13.3 If $f(t) = e^{-at}$, show that $\mathbf{F}(s) = \dfrac{1}{s+a}$.

SOLUTION:

$$F(s) = \int_0^\infty e^{-at} e^{-st}\,dt = \int_0^\infty e^{-(s+a)t}\,dt = \left.\frac{e^{-(s+a)t}}{-(s+a)}\right|_0^\infty$$

$$F(s) = \frac{e^{-\infty(s+a)}}{-(s+a)} - \frac{e^{-0}}{-(s+a)} = \frac{1}{s+a}$$

$$\boxed{F(s) = \frac{1}{s+a}} \text{ for } \sigma > -a$$