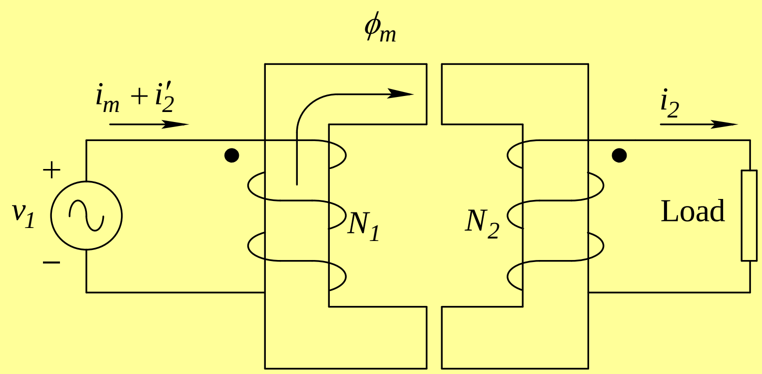
Induction Motors: Balanced, Sinusoidal Steady State Operation

- Principle of Operation
  - Induced voltages
  - Induced Currents
  - Torque Produced

Short-circuiting the Rotor (R_s, $L_{s,leakage} = 0$)

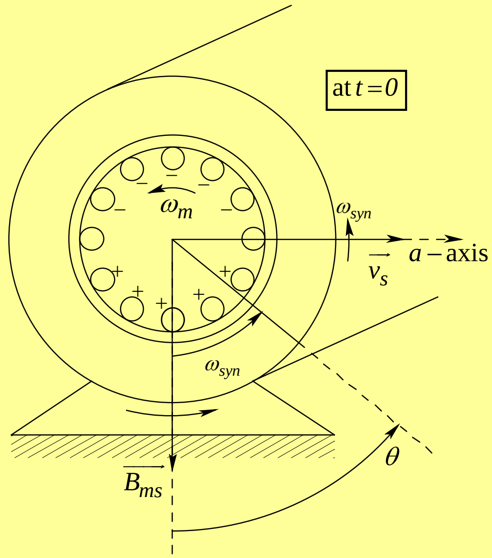
- Transformer Analogy



- Assuming no resistances or leakage inductance in the stator windings, the stator voltages completely determine the motor flux regardless of any rotor currents

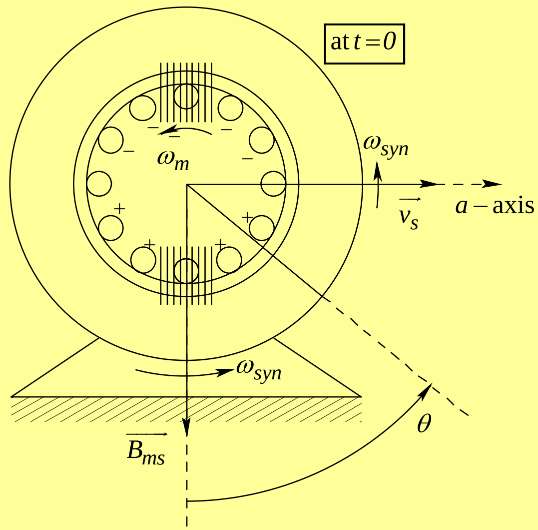
- Flux Φ_m is unaffected by the load

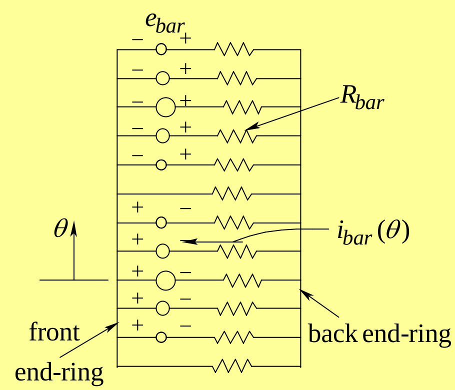
Induced Voltages on Rotor



Flux rotating at speed ω_{syn}
Rotor rotating at speed ω_m
Rotor conductors cutting flux at speed:
$\omega_{syn} - \omega_m = \omega_{slip}$ (slip speed)
Cutting flux generates voltage across rotor conductors: $e_{bar}(\theta) = \ell r \, \omega_{slip} \, \hat{B}_{ms} \cos\theta$

