

13.40 Use convolution to find $f(t)$ if

$$\mathbf{F}(s) = \frac{10}{(s+1)(s+3)^2}$$

SOLUTION:

$$F_1(s) = \frac{10}{s+1} \Rightarrow f_1(t) = 10e^{-t}$$

$$F_2(s) = \frac{1}{(s+3)^2} \Rightarrow te^{-3t} = f_2(t)$$

$$f(t) = \int_0^t 10e^{-(t-\lambda)} \lambda e^{-3\lambda} d\lambda = 10e^{-t} \int_0^t \lambda e^{-2\lambda} d\lambda$$

$$= 10e^{-t} \left[\frac{\lambda}{2} e^{-2\lambda} + \frac{e^{-2\lambda}}{4} \right] \Big|_0^t = \left(\frac{10}{4} e^{-t} - 5te^{-3t} - \frac{10}{4} e^{-3t} \right) u(t)$$

$$\boxed{f(t) = [2.5e^{-t} - 5te^{-3t} - 2.5e^{-3t}] u(t)}$$