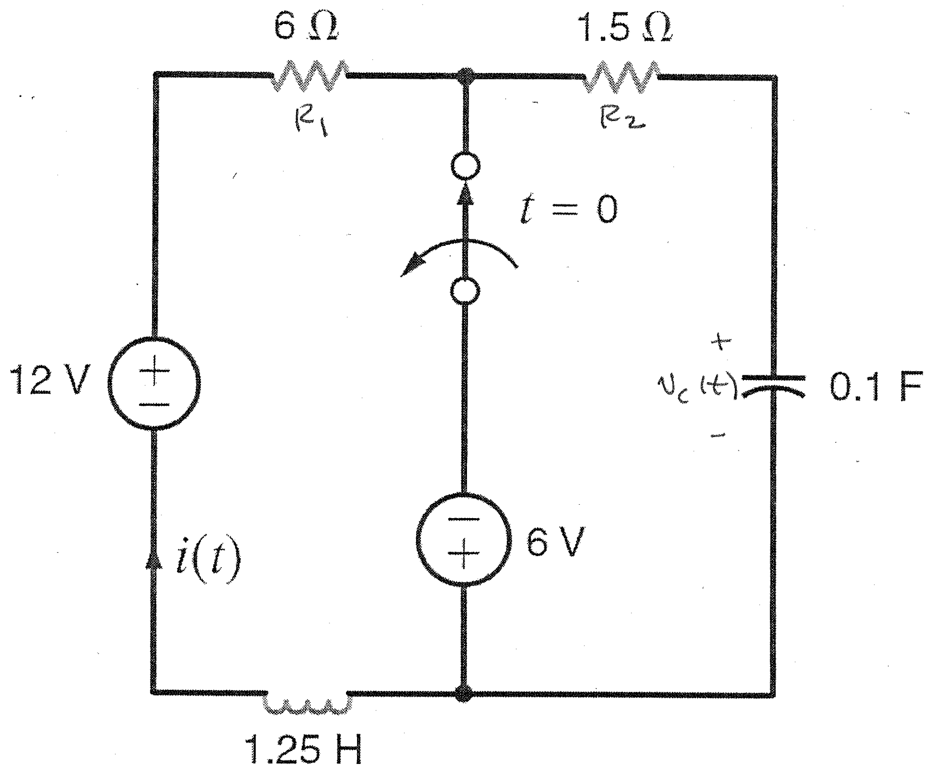


Figure P7.95

SOLUTION: $\underline{t=0^-}$: $i(0^-) = 18/R_1 = 3A$ $\quad v_c(0^-) = -6V = v_c(0^+)$

$\underline{t>0}$: $12 = Ri + \frac{1}{C}\int i\,dt + L\,di/dt$ $\quad R = R_1 + R_2 = 7.5\Omega$

$s^2 + (R/L)s + \frac{1}{LC} = s^2 + 6s + 8 \Rightarrow \quad \sigma_1 = 2 \quad \sigma_2 = 4$

$i(t) = K_1 e^{-2t} + K_2 e^{-4t}$ & $i(0^+) = 3 = K_1 + K_2$

$\underline{\text{at } t=0^+}$: $12 = R(K_1+K_2) + \frac{1}{C}\left[-\frac{K_1}{2} - \frac{K_2}{4}\right] + K + L\left[-2K_1 - 4K_2\right] \Rightarrow K = 12$

and $v_c(0^+) = \frac{1}{C}\left[-K_1/2 - K_2/4\right] + K = -6 \Rightarrow +5K_1 + 2.5K_2 = 18$

yields $K_1 = 4.2$ & $K_2 = -1.2$ $\quad \boxed{i(t) = 4.2e^{-2t} - 1.2e^{-4t}\ A}$

7.96 Using the PSPICE *Schematics* editor, draw the circuit in Fig. P7.96, and use the PROBE utility to plot $v_C(t)$ and determine the time constants for $0 < t < 1$ ms and $1 \text{ ms} < t < \infty$. Also, find the maximum voltage on the capacitor.