Induced Currents in Rotor

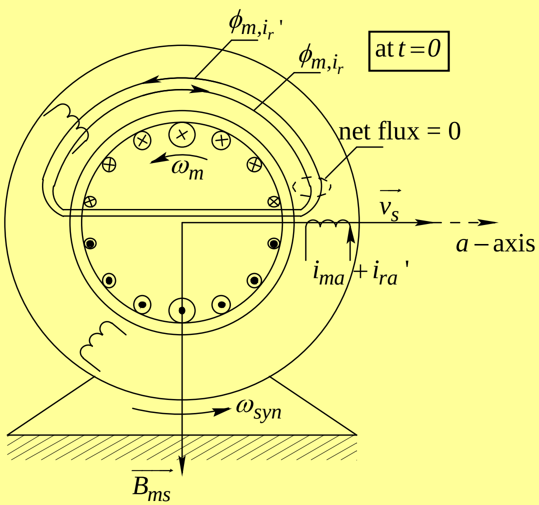


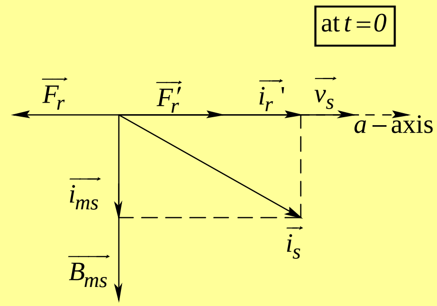


- Rotor conductors (bars) shorted together by end rings
- Because of symmetry of induced bar voltages, end rings are at same potential, therefore bar voltage is dropped across bar resistance (assuming $L_{r,l}$ = 0) generating currents by Ohms Law

$$i_{bar} = \frac{e_{bar}}{R_{bar}} = \frac{B_m \ell r \omega_{slip}}{R_{bar}}$$

Rotor MMF – Reflected Rotor MMF – Reflected Rotor Current





$$\vec{F}_s(t) = \vec{F}_{ms}(t) + \vec{F}_r'(t)$$

$$\vec{i}_s(t) = \vec{i}_{ms}(t) + \vec{i}_r'(t)$$

$$\hat{I}_r' = k_i \hat{B}_{ms} \omega_{slip}$$

- $\vec{F}_r$ produced by rotor currents
- $\vec{F}_r'$ produced by additional stator currents to keep total flux unchanged (transformer analogy)
- These currents are viewed as a current space vector
- Total stator current is magnetizing current plus this reflected rotor current $\vec{i}_r'$

