Digital communications I: Modulation and Coding Course

Spring - 2015

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Lecture 3d: ISI and Equalization

Last time we talked about:

- Signal detection in AWGN channels
  - Minimum distance detector
  - Maximum likelihood

- Average probability of symbol error
  - Union bound on error probability
  - Upper bound on error probability based on the minimum distance

Today we are going to talk about:

- Another source of error:
  - Inter-symbol interference (ISI)
- Nyquist theorem
- The techniques to reduce ISI
  - Pulse shaping
  - Equalization

Inter-Symbol Interference (ISI)

- ISI in the detection process due to the filtering effects of the system
- Overall equivalent system transfer function

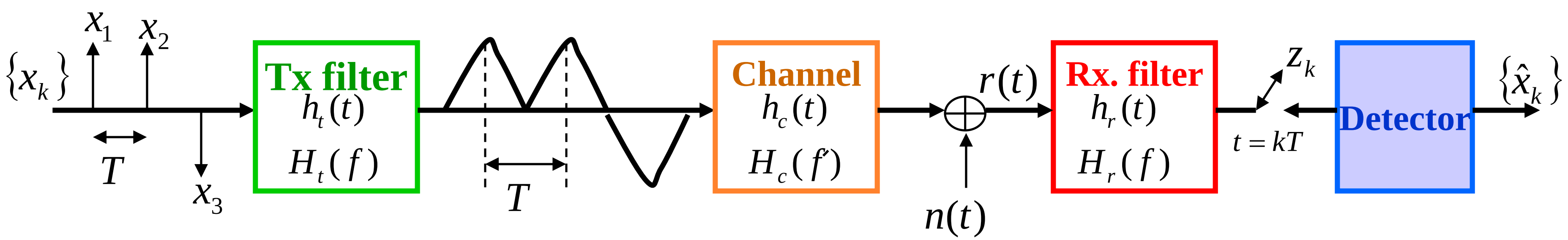
$$H(f) = H_t(f)H_c(f)H_r(f)$$

  - creates echoes and hence time dispersion
  - causes ISI at <u>sampling time</u>

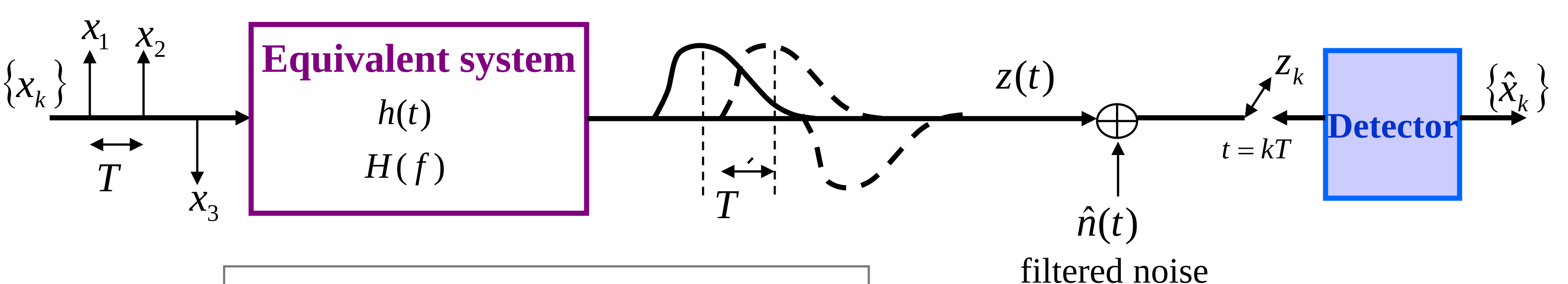
$$z_k = s_k + n_k + \sum_{i \neq k} \alpha_i s_i$$

Inter-symbol interference

- Baseband system model



- Equivalent model



$$H(f) = H_t(f)H_c(f)H_r(f)$$

Nyquist bandwidth constraint

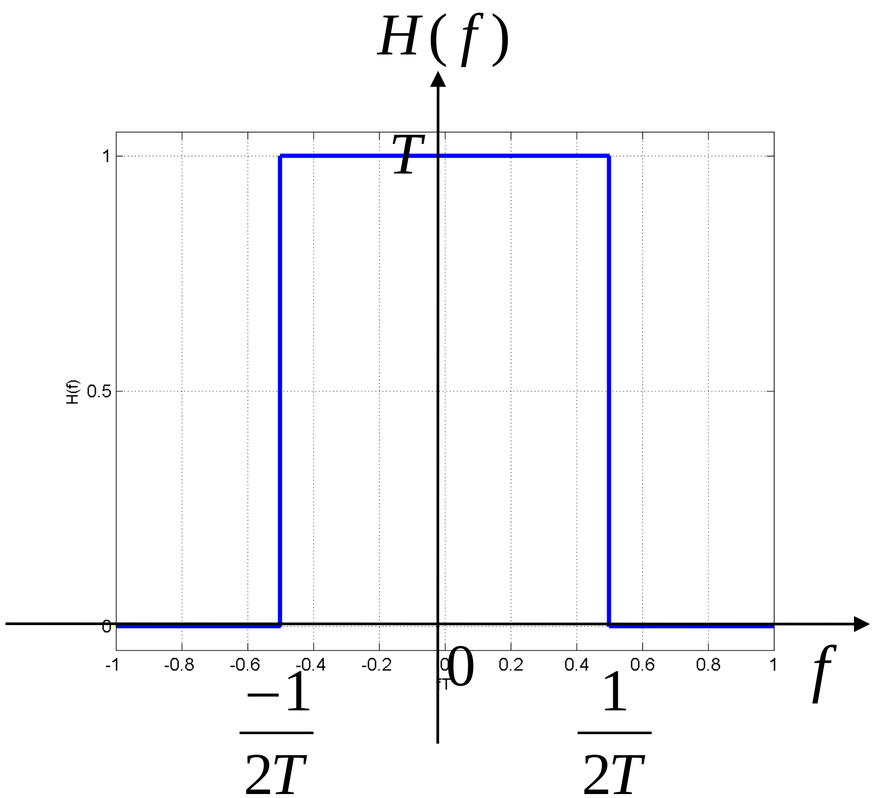
- Nyquist bandwidth constraint:
  - The theoretical minimum required system bandwidth to detect Rs [symbols/s] without ISI is $Rs/2$ [Hz].
  - Equivalently, a system with bandwidth $W=1/2T=Rs/2$ [Hz] can support a maximum transmission rate of $2W=1/T=Rs$ [symbols/s] without ISI.