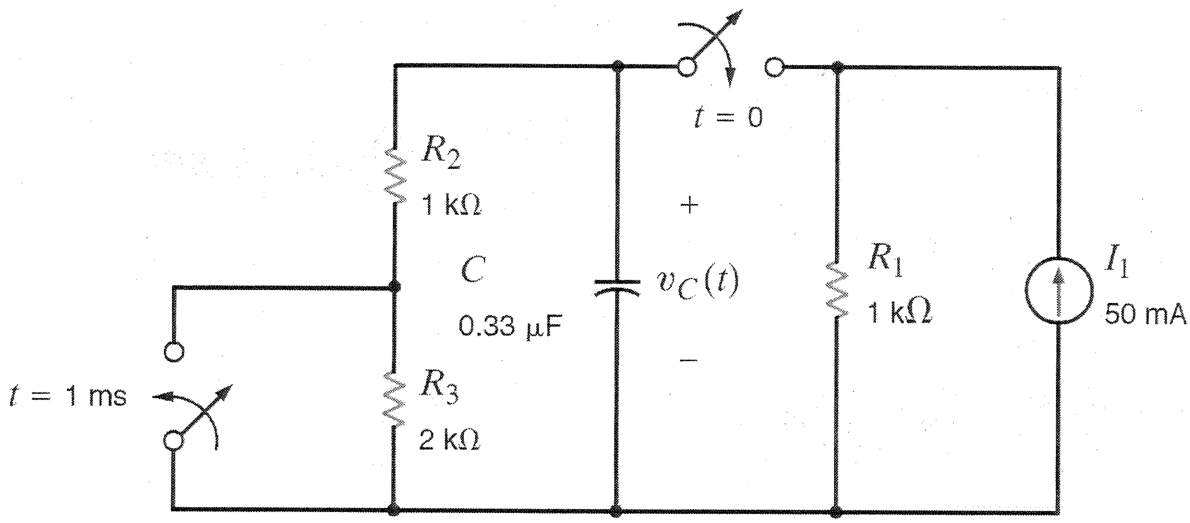
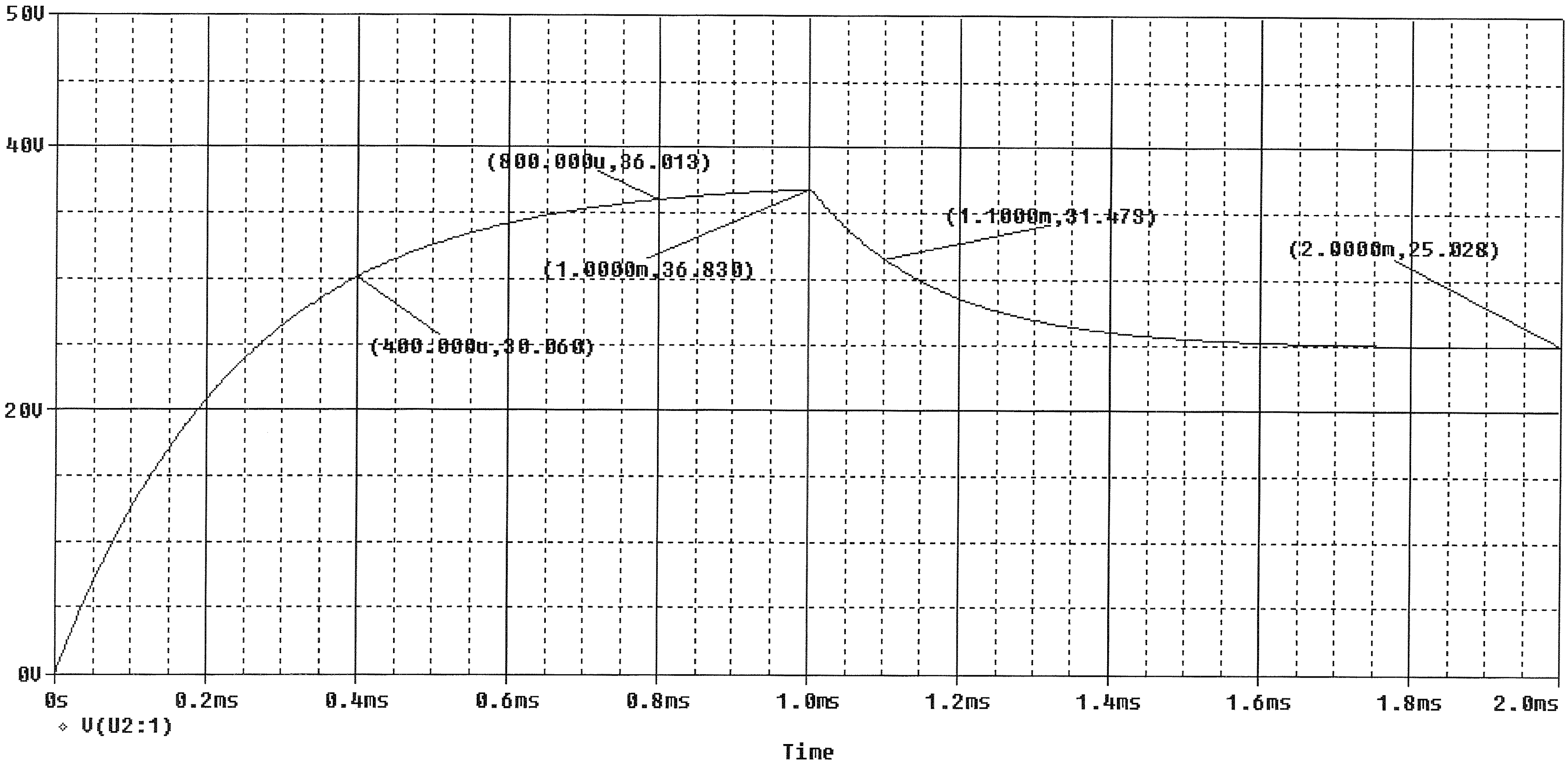


