Week 5: BJT Biasing and Small Signal Model

Topics to cover …

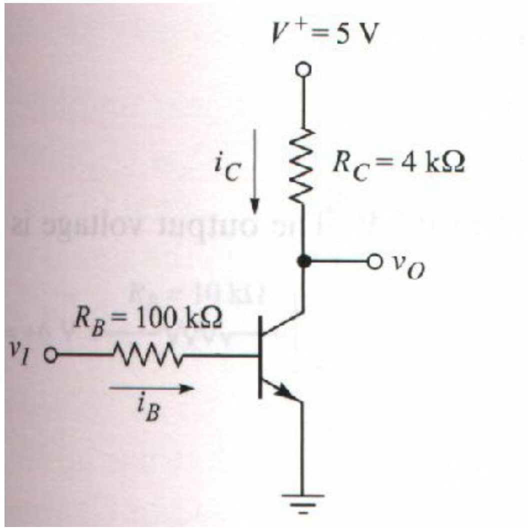
- BJT Amplifier Biasing Circuits

- Small Signal Operation and Equivalent Circuits

Reading Assignment:
Chap 13.1 – 13.6 of Jaeger and Blalock , or
Chap 5.5 - 5.7 of Sedra & Smith

BJT as Amplifier: Example 1

- **Problem:** Determine the dc voltage transfer characteristic of the circuit for $0 < v_I < 5$ V
- **Analysis:**



For $v_I \le 0.7$ V, Q is cut off and $v_O = 5$ V.

For $v_I > 0.7$ V, Q turns on and is in the active mode, so that

$$i_B = \frac{v_I - V_{BE}}{R_B} = \frac{v_I - 0.7}{100\text{k}\Omega}$$

The output voltage is

$$v_O = V^+ - i_C R_C = V^+ - \beta i_B R_C$$

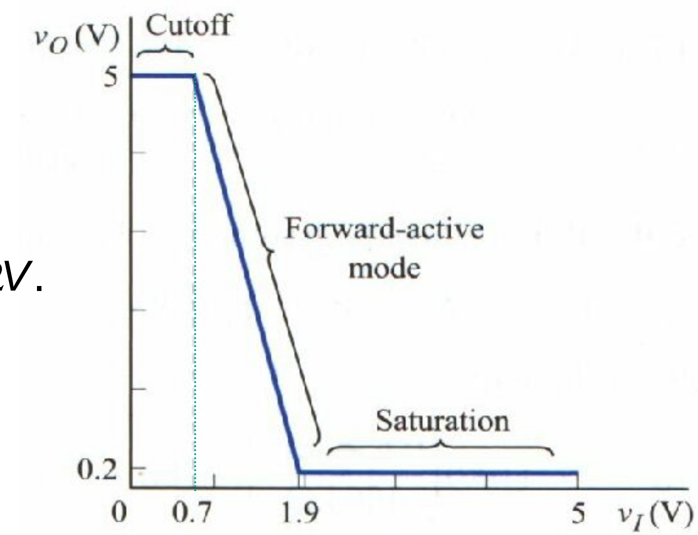
or

$$v_O = 5 - (100)\left[\frac{v_I - 0.7}{100\text{k}\Omega}\right] 4\text{k}\Omega$$

This equation is valid for $v_I \ge 0.7$ V and $v_O \ge v_{CE}(\text{sat}) = 0.2V$.

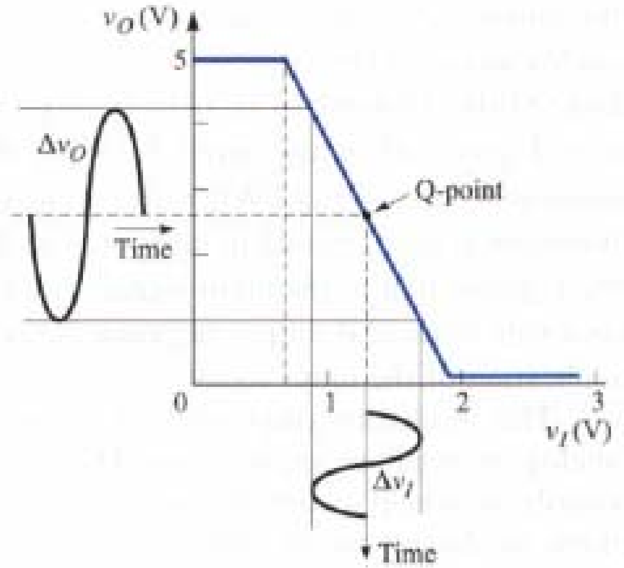
The input voltage for $v_O = 0.2$ V is found to be $v_I = 1.9$ V.

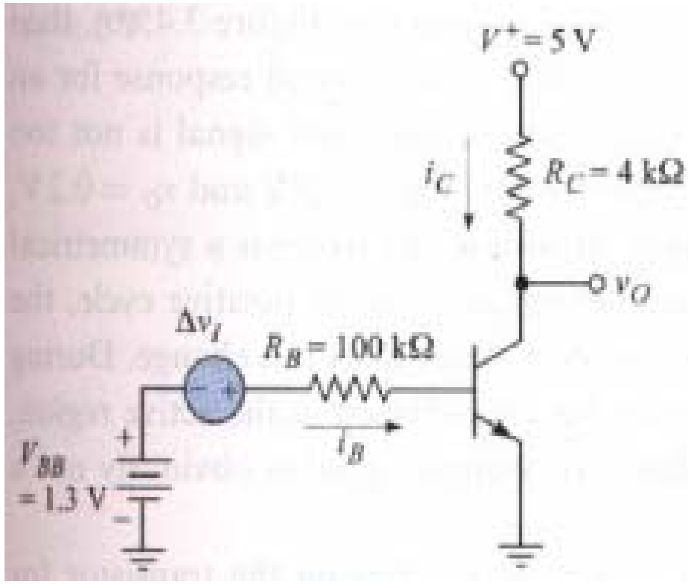
Now, for $v_I > 1.9$ V, the transistor is in saturation.



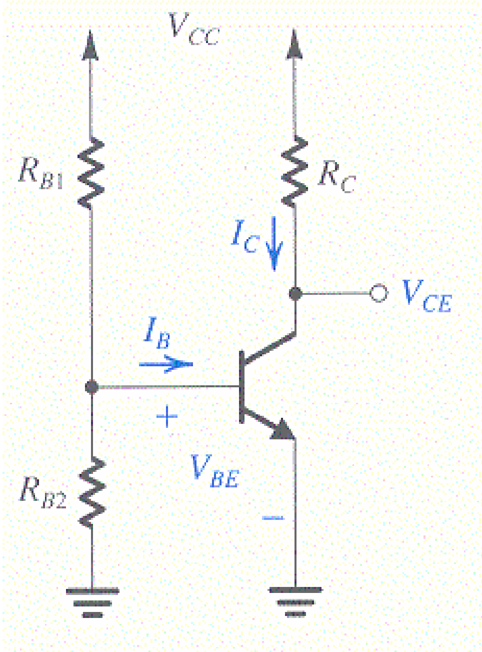
Conceptual Bias Circuit for BJT

- Keep the transistor in the active mode;
- Establish a Q-point near the center of the active region;
- Couple the time-varying signal to the base.