$$\frac{1}{2T} = \frac{R_s}{2} \leq W \Rightarrow \frac{R_s}{W} \geq 2 \quad \text{[symbol/s/Hz]}$$

- Bandwidth efficiency, R/W [bits/s/Hz] :
  - An important measure in DCs representing data throughput per hertz of bandwidth.
  - Showing how efficiently the bandwidth resources are used by signaling techniques.

Ideal Nyquist pulse (filter)

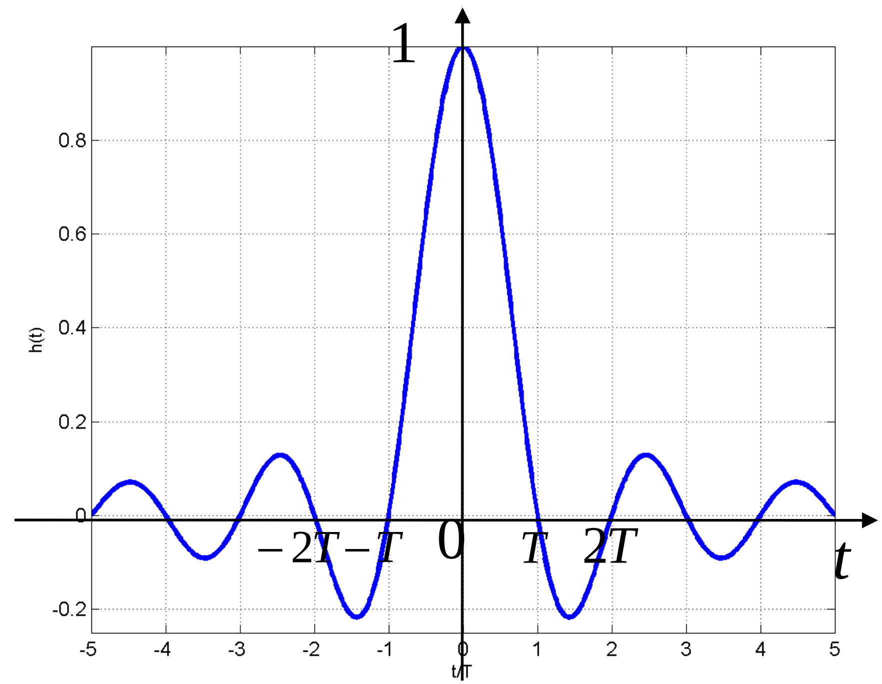
Ideal Nyquist filter



$$W = \frac{1}{2T}$$

Ideal Nyquist pulse

$$h(t) = \mathrm{sinc}(t/T)$$



Nyquist pulses (filters)

- Nyquist pulses (filters):
  - Pulses (filters) which results in no ISI at the <u>sampling time</u>.
- Nyquist filter:
  - Its transfer function in frequency domain is obtained by convolving a rectangular function with any real even-symmetric frequency function
- Nyquist pulse:
  - Its shape can be represented by a sinc(t/T) function multiply by another time function.
- Example of Nyquist filters: Raised-Cosine filter

Pulse shaping to reduce ISI

- Goals and trade-off in pulse-shaping
  - Reduce ISI
  - Efficient bandwidth utilization
  - Robustness to timing error (small side lobes)

The raised cosine filter

- Raised-Cosine Filter
  - A Nyquist pulse (No ISI at the sampling time)

$$H(f) = \begin{cases} 1 & \text{for} \ |f| < 2W_0 - W \\ \cos^2\left[\dfrac{\pi}{4}\dfrac{|f| + W - 2W_0}{W - W_0}\right] & \text{for} \ 2W_0 - W < |f| < W \\ 0 & \text{for} \ |f| > W \end{cases}$$

$$h(t) = 2W_0(\text{sinc}(2W_0 t)) \frac{\cos[2\pi(W - W_0)t]}{1 - [4(W - W_0)t]^2}$$