Figure P7.96

SOLUTION:

PROBLEM 7.96 PSPICE RESULTS



7.96 for $0 \le t \le 1ms$ $\quad v_c(t) = K_1 + K_2 e^{-t/\tau_1}$

$v_c(0) = 0 = K_1 + K_2 \Rightarrow K_2 = -K_1$

$v_c(t_1) = 30.6 = K_1 + K_2 e^{-t_1/\tau_1} \quad (t_1 = 0.4ms)$

$v_c(t_2) = 36.03 = K_1 + K_2 e^{-t_2/\tau_1} \quad (t_2 = 0.8ms)$

$$\frac{36.03}{30.60} = \frac{1 - e^{-t_2/\tau_1}}{1 - e^{-t_1/\tau_1}} = 1.177$$

$$1.177\, e^{-t_1/\tau_1} - e^{-t_2/\tau_1} = 0.177 = K$$

iterate!

$\tau_1 (ms)$	K
0.1	0.141
0.3	0.982
0.25	0.177 ✓

$\boxed{\tau_1 = 0.25\, ms}$

for $t > 1ms$, $\quad v_c(t) = K_3 + K_4 e^{-(t-10^{-3})/\tau_2}$

$v_c(10^{-3}) = K_3 + K_4 = 36.83 \qquad v_c(\infty) = 25 = K_1 \Rightarrow K_2 = 11.83$

$v_c(t_3) = K_3 + K_4 e^{-(t_3 - 10^{-3})/\tau_2} = 31.48 \qquad (t_3 = 1.1ms)$

yields $\boxed{\tau_3 = 0.166\, ms}$

v_c max occus at $t = 1ms$ $\qquad \boxed{v_{c\,max} = 36.83V}$

7.97 Using the PSPICE *Schematics* editor, draw the circuit in Fig. P7.97, and use the PROBE utility to find the maximum values of $v_L(t)$, $i_C(t)$, and $i(t)$.