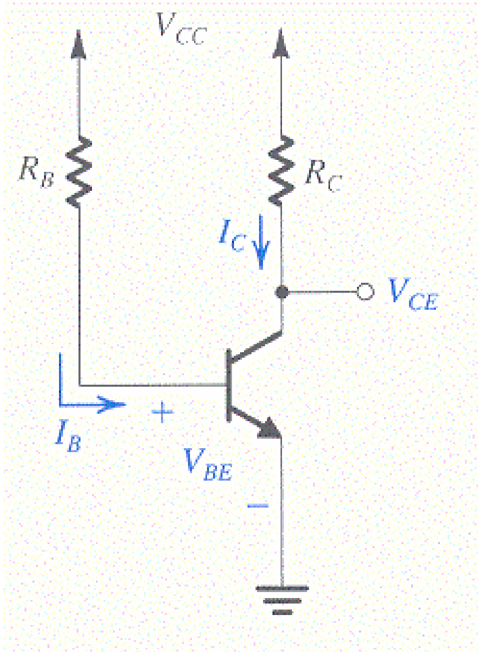


Two Obvious Bias Circuits



Fix V_{BE}

→ I_C is an exponential function of V_{BE} and thus V_{CC}



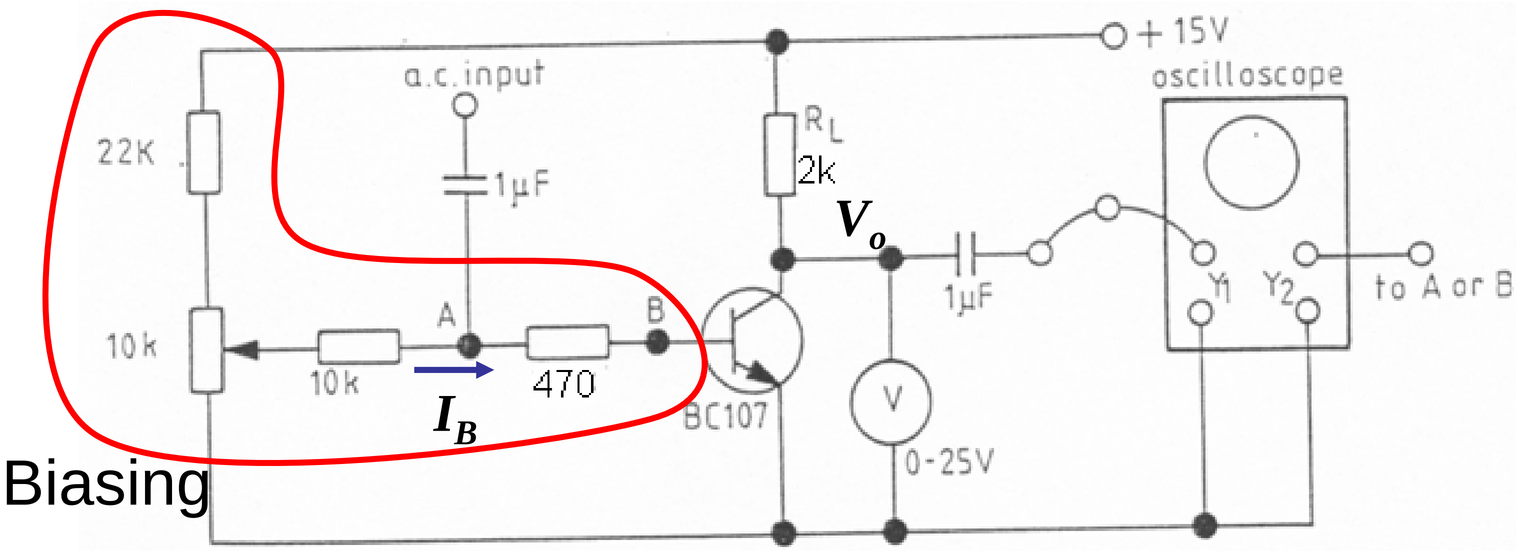
Fix I_B

→ $I_C=\beta I_B$, but β is a poorly controlled parameter

→ Wide variations in I_C and hence in V_{CE}

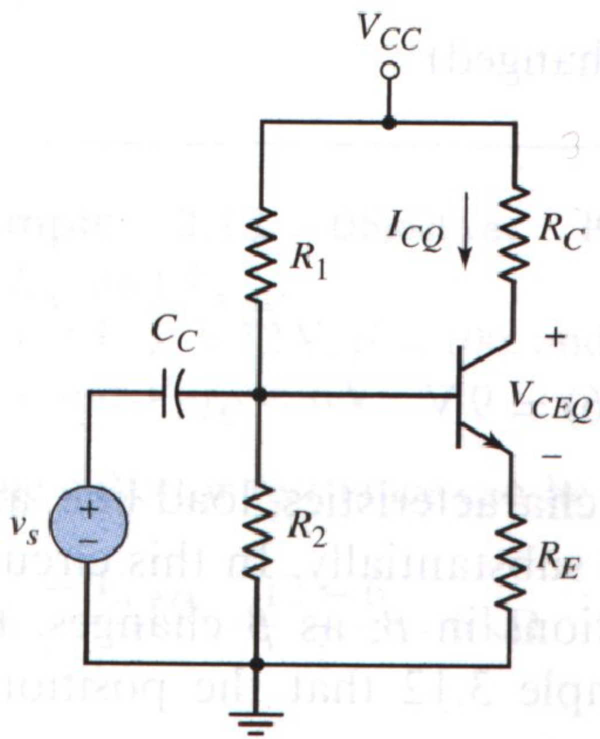
→ "Bad" biasing schemes

Biasing Circuit in ERG2810 Experiment A3



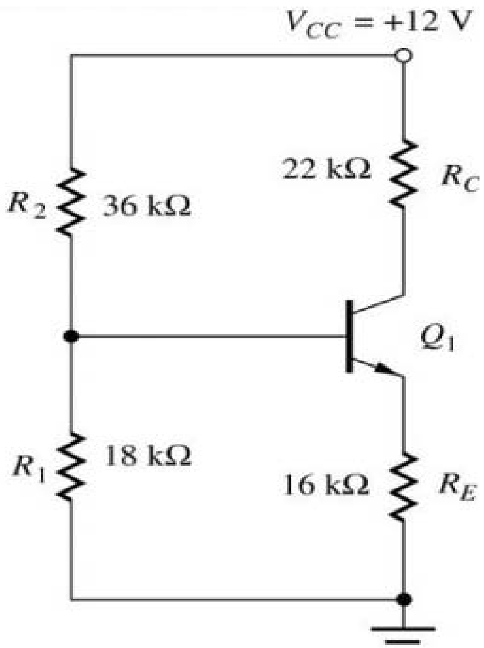
- Adjusting the variable resistor to give a desired amount of I_B such that V_o is at about 8V (half the supply voltage).
- The transistor operates at the middle of active region.

