13.34 Find $f(t)$ if $\mathbf{F}(s)$ is given by the following functions:

(a) $\mathbf{F}(s) = \dfrac{2(s+1)e^{-s}}{(s+2)(s+4)}$

(b) $\mathbf{F}(s) = \dfrac{10(s+2)e^{-2s}}{(s+1)(s+4)}$

SOLUTION:

a) $G(s) = \dfrac{2(s+1)}{(s+2)(s+4)} = \dfrac{K_1}{s+2} + \dfrac{K_2}{s+4}$ $\begin{cases} K_1 = \dfrac{2(-1)}{2} = -1 \\ K_2 = \dfrac{2(-3)}{-2} = 3 \end{cases}$

$G(s) = \dfrac{3}{s+4} - \dfrac{1}{s+2} \Rightarrow g(t) = (3e^{-4t} - e^{-2t})u(t)$

$F(s) = G(s)e^{-s} \Rightarrow \boxed{f(t) = [3e^{-4(t-1)} - e^{-2(t-1)}]u(t-1)}$

b) $G(s) = \dfrac{10(s+2)}{(s+1)(s+4)} = \dfrac{K_1}{s+1} + \dfrac{K_2}{s+4}$ $\begin{cases} K_1 = 10/3 \\ K_2 = -20/-3 = 20/3 \end{cases}$

$G(s) = \dfrac{10}{3}\left[\dfrac{1}{s+1} + \dfrac{2}{s+4}\right] \Rightarrow g(t) = \left[\dfrac{10}{3}e^{-t} + \dfrac{20}{3}e^{-4t}\right]u(t)$

$F(s) = e^{-2s}G(s) \Rightarrow \boxed{f(t) = \left[\dfrac{10}{3}e^{-(t-2)} + \dfrac{20}{3}e^{-4(t-2)}\right]u(t-2)}$