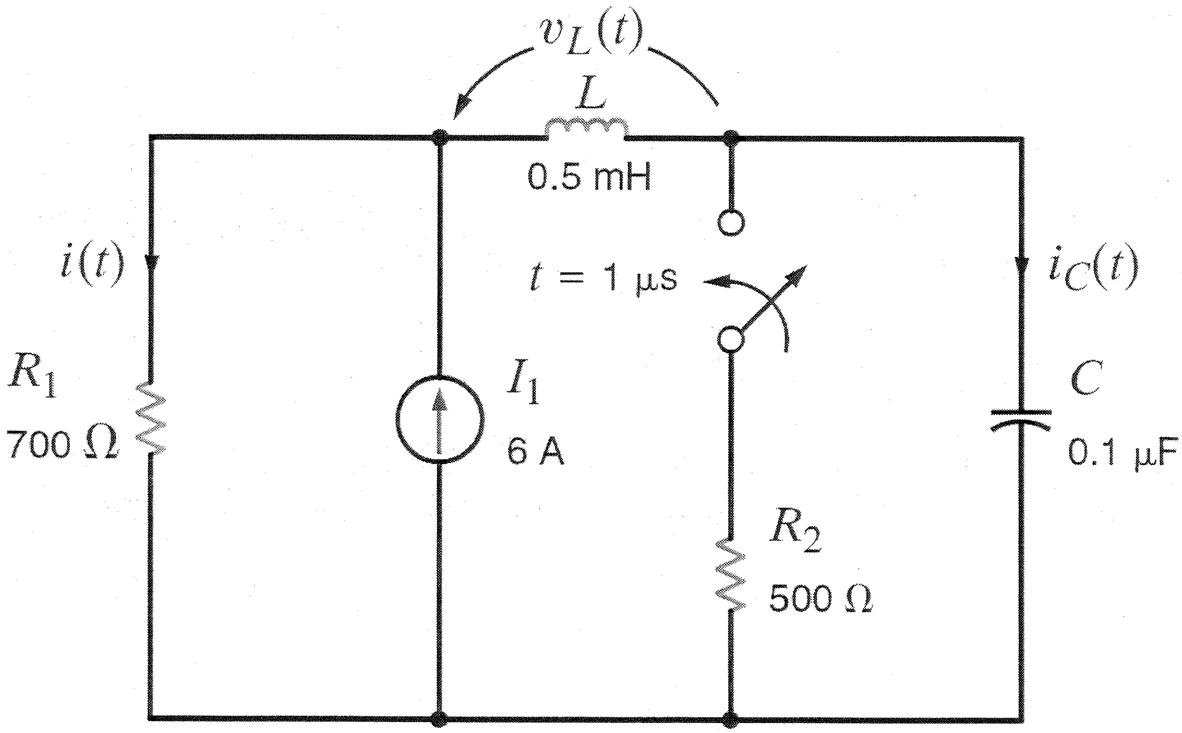


Figure P7.97

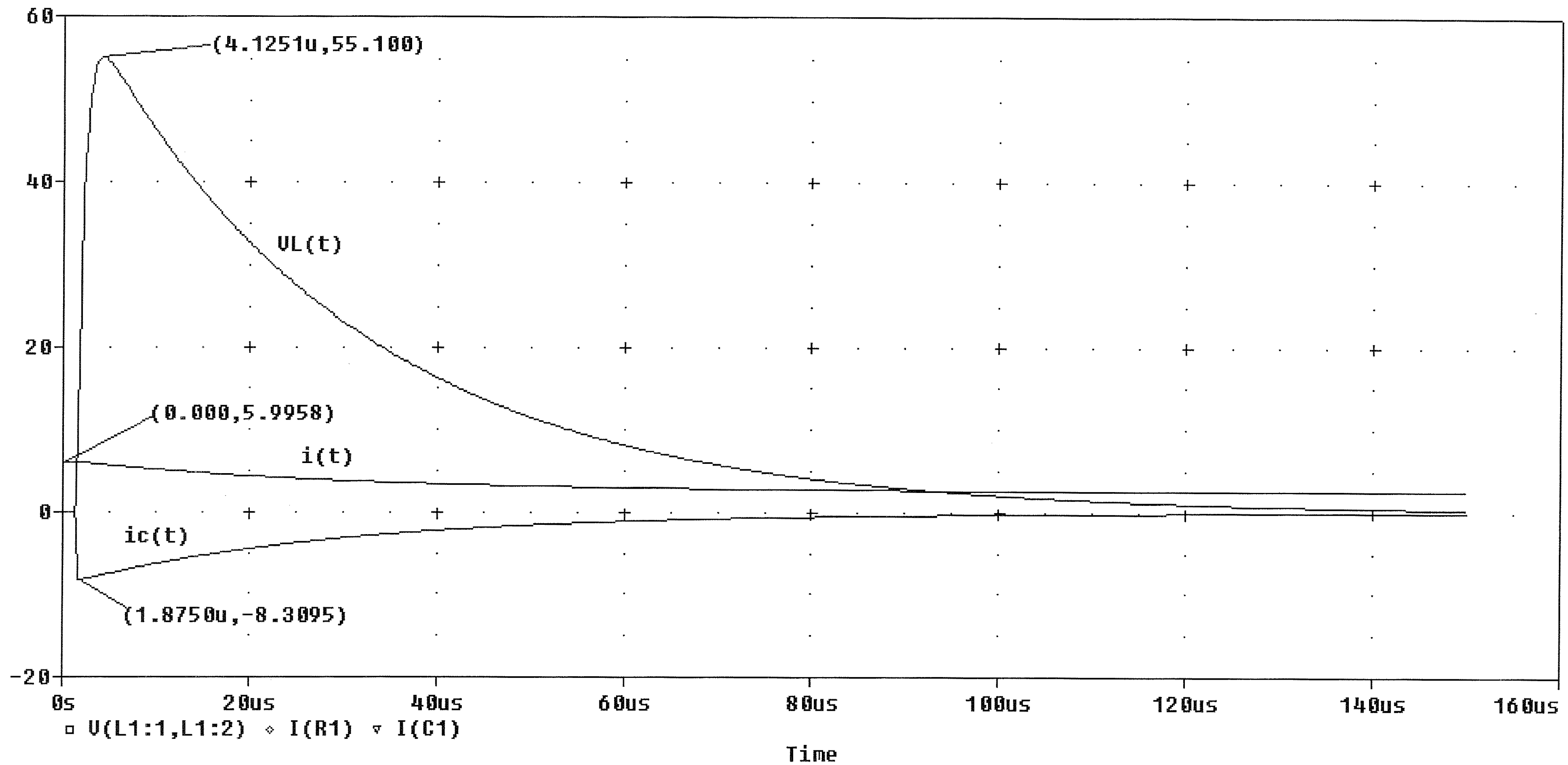
SOLUTION:

From the PSPICE simulation results,

v_L max = 55.1 V i_c max = − 8.31 A (actually the greatest deviation from ∅!)

i max = 6.00 A

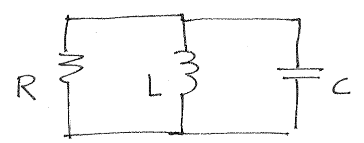
PROBLEM 7.97 PSPICE RESULTS



7.98 Design a parallel *RLC* circuit with $R \geqq 1\ \text{k}\Omega$ that has the characteristic equation

$$s^2 + 4 \times 10^7 s + 4 \times 10^{14} = 0$$

SOLUTION:



$$s^2 + \frac{1}{RC} s + \frac{1}{LC} = 0$$

$$RC = 2.5 \times 10^{-8} \qquad LC = 2.5 \times 10^{-15}$$

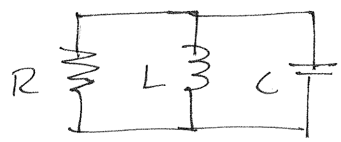
$$\frac{L}{R} = \frac{LC}{RC} = 10^{-7}$$

arbitrarily select $\boxed{L = 1\,\mu H \rightarrow R = 10\,\Omega,\ C = 2.5\,nF}$

7.99 Design a parallel *RLC* circuit with $R \geqq 1\ \text{k}\Omega$ that has the characteristic equation

$$s^2 + 4 \times 10^7 s + 3 \times 10^{14} = 0$$

SOLUTION:



$$s^2 + \frac{1}{RC} s + \frac{1}{LC} = 0$$

$RC = 0.25 \times 10^{-7}$ $\quad LC = 0.33 \times 10^{-14}$

Arbitrarily choose $\boxed{L = 10\mu H \rightarrow R = 75\Omega \ \& \ C = 0.33\,\text{nF}}$

7.100 The curve shown in Fig. P7.100 is used to model the pressure in a vessel located in a chemical plant. We wish to design a circuit to realize this function so that we can study various parameters in the vessel, such as volume.

