

13.27 Given the following functions $F(s)$, find $f(t)$. **CS**

$$(a) \quad F(s) = \frac{s^2}{(s+1)^2(s+2)}$$

$$(b) \quad F(s) = \frac{s^2 + 9s + 20}{s(s+4)^3(s+5)}$$

SOLUTION:

$$a) \quad F(s) = \frac{k_1}{(s+1)^2} + \frac{k_2}{s+1} + \frac{k_3}{s+2} \quad k_1 = 1 \quad k_3 = \frac{4}{1} = 4$$

$$\text{let } s = 0, \quad F(0) = 0 = 1 + k_2 + 2 \Rightarrow k_2 = -3$$

$$F(s) = \frac{1}{(s+1)^2} - \frac{3}{s+1} + \frac{4}{s+2} \Rightarrow f(t) = [te^{-t} - 3e^{-t} + 4e^{-2t}]u(t)$$

$$b) \quad F(s) = \frac{(s+4)(s+5)}{s(s+4)^3(s+5)} = \frac{1}{s(s+4)^2} = \frac{k_1}{s} + \frac{k_2}{(s+4)^2} + \frac{k_3}{s+4}$$

$$k_1 = 1/16 \quad k_2 = -1/4$$

$$\text{let } s = -2; \quad F(-2) = \frac{1}{-2(2)^2} = -\frac{1}{8} = -\frac{1}{32} - \frac{1}{16} + \frac{k_3}{2} \Rightarrow k_3 = -1/16$$

$$F(s) = \frac{1}{16} \left[\frac{1}{s} - \frac{4}{(s+4)^2} - \frac{1}{s+4} \right] \Rightarrow f(t) = \frac{1}{16} [1 - 4te^{-4t} - e^{-4t}]u(t)$$