13.33 Find $f(t)$ if $\mathbf{F}(s)$ is given by the following functions:

(a) $\mathbf{F}(s) = \dfrac{(s^2 + 2s + 1)e^{-2s}}{s(s+1)(s+2)}$

(b) $\mathbf{F}(s) = \dfrac{(s+1)e^{-4s}}{s^2(s+2)}$

SOLUTION:

a) Let $G(s) = \dfrac{s^2+2s+1}{s(s+1)(s+2)} = \dfrac{s+1}{s(s+2)} = \dfrac{K_1}{s} + \dfrac{K_2}{s+2}$

$K_1 = 1/2 \qquad K_2 = -1/-2 = 1/2 \qquad G(s) = \frac{1}{2}\left[\frac{1}{s} + \frac{1}{s+2}\right]$

$F(s) = e^{-2s}G(s) \qquad g(t) = \frac{1}{2}\left[1 + e^{-2t}\right]u(t)$

$$\boxed{f(t) = \left[\tfrac{1}{2}\left(1 + e^{-2(t-2)}\right)u(t-2)\right.}$$

b) $G(s) = \dfrac{s+1}{s^2(s+2)} = \dfrac{K_1}{s^2} + \dfrac{K_2}{s} + \dfrac{K_3}{s+2} \qquad K_1 = \frac{1}{2} \qquad K_3 = \dfrac{-1}{(-2)^2} = -1/4$

let $s = -1$, $G(-1) = 0 = K_1 - K_2 + K_3 \Rightarrow K_2 = 1/4$

$G(s) = \frac{1}{4}\left[\frac{2}{s^2} + \frac{1}{s} - \frac{1}{s+2}\right] \Rightarrow g(t) = \frac{1}{4}\left[2t + 1 - e^{-2t}\right]u(t)$

$F(s) = e^{-4s}G(s) \rightarrow$ $$\boxed{f(t) = \frac{1}{4}\left[2(t-4) + 1 - e^{-2(t-4)}u(t-4)\right]}$$