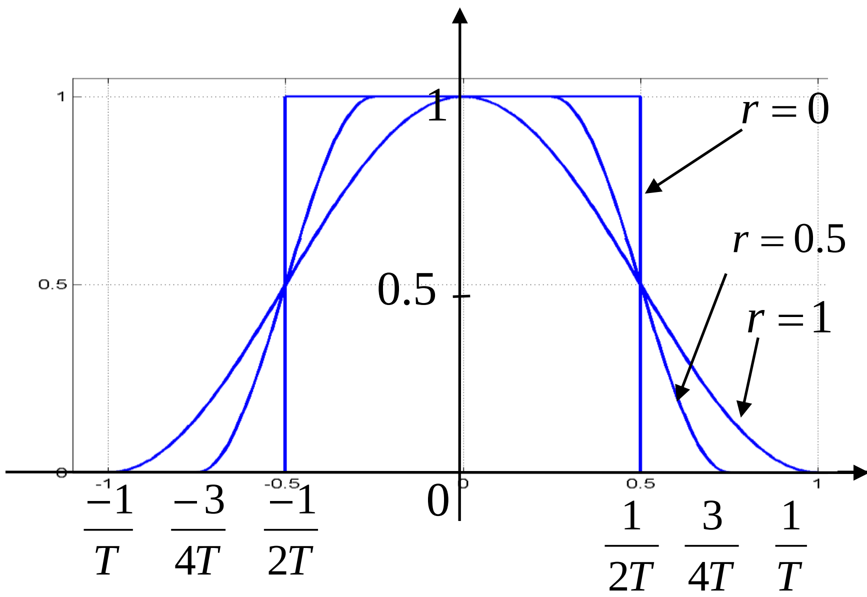
Excess bandwidth: $W - W_0$

Roll-off factor $r = \dfrac{W - W_0}{W_0}$

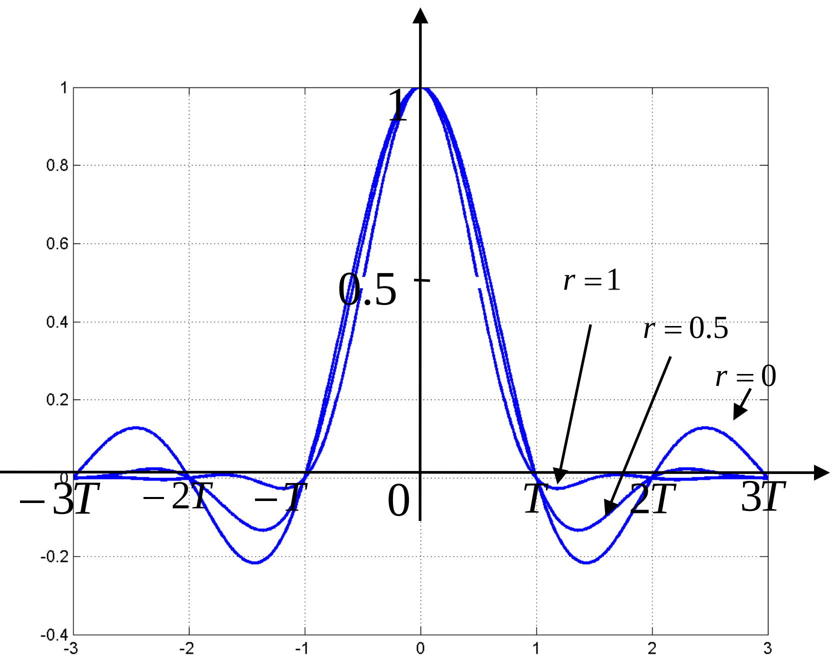
$0 \le r \le 1$

The Raised cosine filter – cont'd

$$|H(f)| = |H_{RC}(f)|$$



$$h(t) = h_{RC}(t)$$



Baseband $W_{\text{sSB}} = (1+r)\dfrac{R_s}{2}$

Passband $W_{\text{DSB}} = (1+r)R_s$

