13.36 Solve the following differential equations using Laplace transforms.

(a) $\dfrac{d^2y(t)}{dt^2} + \dfrac{2dy(t)}{dt} + y(t) = e^{-2t},$

$$y(0) = y'(0) = 0$$

(b) $\dfrac{d^2y(t)}{dt^2} + \dfrac{4dy(t)}{dt} + 4y(t) = u(t), \quad y(0) = 0,$

$$y'(0) = 1$$

SOLUTION:

a) $s^2 Y(s) + 2s Y(s) + Y(s) = \dfrac{1}{s+2} \Rightarrow Y(s)\left[s^2 + 2s + 1\right] = \dfrac{1}{s+2}$

$Y(s) = \dfrac{1}{(s+2)(s+1)^2} = \dfrac{K_1}{s+2} + \dfrac{K_2}{(s+1)^2} + \dfrac{K_3}{s+1} \qquad K_1 = 1, \; K_2 = 1$

$\text{let } s = 0, \quad Y(0) = \dfrac{1}{2} = \dfrac{K_1}{2} + K_2 + K_3 \Rightarrow K_3 = -1$

$Y(s) = \dfrac{1}{s+2} + \dfrac{1}{(s+1)^2} - \dfrac{1}{s+1} \Rightarrow \boxed{y(t) = \left[e^{-2t} + te^{-t} - e^{-t}\right]u(t)}$

b) $s^2 Y(s) - sy'(0) + 4s Y(s) + 4 Y(s) = \dfrac{1}{s} \Rightarrow Y(s)\left[s^2 + 4s + 4\right] = \dfrac{1}{s} + s = \dfrac{s^2+1}{s}$

$Y(s) = \dfrac{s^2+1}{s(s+2)^2} = \dfrac{K_1}{s} + \dfrac{K_2}{(s+2)^2} + \dfrac{K_3}{s+2} \qquad K_1 = \dfrac{1}{4} \qquad K_2 = -\dfrac{5}{2}$

$\text{let } s = -1, \quad Y(-1) = \dfrac{2}{-1} = -2 = -K_1 + K_2 + K_3 \Rightarrow K_3 = 3/4$

$Y(s) = \dfrac{1}{4}\left[\dfrac{1}{s} - \dfrac{10}{(s+2)^2} + \dfrac{3}{s+2}\right] \Rightarrow \boxed{y(t) = \dfrac{1}{4}\left[1 - 10te^{-2t} + 3e^{-2t}\right]u(t)}$