13.25 Given the following functions $\mathbf{F}(s)$, find $f(t)$.

(a) $\mathbf{F}(s) = \dfrac{s+8}{s^2(s+4)}$

(b) $\mathbf{F}(s) = \dfrac{1}{s^2(s+1)^2}$

SOLUTION:

a) $F(s) = \dfrac{K_1}{s^2} + \dfrac{K_2}{s} + \dfrac{K_3}{s+4}$ $\quad K_1 = 2 \quad K_3 = \dfrac{4}{(-4)^2} = 1/4$

let $s=-2$, $F(-2) = \dfrac{6}{4(2)} = \dfrac{3}{4} = \dfrac{2}{4} - \dfrac{K_2}{2} + \dfrac{1/4}{2} \Rightarrow K_2 = -1/4$

$F(s) = \dfrac{2}{s^2} - \dfrac{1/4}{s} + \dfrac{1/4}{s+4} \Rightarrow \boxed{f(t) = \left[2t - 1/4 + 1/4\, e^{-4t}\right] u(t)}$

b) $F(s) = \dfrac{K_1}{s^2} + \dfrac{K_2}{s} + \dfrac{K_3}{(s+1)^2} + \dfrac{K_4}{s+1}$ $\quad K_1 = 1 \quad K_3 = 1$

let $s=1$, $F(1) = \dfrac{1}{4} = 1 + K_2 + \dfrac{1}{4} + \dfrac{K_4}{2} \Rightarrow K_2 + \dfrac{K_4}{2} = -1$

let $s=-2$, $F(-2) = \dfrac{1}{4} = \dfrac{1}{4} - \dfrac{K_2}{2} + \dfrac{1}{1} - K_4 \Rightarrow K_2 + 2K_4 = 2$

$\Rightarrow K_2 = -2, \quad K_4 = 2$

$F(s) = \dfrac{1}{s^2} - \dfrac{2}{s} + \dfrac{1}{(s+1)^2} + \dfrac{2}{s+1} \Rightarrow \boxed{f(t) = \left[t - 2 + te^{-t} + 2e^{-t}\right] u(t)}$