Classical Four-Resistor Bias Circuit



- V_B established by the voltage divider formed by R_1 and R_2.
- R_E added → reduce sensitivity to supply voltage, process, and temperature variations
  → to be discussed later
- Coupling capacitor C_C:
  - open circuit to DC, isolating the signal source from the dc biasing current.
  - short circuit to AC signal (if the signal frequency is large enough and C_C is large enough).
- The Q-point is usually specified by (I_C, V_{CE}) for *npn* transistor or (I_C, V_{EC}) for *pnp* transistor.

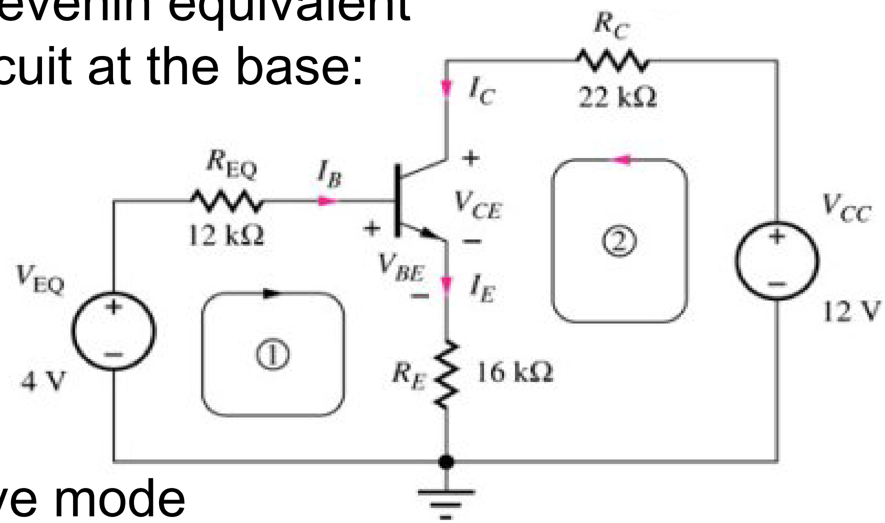
Example 2



Given β = 200.

Problem: find Q-point.

Thevenin equivalent circuit at the base:



$$V_{EQ} = V_{CC}\frac{R_1}{R_1 + R_2}$$

$$R_{EQ} = \frac{R_1 R_2}{R_1 + R_2}$$

Assume BJT in active mode

KVL at loop 1: $V_{EQ} = R_{EQ} I_B + V_{BE} + R_E I_E$

$$4 = 12{,}000 I_B + 0.7 + 16{,}000(\beta + 1) I_B$$

$$\therefore I_B = \frac{4\text{V} - 0.7\text{V}}{1.23 \times 10^6 \Omega} = 2.68\mu\text{A}$$

$$I_C = \beta I_B = 201\mu\text{A} \qquad I_E = (\beta + 1) I_B = 204\mu\text{A}$$

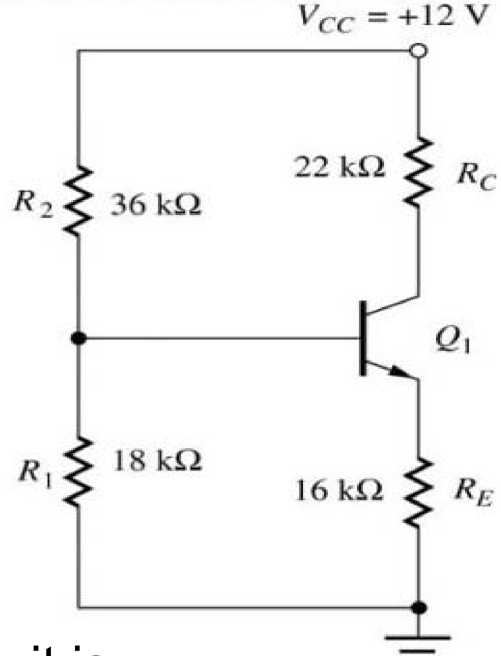
$$V_{CE} = V_{CC} - R_C I_C - R_E I_E$$

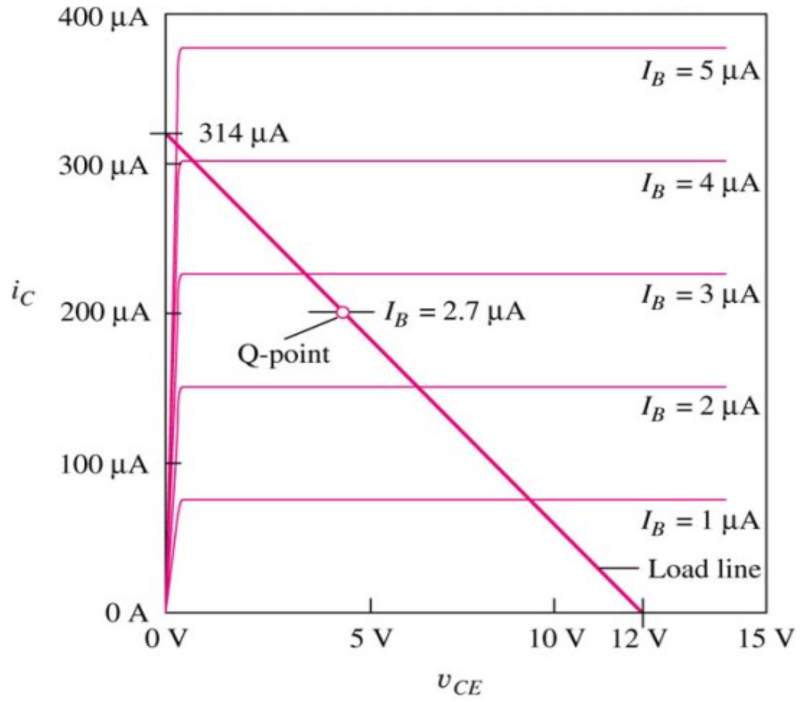
$$= V_{CC} - \left(R_C + \frac{R_F}{\alpha_F} \right) I_C = 4.32\text{ V}$$

Q-point is (201 μA, 4.32 V)

Example 2 (Cont')

- All calculated currents > 0,
  $V_{BC} = V_{BE} - V_{CE} = 0.7 - 4.32 = -3.62$ V
- Hence, base-collector junction is reverse-biased, assumption of active region operation is correct.