Pulse shaping and equalization to remove ISI

No ISI at the sampling time

$$H_{\mathrm{RC}}(f) = H_t(f)H_c(f)H_r(f)H_e(f)$$

- Square-Root Raised Cosine (SRRC) filter and Equalizer

$$H_{\mathrm{RC}}(f) = H_t(f)H_r(f)$$

$$H_r(f) = H_t(f) = \sqrt{H_{\mathrm{RC}}(f)} = H_{\mathrm{SRRC}}(f)$$

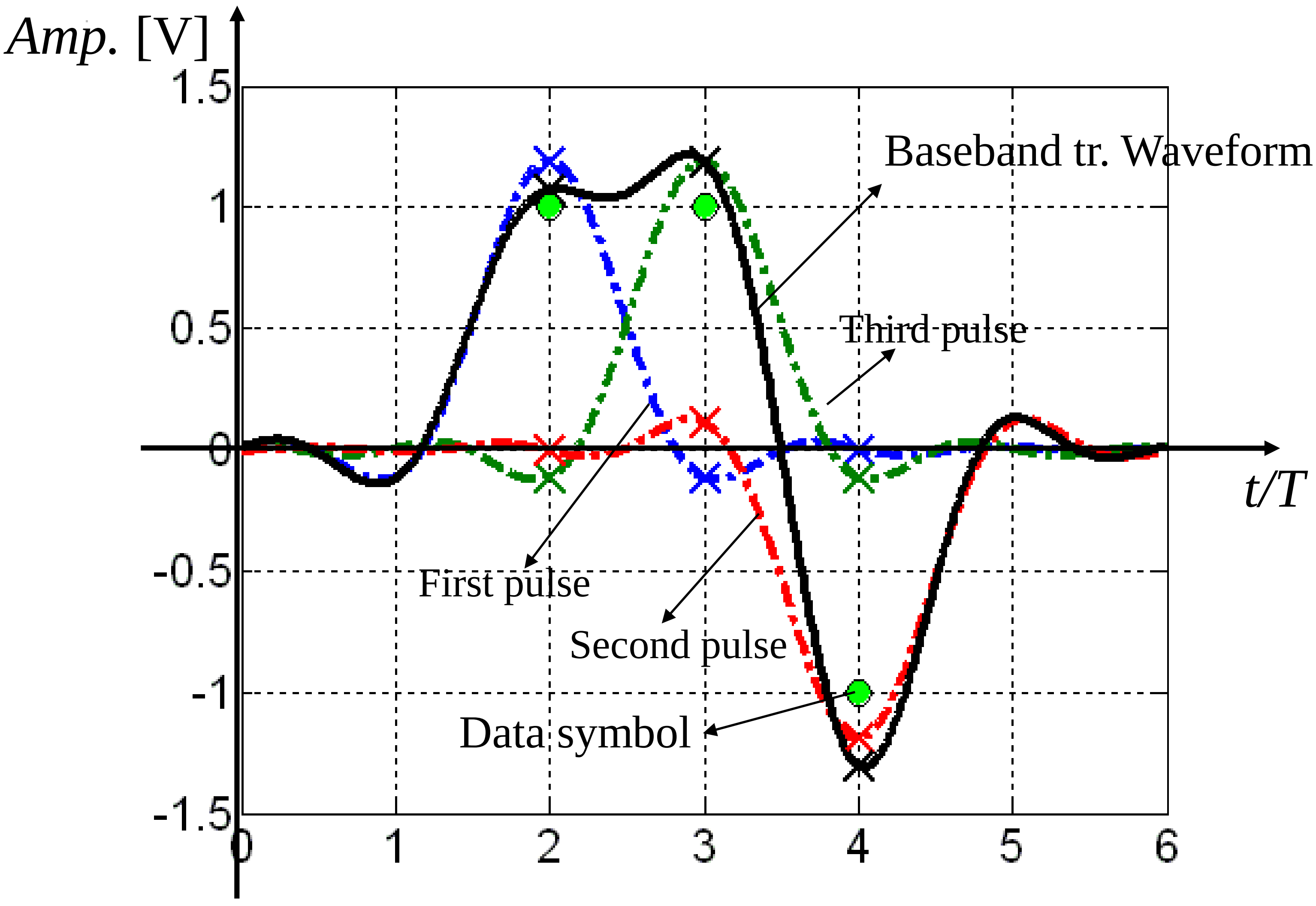
Taking care of ISI caused by tr. filter

$$H_e(f) = \frac{1}{H_c(f)}$$

Taking care of ISI caused by channel

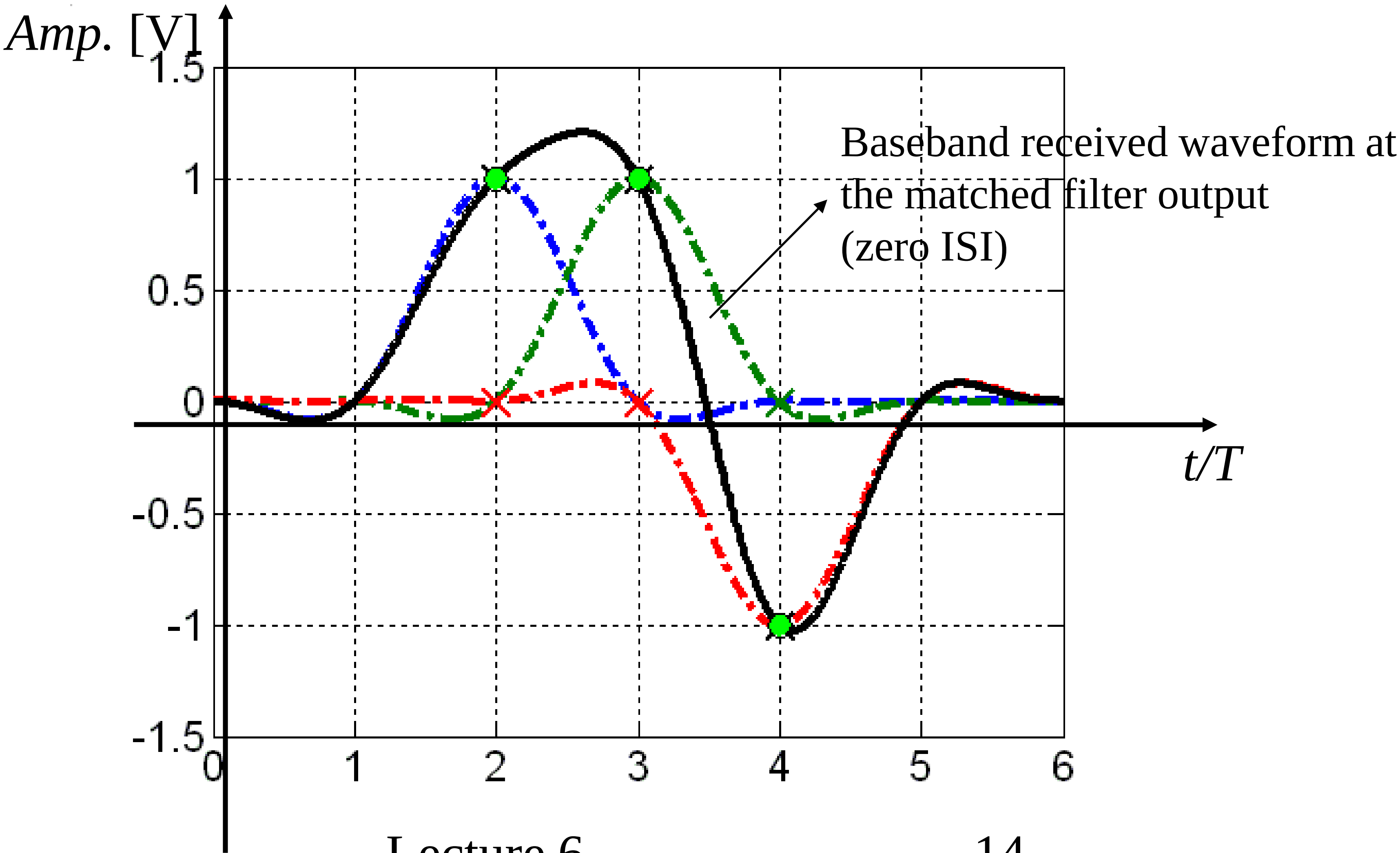
Example of pulse shaping

- Square-root Raised-Cosine (SRRC) pulse shaping



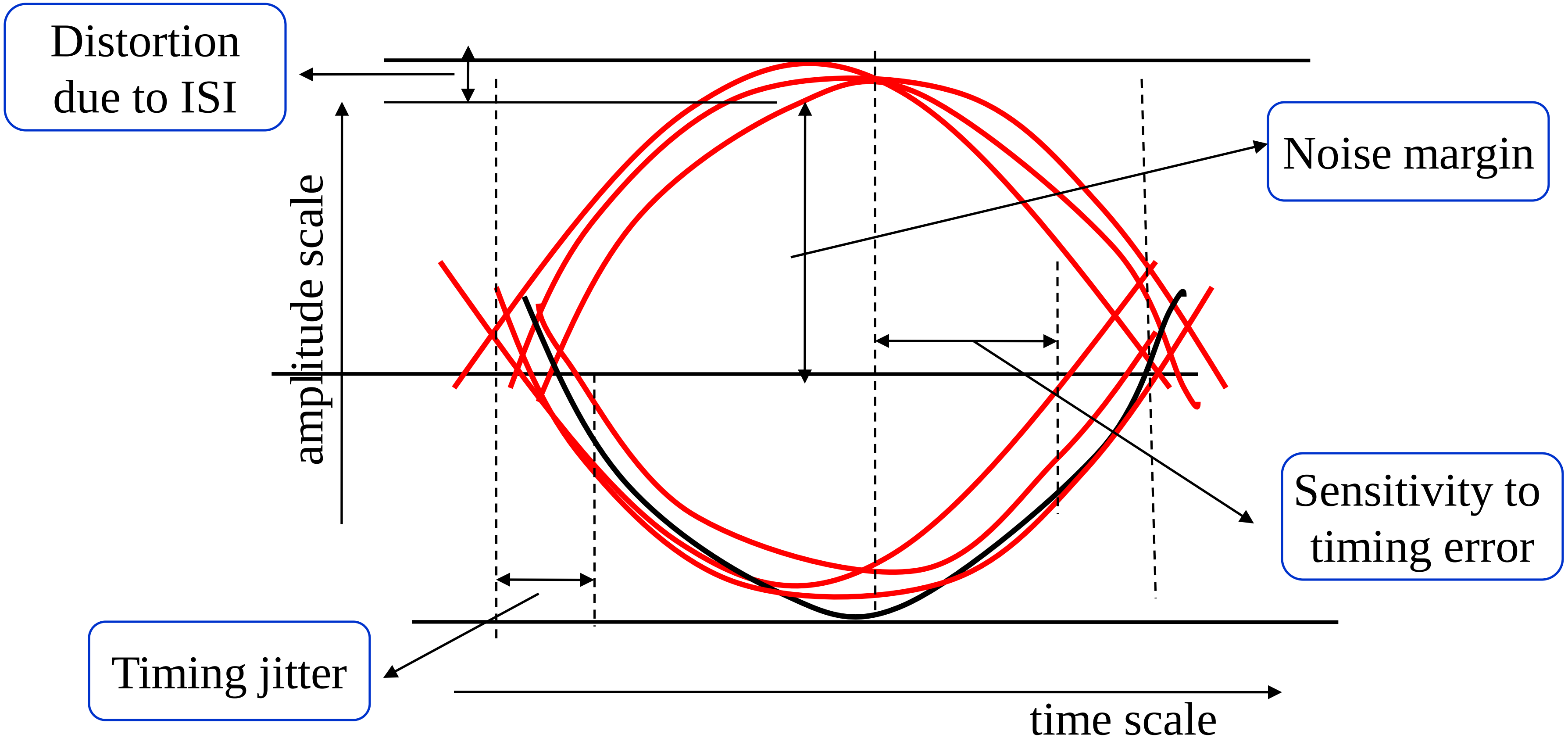
Example of pulse shaping …