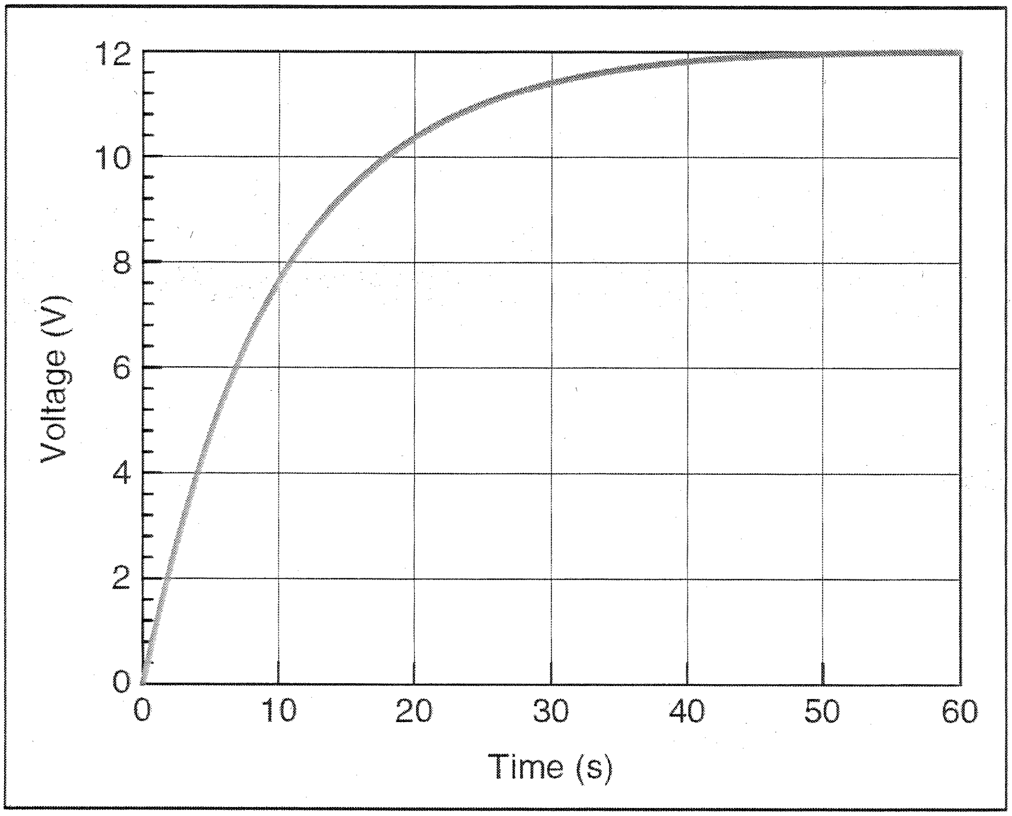
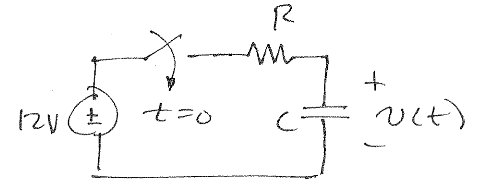


Figure P7.100

SOLUTION:

$v(t) = K_1 + K_2 e^{-t/\tau}$ $\quad V(0) = K_1 + K_2 = 0$ $\quad v(\infty) = 12 = K_1$

$t_1 = 10\,s$ $\quad v(t_1) = 7.6\,V = 12\,(1 - e^{-t_1/\tau}) \Rightarrow \tau = 10\,s$

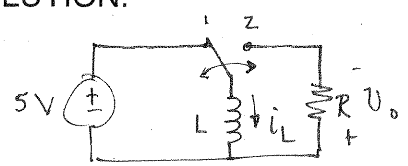


$RC = \tau = 10$

select $\boxed{C = 100\mu F \rightarrow R = 100k\Omega}$

7.101 Let us redesign the pulse generator in Example 7.14 such that a voltage with the following characteristics is created across a 10-kΩ resistor: a peak value of 250 V, a cycle time of 10,000 pulses/second, and a T_1 value of one-half the cycle time.