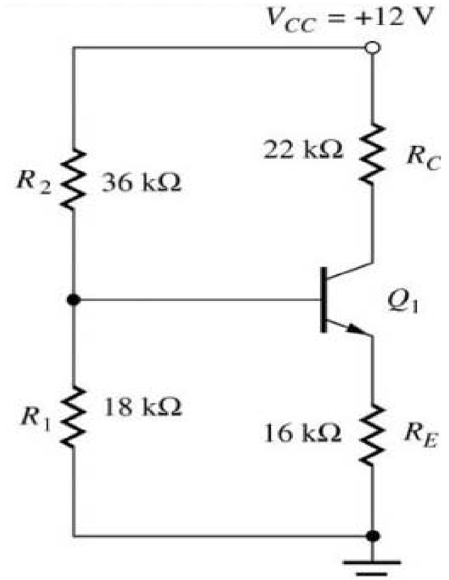
- Load-line for the circuit is:

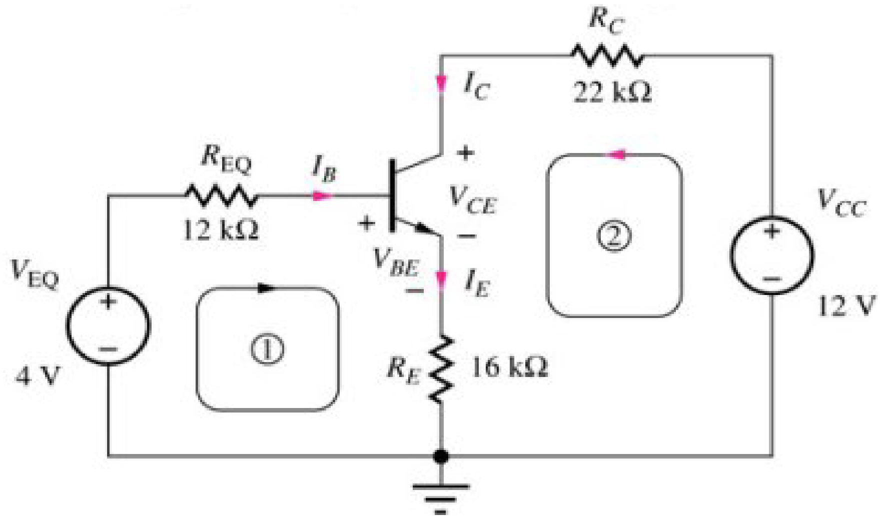
$$V_{CE} = V_{CC} - \left(R_C + \frac{R_E}{\alpha} \right) I_C = 12 - 38{,}200\ I_C$$

$I_B = 2.7$ μA

intersection point → Q-point.

Example 2 (Cont')





$$V_{EQ} = V_{CC} \frac{R_1}{R_1 + R_2}$$

$$R_{EQ} = \frac{R_1 R_2}{R_1 + R_2}$$

$$V_{EQ} = R_{EQ} I_B + V_{BE} + R_E I_E$$

V_{CC} Temp. Process (β)

$$I_E = \frac{V_{EQ} - V_{BE} - R_{EQ} I_B}{R_E}$$

Design Objectives:
Q-point Insensitive to PVT Variations

$$I_E = \frac{V_{EQ} - V_{BE} - R_{EQ} I_B}{R_E}$$

1) I_E linearly (not exponentially) related to V_{CC}
2) For I_E to be less sensitive to I_B (thus β):

   → $R_{EQ} I_B << (V_{EQ} - V_{BE})$
   → Need small R_{eq}
   → Large currents through R_1 and R_2 ($I_2 >> I_B$)

3) For I_E to be less sensitive to V_{BE} (due to temperature change):
   → $V_{EQ} >> V_{BE}$

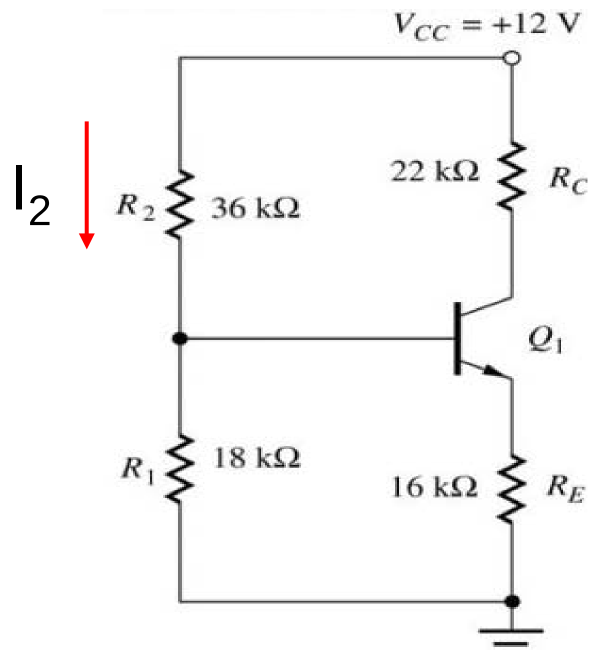
Design Objectives: Low Power and Large Signal Swing

But …

- For low power consumption:
  → needs small I_2
  → contradict with constraint (2)
  → set $I_2 = 10I_B$ typically

- For large output signal swing:
  → V_{EQ} cannot be too high
  (because $V_C \in [V_{EQ}, V_{CC}]$)

  → $V_{EQ} = 1/3\ V_{CC}$ typically



Design Guidelines

- Choose Thevenin equivalent base voltage

$$\frac{V_{CC}}{4} \le V_{EQ} \le \frac{V_{CC}}{2}$$

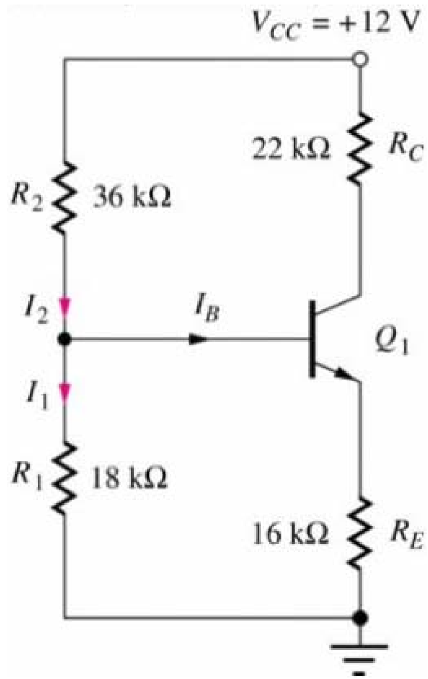
- Select R_1 to set $I_1 = 9I_B$ $\quad R_1 = \frac{V_{EQ}}{9 I_B}$

- Select R_2 to set $I_2 = 10I_B$ $\quad R_2 = \frac{V_{CC} - V_{EQ}}{10 I_B}$

- R_E is determined by V_{EQ} and desired I_C

$$R_E \approx \frac{V_{EQ} - V_{BE}}{I_C}$$

- R_C is determined by desired V_{CE} $\quad R_C \approx \frac{V_{CC} - V_{CE}}{I_C} - R_E$



Example 3

- **Problem:** Design a 4-resistor bias circuit with given parameters.
- **Given data:** I_C =750 µA, V_{CE} = 5 V, β = 100, V_{CC} = 15 V, V_{BE} = 0.7 V
- **Unknowns:** V_B, voltages across R_E and R_C; values for R_1, R_2, R_C and R_E.
- **Analysis:** A common approach is to divide **(**V_{CC} **-** V_{CE}**)** equally between R_E and R_C. Thus, V_E =5 V and V_C =10 V.