- Raised Cosine pulse at the output of matched filter



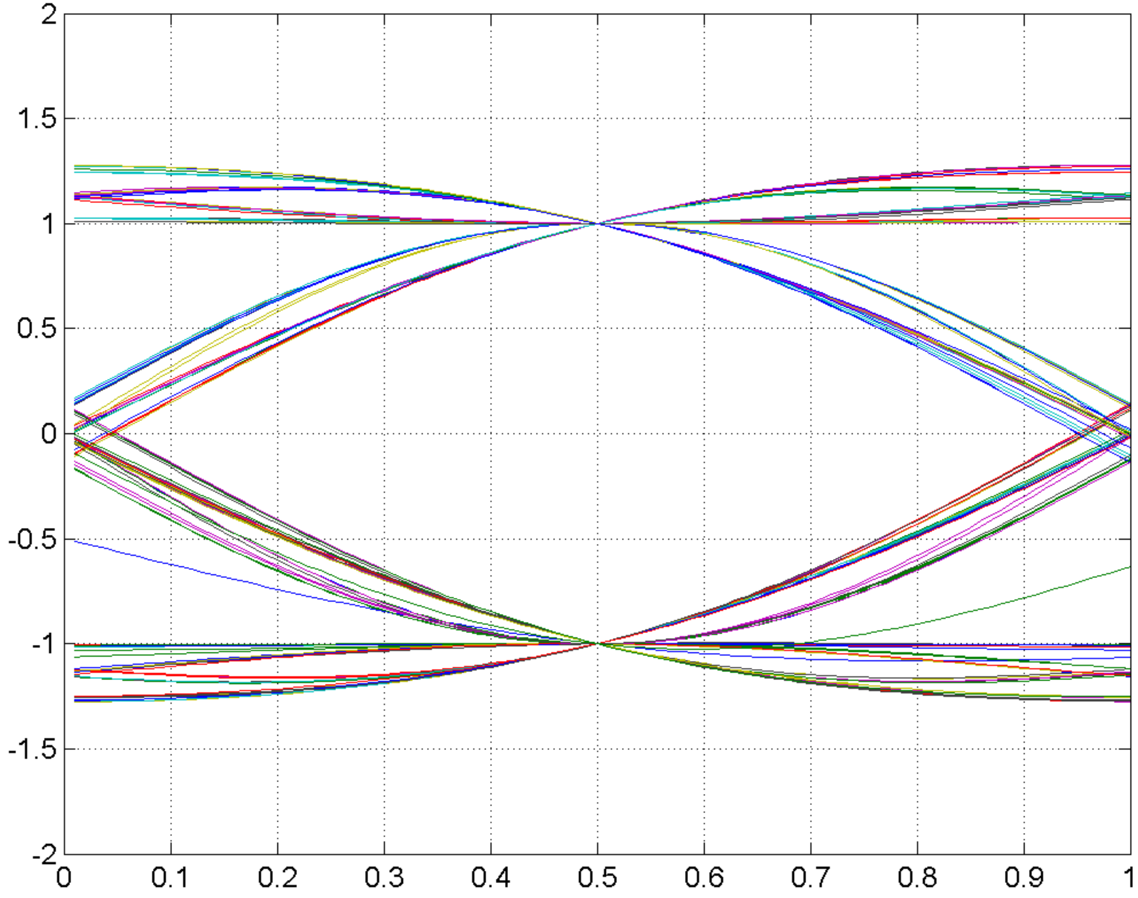
Eye pattern

- **Eye pattern:** Display on an oscilloscope which sweeps the system response to a baseband signal at the rate $1/T$ (T symbol duration)



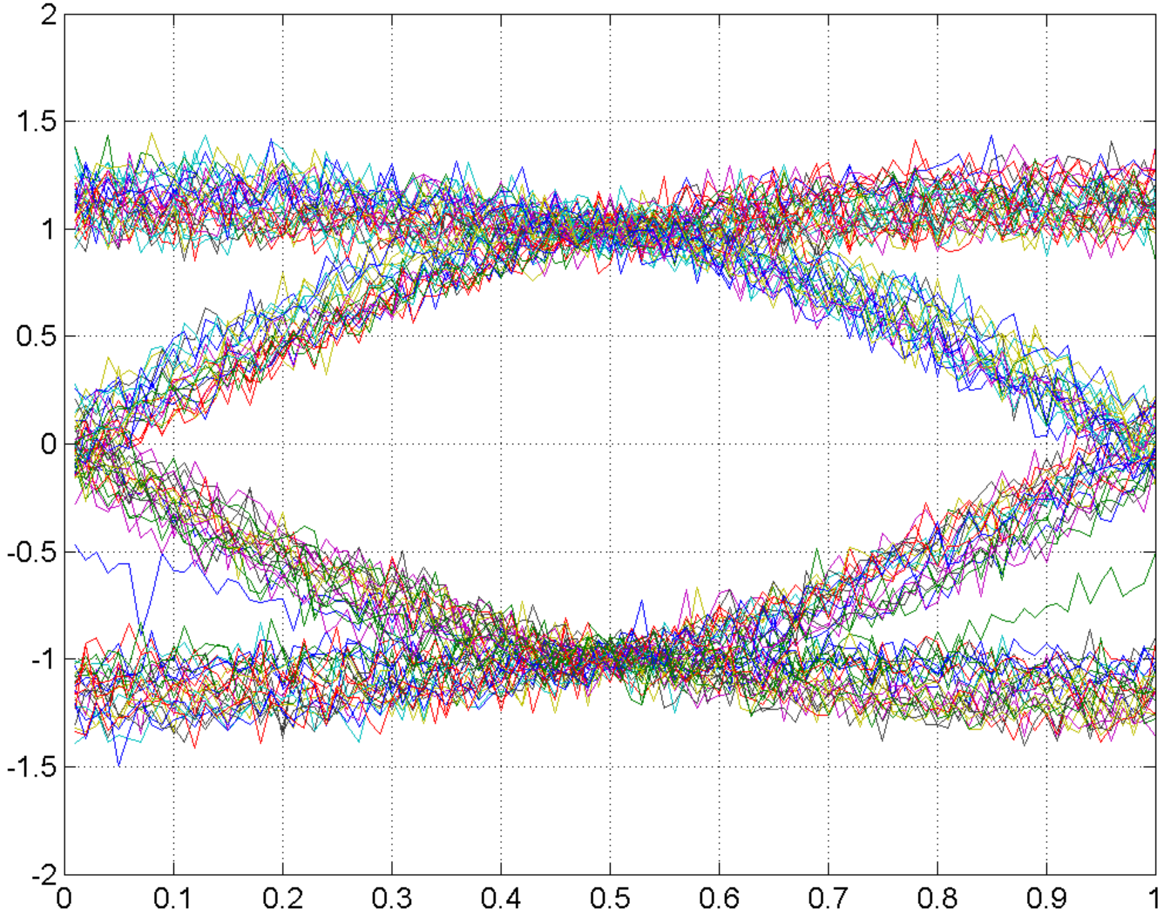
Example of eye pattern:
Binary-PAM, SRRQ pulse