SOLUTION:



frequency = 10 kHz

period $= \frac{1}{f} = 0.1\,ms$

$T_1 = \frac{period}{2} = 50\mu s$

$i_{L_{max}} = \frac{5}{L} T_1$

$v_{o_{max}} = i_{L_{max}} R = \frac{5}{L}(50\times10^{-6})(10^4) = 250$

$\boxed{L = 10\,mH}$

Check repeatability: $\tau = \frac{L}{R} = 1\mu s$

$5\tau < T_1$? yes!!

7FE-1 In the circuit in Fig. 7PFE-1, the switch, which has been closed for a long time, opens at $t = 0$. Find the value of the capacitor voltage $v_C(t)$ at $t = 2$ s. **CS**

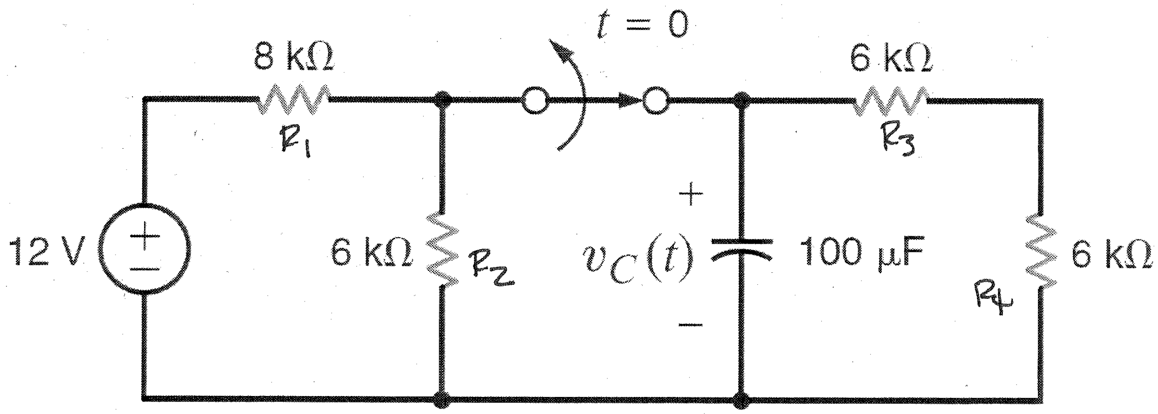
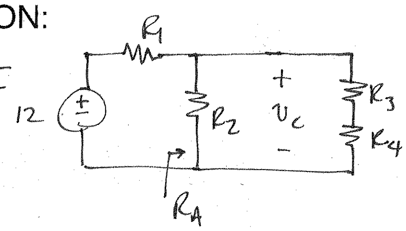


Figure 7PFE-1

SOLUTION:

$t = 0^-$



$R_A = R_2 // (R_3 + R_4) = 4k\Omega$

$v_c(0^-) = \dfrac{12 R_A}{R_A + R_1} = 4V$

$t = 0^+$: $K_1 + K_2 = v_c(0^-) = 4$

$t = \infty$ $K_1 = 0$

$\tau = CR_{eq}$ $R_{eq} = R_3 + R_4 = 12k\Omega$ $\tau = 1.2s$

$v_c(t) = 4e^{-t/1.2}$ V $\boxed{v_c(2) = 0.756V}$

7FE-2 In the network in Fig. 7PFE-2, the switch closes at $t = 0$. Find $v_o(t)$ at $t = 1$ s.