$$R_C = \frac{V_{CC} - V_C}{I_C} = 6.67\,\text{k}\Omega$$

$$R_E = \frac{V_E}{I_E} = 6.60\,\text{k}\Omega$$

$$V_B = V_E + V_{BE} = 5.7\text{V}$$

$$I_B = \frac{I_C}{\beta} = 7.5\mu\text{A}$$

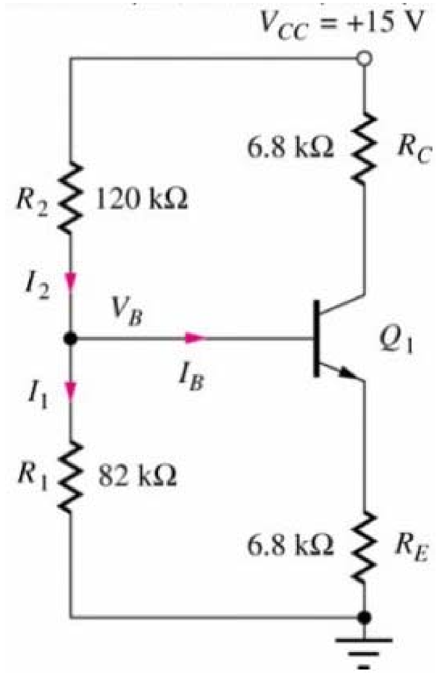
Now choose $I_2 = 10I_B$:

$$I_2 = 10I_B = 75\mu\text{A}$$

$$I_1 = 9I_B = 67.5\mu\text{A}$$

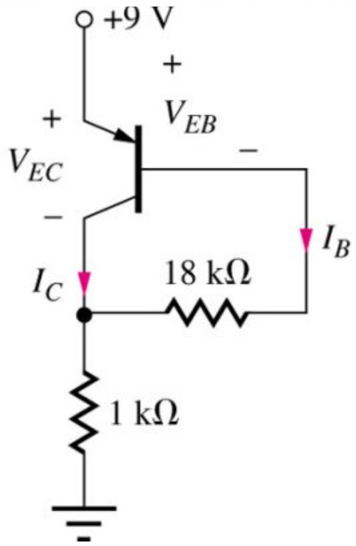
$$R_1 = \frac{V_B}{9I_B} = 84.4\text{k}\Omega$$

$$R_2 = \frac{V_{CC} - V_B}{10I_B} = 124\ \text{k}\Omega$$



Example 4: Two-Resistor Bias Network

- **Problem:** Find Q-point for *pnp* transistor in 2-resistor bias circuit with given parameters.
- **Given data:** β_F **=** 50, $V_{CC} = 9$ V
- **Assumptions:** Forward-active operation region, $V_{EB} = 0.7$ V
- **Analysis:**



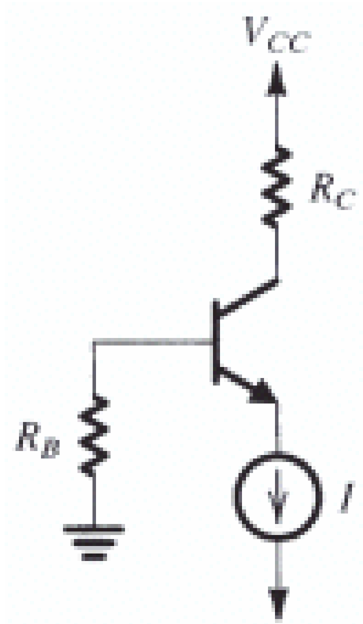
$$9 = V_{EB} + 18{,}000\, I_B + 1000\,(I_C + I_B)$$
$$\therefore 9 = V_{EB} + 18{,}000\, I_B + 1000\,(51) I_B$$

$$\therefore I_B = \frac{9\text{V} - 0.7\text{V}}{69{,}000\ \Omega} = 120\ \mu\text{A}$$
$$I_C = 50\, I_B = 6.01\,\text{mA}$$

$$V_{EC} = 9 - 1000\,(I_C + I_B) = 2.88\,\text{V}$$
$$V_{BC} = 2.18\,\text{V}$$

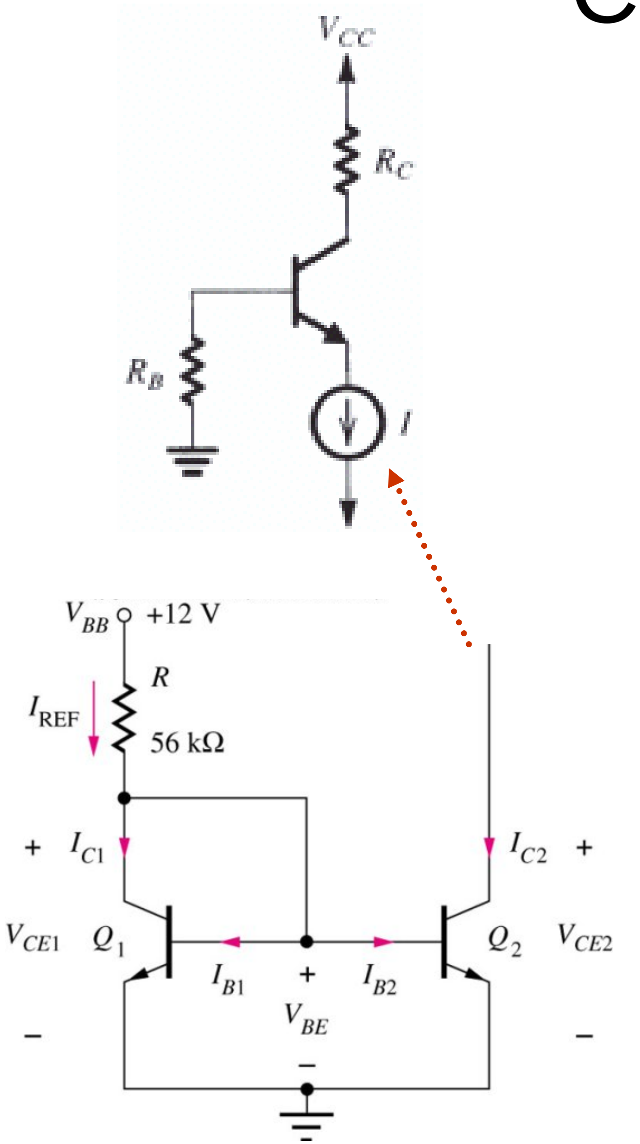
Q-point is : (6.01 mA, 2.88 V)

Current Source Biasing



- $I_E = I$ → independent of the value of β and temperature

Current Source Implementation: Current Mirror



- Use collector current of a transistor in active mode
- Neglect Early effect, I_{C2} is independent of V_{CE2} as long as $V_{CE2} > V_{CEsat}$ (i.e. BJT in active mode)
- For matched Q_1 and Q_2 , i.e., having identical I_S, β, V_A), we have

$$I_{C2} = I_{REF}$$

$$I_{REF} = \frac{V_{BB} - V_{BE}}{R}$$

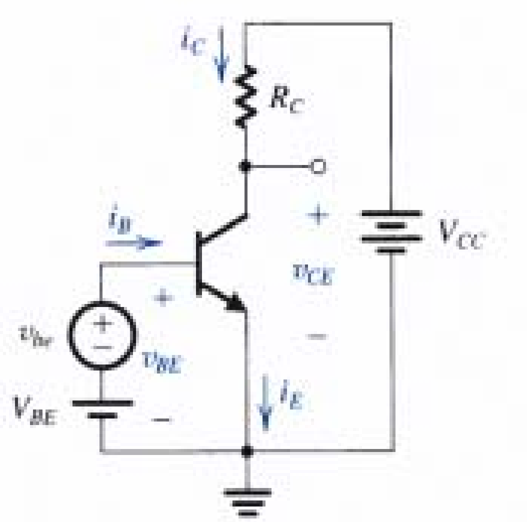
Topics to cover …

- BJT Amplifier Biasing Circuits

- Small Signal Operation and Equivalent Circuits

BJT as an Amplifier

Conceptual Amplifier circuit:



Superposition of DC with AC signal

If an ac+dc input signal the total v_{BE} becomes

$$v_{BE} = V_{BE} + v_{be}$$

The collector current becomes

$$i_C = I_S e^{(V_{BE} + v_{be})/V_T}$$

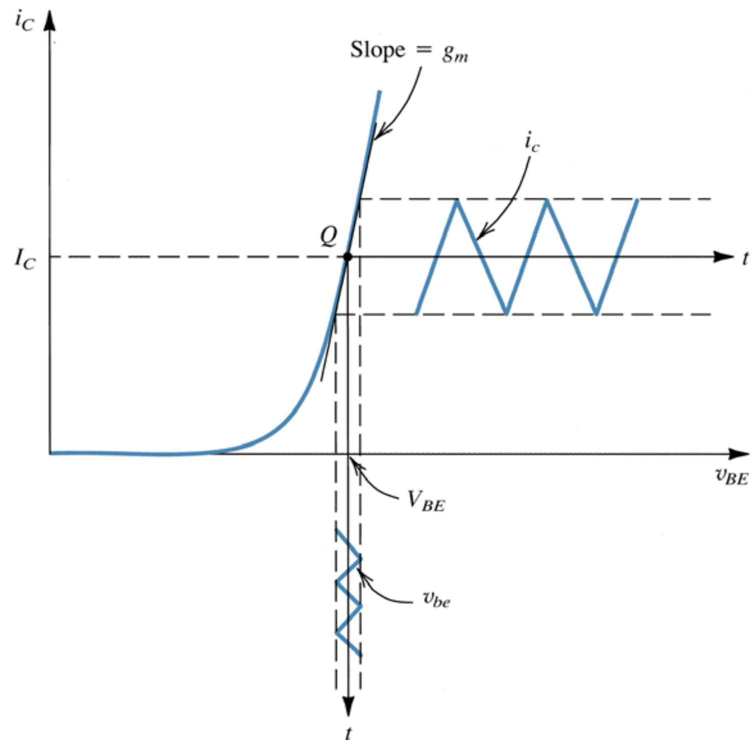
$$= I_S e^{V_{BE}/V_T} e^{v_{be}/V_T} = I_C e^{v_{be}/V_T}$$

Small-signal Transconductance

For small ac signal, i.e., $v_{be} << V_T$:

$$i_C = I_C e^{v_{be}/V_T} \cong I_C(1 + v_{be}/V_T)$$

$$= \underbrace{I_C}_{DC} + \underbrace{\frac{I_C}{V_T} v_{be}}_{AC}$$



The ac (or signal) component of the collector current is:

$$i_c = \frac{I_C}{V_T} v_{be}$$

We define:

$$g_m \equiv \frac{i_c}{v_{be}} = \frac{I_C}{V_T}$$

$$g_m \equiv \left.\frac{i_c}{v_{be}}\right|_{v_{be}\to 0} = \left.\frac{\partial i_C}{\partial v_{BE}}\right|_{i_C = I_C}$$

g_m is called the small signal ***transconductance***.
It represents the slope of i_C-v_{BE} curve at the Q-point.

Signal Component of Base Current

Total base current:
$$i_B = \frac{i_C}{\beta} = \underbrace{\frac{I_C}{\beta}}_{DC} + \underbrace{\frac{1}{\beta}\frac{I_C}{V_T}v_{be}}_{AC}$$

Signal component of base current:
$$i_b = \frac{1}{\beta}\frac{I_C}{V_T}v_{be} = \frac{g_m}{\beta}v_{be}$$

Define:
$$r_\pi \equiv \frac{v_{be}}{i_b} = \frac{\beta}{g_m} \quad \text{or} \quad r_\pi = \frac{V_T}{I_B}$$

r_π is the small-signal input resistance between base and emitter, *looking into the base*.

Signal Component of Emitter Current

The total emitter current i_E:

$$i_E = \frac{i_C}{\alpha} = \underbrace{\frac{I_C}{\alpha}}_{DC} + \underbrace{\frac{1}{\alpha}\frac{I_C}{V_T}v_{be}}_{AC}$$

Signal component of emitter current:

$$i_e = \frac{1}{\alpha}\frac{I_C}{V_T}v_{be} = \frac{I_E}{V_T}v_{be}$$

Define:

$$r_e \equiv \frac{v_{be}}{i_e} = \frac{\alpha}{g_m} \quad \text{or} \quad r_e = \frac{V_T}{I_E}$$

r_e is the small-signal input resistance between base and emitter, *looking into the emitter*.

It is easy to find out that $r_\pi = (i_e / i_b) r_e = (\beta + 1) r_e$

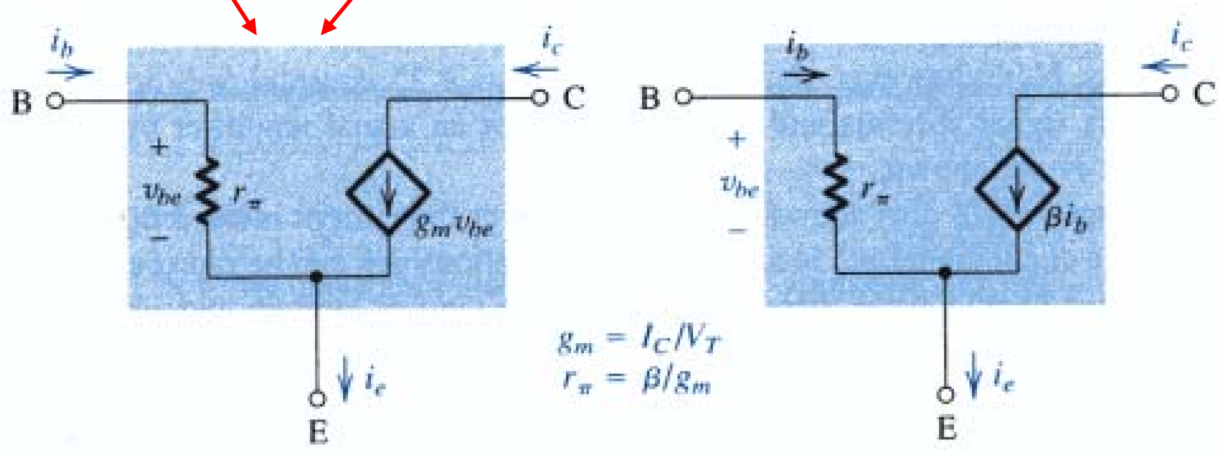
Small Signal I-V Expressions

$$\frac{i_c}{v_{be}} = g_m = \frac{I_C}{V_T}$$

$$\frac{v_{be}}{i_b} = r_\pi = \frac{V_T}{I_B}$$

$$\frac{v_{be}}{i_e} = r_e = \frac{V_T}{I_E}$$

Can be modeled by equivalent circuits:



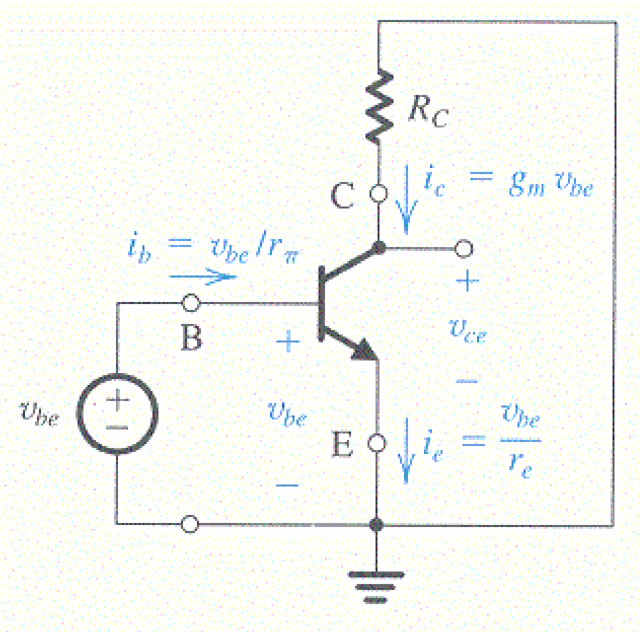
(***Hybrid-π*** small signal model of BJT)

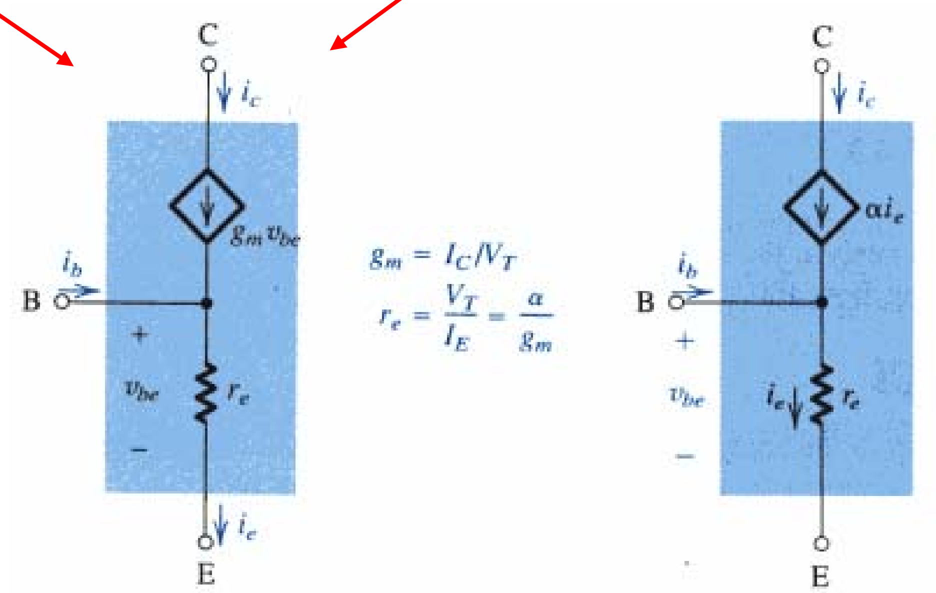
Another model: T-model

$$\frac{i_c}{v_{be}} = g_m = \frac{I_C}{V_T}$$

$$\frac{v_{be}}{i_b} = r_\pi = \frac{V_T}{I_B}$$

$$\frac{v_{be}}{i_e} = r_e = \frac{V_T}{I_E}$$

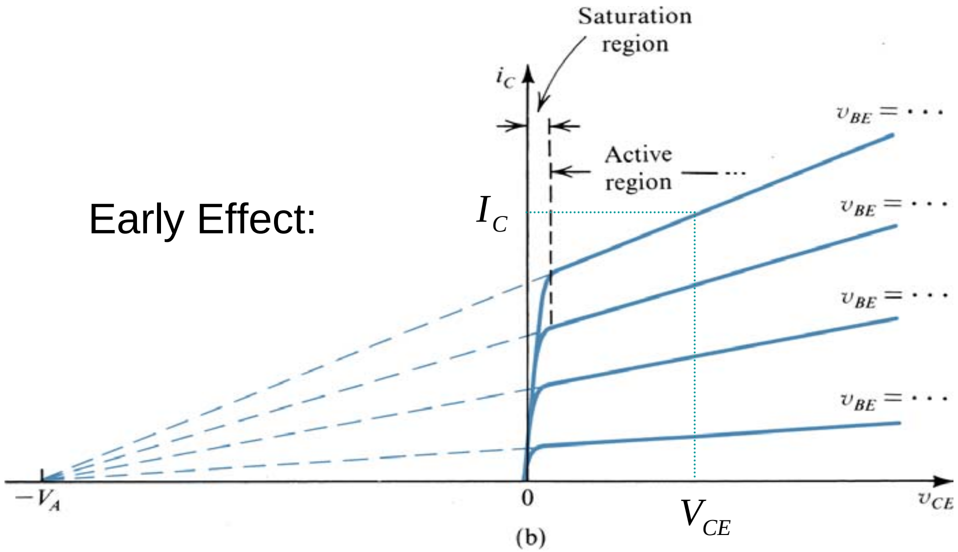




T-model

Hybrid-π model including Early effect

Early Effect:

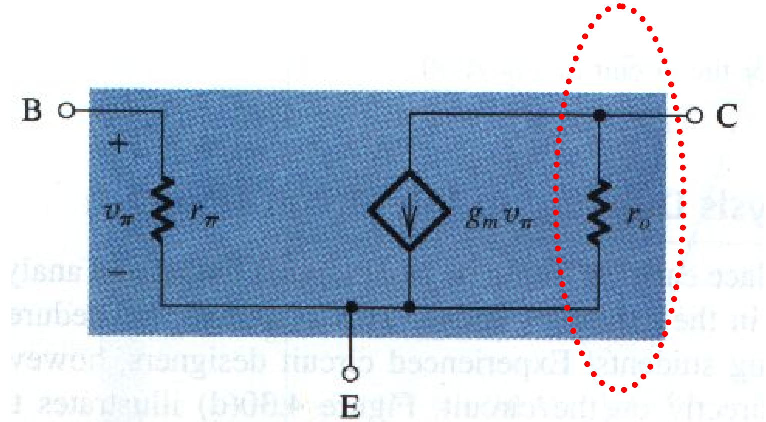


$$i_C = I_S e^{v_{BE}/V_T}\left(1 + \frac{v_{CE}}{V_A}\right)$$

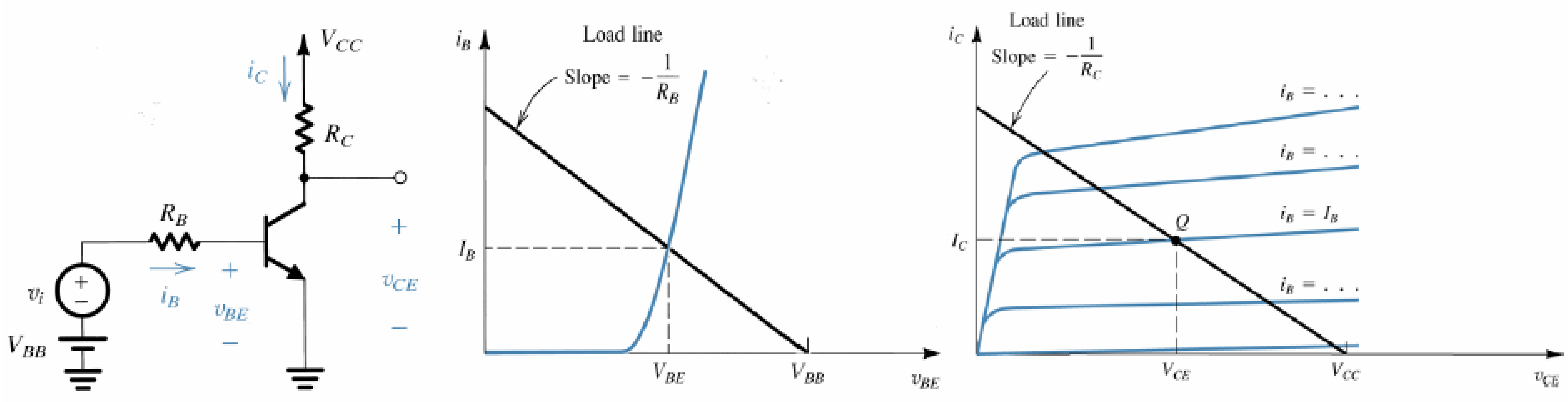
$$\text{Define } r_o = \left(\left.\frac{\partial i_C}{\partial v_{CE}}\right|_{\text{fixed } v_{BE}}\right)^{-1}$$

$$= \frac{V_A + V_{CE}}{I_C} \approx \frac{V_A}{I_C}$$

Include Early Effect in the model:



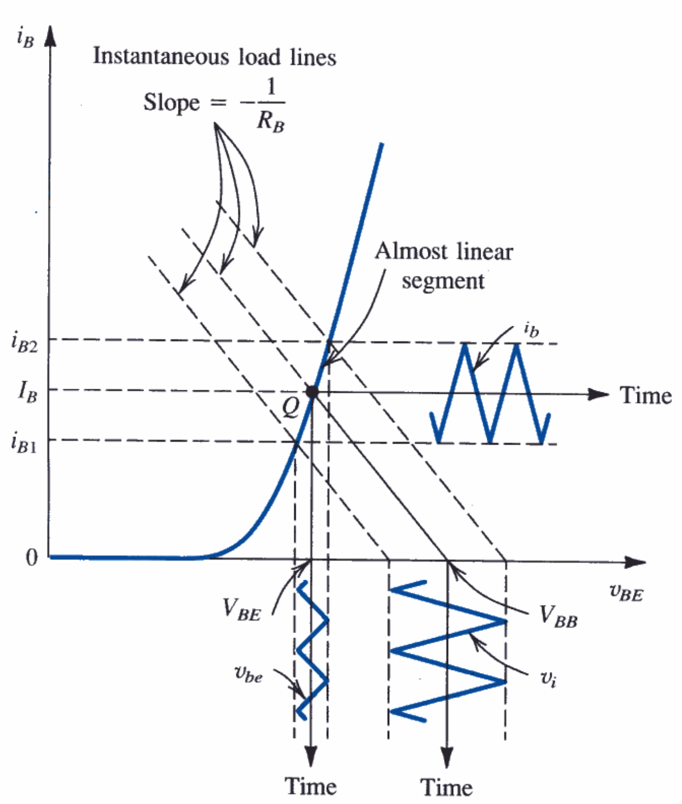
Graphic Analysis

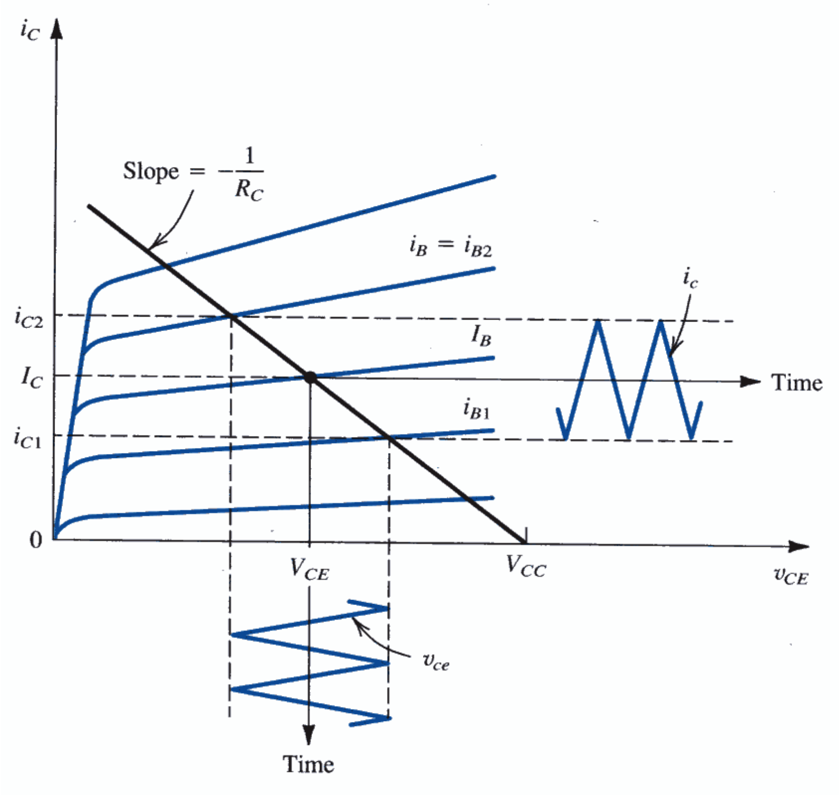


- Can be useful to understand the operation of BJT circuits
- First, establish DC conditions by finding I_B (or V_{BE})
  - Input load line: $v_{BE} = V_{BB} - R_B i_B$
- Second, figure out the DC operating point for I_C
  - Output load line:

$$v_{CE} = V_{CC} - i_C R_C \Rightarrow i_C = \frac{V_{CC}}{R_C} - \frac{1}{R_C} v_{CE}$$

Graphic Analysis (Cont.)





- Apply a small signal input voltage and see i_b
- See how i_b translates into V_{CE}
- Can get a feel for whether the BJT will stay in active region of operation
  - What happens if R_C is larger or smaller?