- Perfect channel (no noise and no ISI)



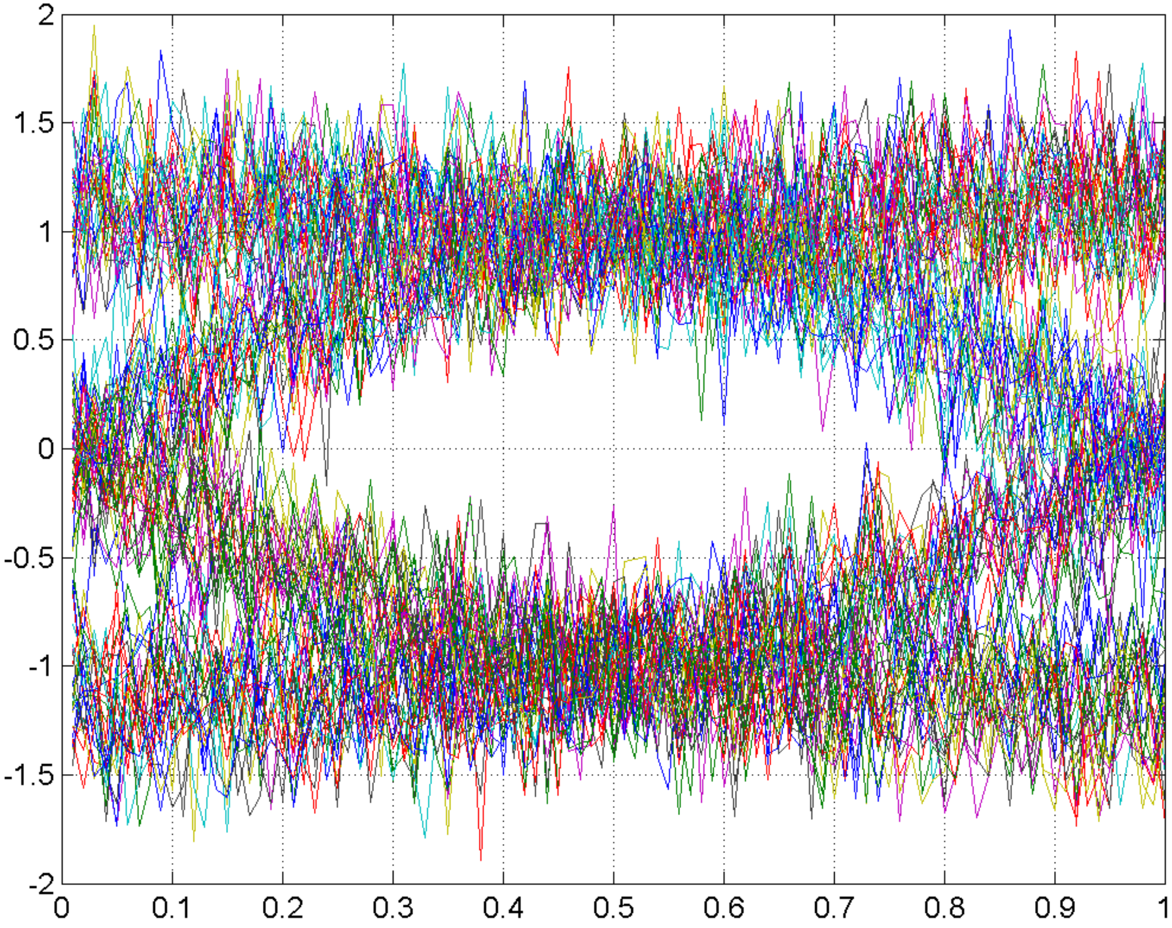
Example of eye pattern: Binary-PAM, SRRQ pulse …