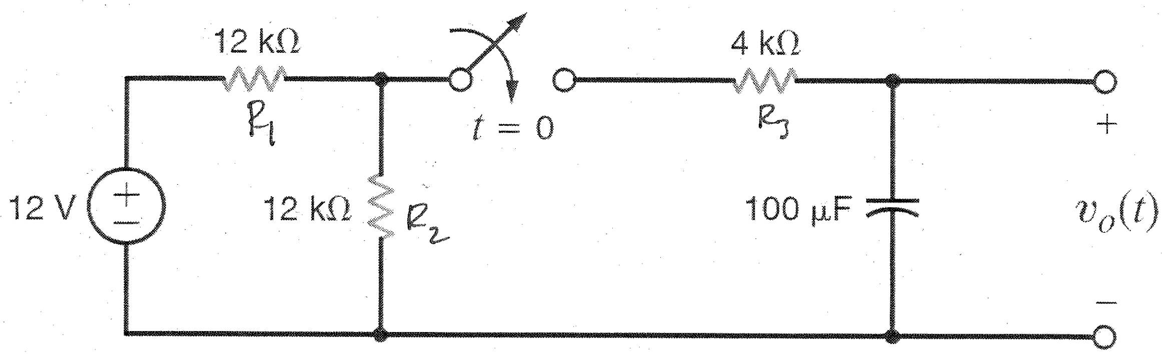


Figure 7PFE-2

SOLUTION: $v_o(t) = K_1 + K_2 e^{-t/\tau}$

$\underline{t=0^-}$: $v_c(0^-) = 0 = v_o(0^-)$ $\quad$ $\underline{t=0^+}$: $v_c(0^+) = 0 = v_o(0^+) = K_1 + K_2$

$\underline{t \to \infty}$: $v_o(\infty) = \dfrac{12 R_2}{R_1 + R_2} = 6\text{V} = K_1$

$\tau = C R_{eq}$ $\quad$ $R_{eq} = R_3 + (R_1 // R_2) = 10\,k\Omega$ $\quad$ $\tau = 1\,s$

$v_o(t) = 6(1 - e^{-t})$ $\quad$ $\boxed{v_o(1) = 3.79\text{V}}$

7FE-3 Assume that the switch in the network in Fig. 7PFE-3 has been closed for some time. At $t = 0$ the switch opens. Determine the time required for the capacitor voltage to decay to one-half of its initially charged value. CS

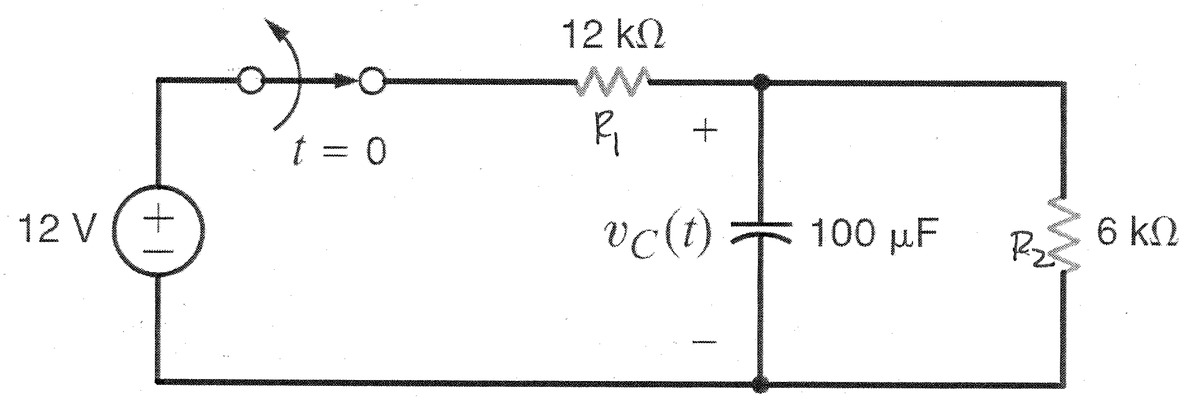


Figure 7PFE-3

SOLUTION: $v_c(t) = K_1 + K_2 e^{-t/\tau}$

$t=0^-$: $v_c(0^-) = \frac{12 R_2}{R_1+R_2} = 4V = v_c(0^+)$

$t=0^+$ $v_c(0^+) = 4 = K_1 + K_2$ $t \to \infty$ $v_c(\infty) = 0 = K_1$

$\tau = C R_{eq} \Rightarrow R_{eq} = R_2$ $\tau = 0.6 s$

$v_c(t) = 4 e^{-t/0.6}$ V $v(t_1) = 2 = 4 e^{-t_1/0.6}$

$\boxed{t_1 = 0.416 \text{ s}}$