Slip frequency (f_{slip}) in the rotor circuit

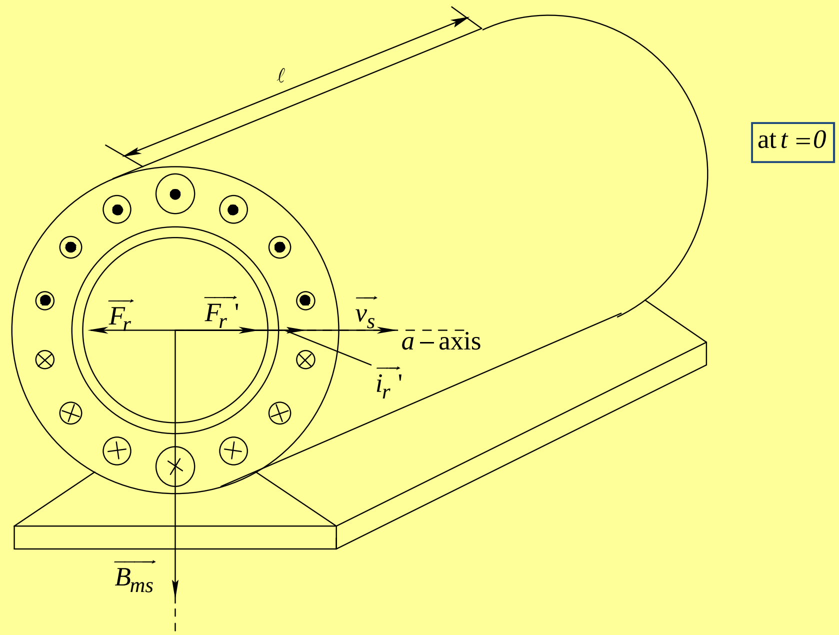
$$slip\ :\ s = \frac{\omega_{slip}}{\omega_{syn}} = \frac{\omega_{syn} - \omega_m}{\omega_{syn}}$$

$$f_{slip} = \frac{\omega_{slip}}{\omega_{syn}} f = sf$$

$$\omega_{slip} + \omega_m = \omega_{syn}$$

- ❑ Slip is rotor speed normalized to synchronous speed
- ❑ Slip generally small (< 3%), therefore rotor current frequency is very low

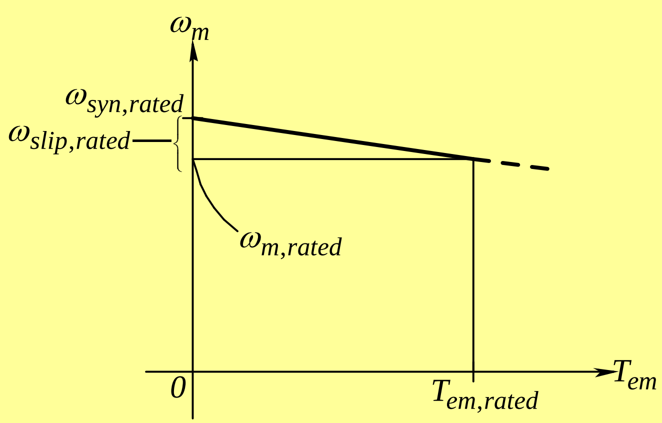
Electromagnetic Torque Production

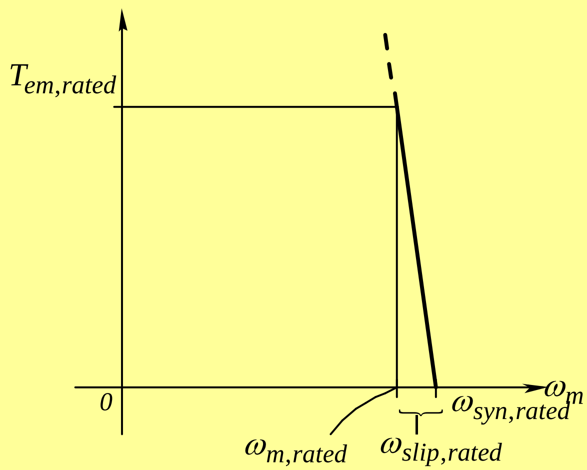


- Current $\vec{i}_r'$, in field $\vec{B}_{ms}$, produces torque T_{em}

$$T_{em} = \underbrace{\pi \frac{N_s}{2} r \ell}_{k_t} \hat{B}_{ms} \hat{I}_r' = k_{e\omega} \hat{B}_{ms}^2 \omega_{slip}$$

Torque – Speed Characteristics (slip small ; ω_m near ω_{syn})





- ❑ Linear relationship
- ❑ These curves are valid up to rated torque

Summary

- Principle of Operation
  - Induced voltages
  - Induced Currents
  - Torque Produced