- AWGN (*Eb/N0=20* dB) and no ISI

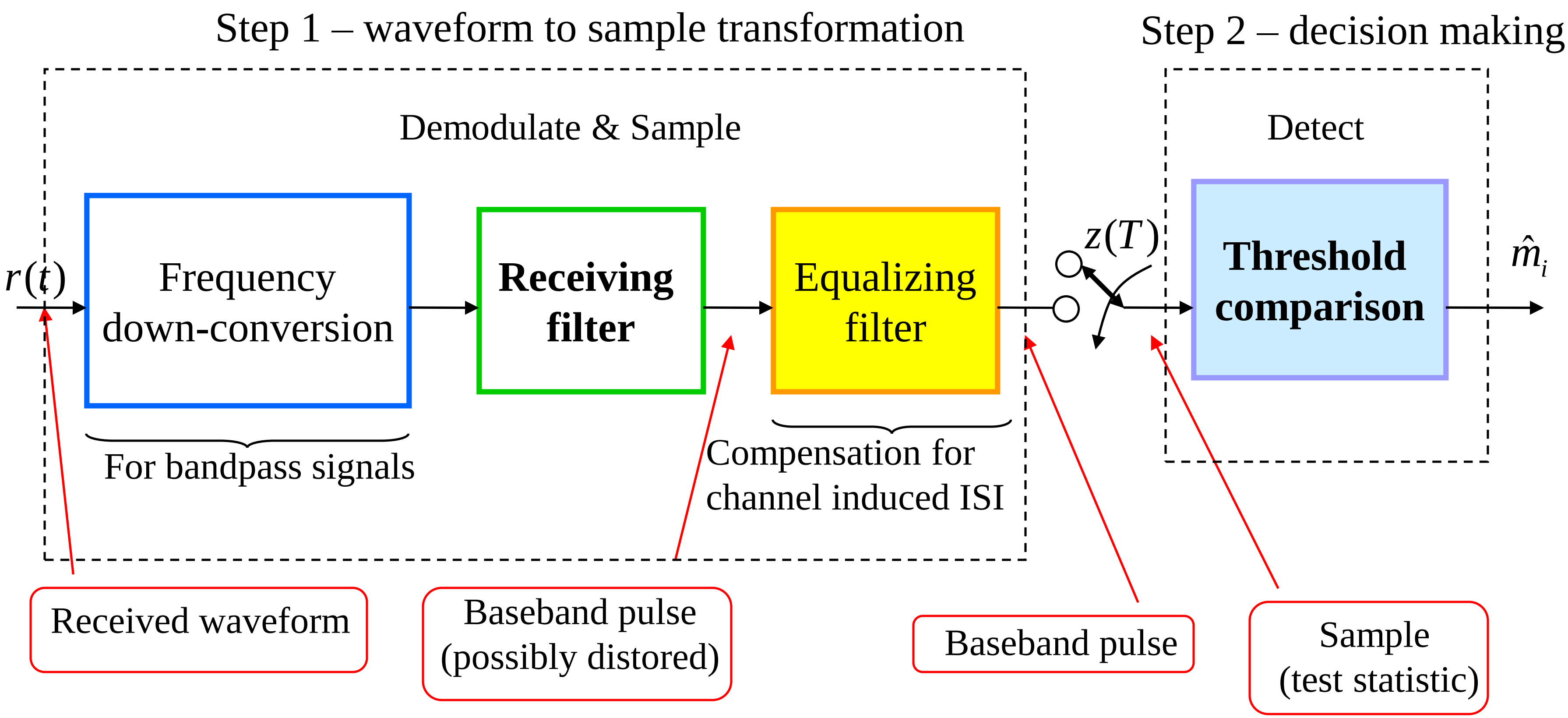


Example of eye pattern: Binary-PAM, SRRQ pulse …

- AWGN ($Eb/N0=10$ dB) and no ISI



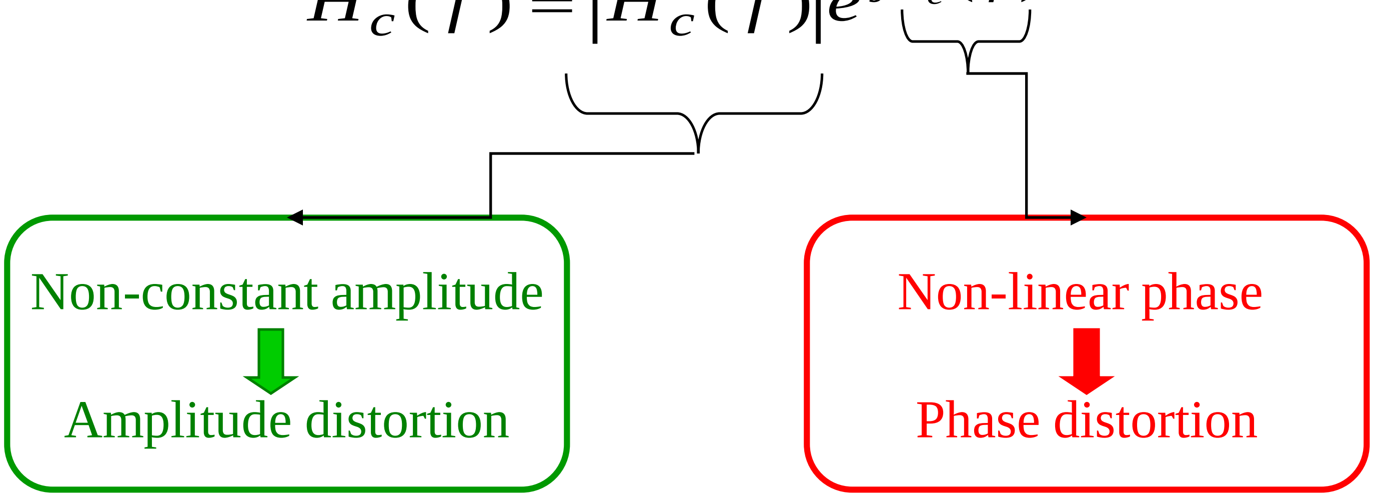
Equalization – cont'd



Equalization

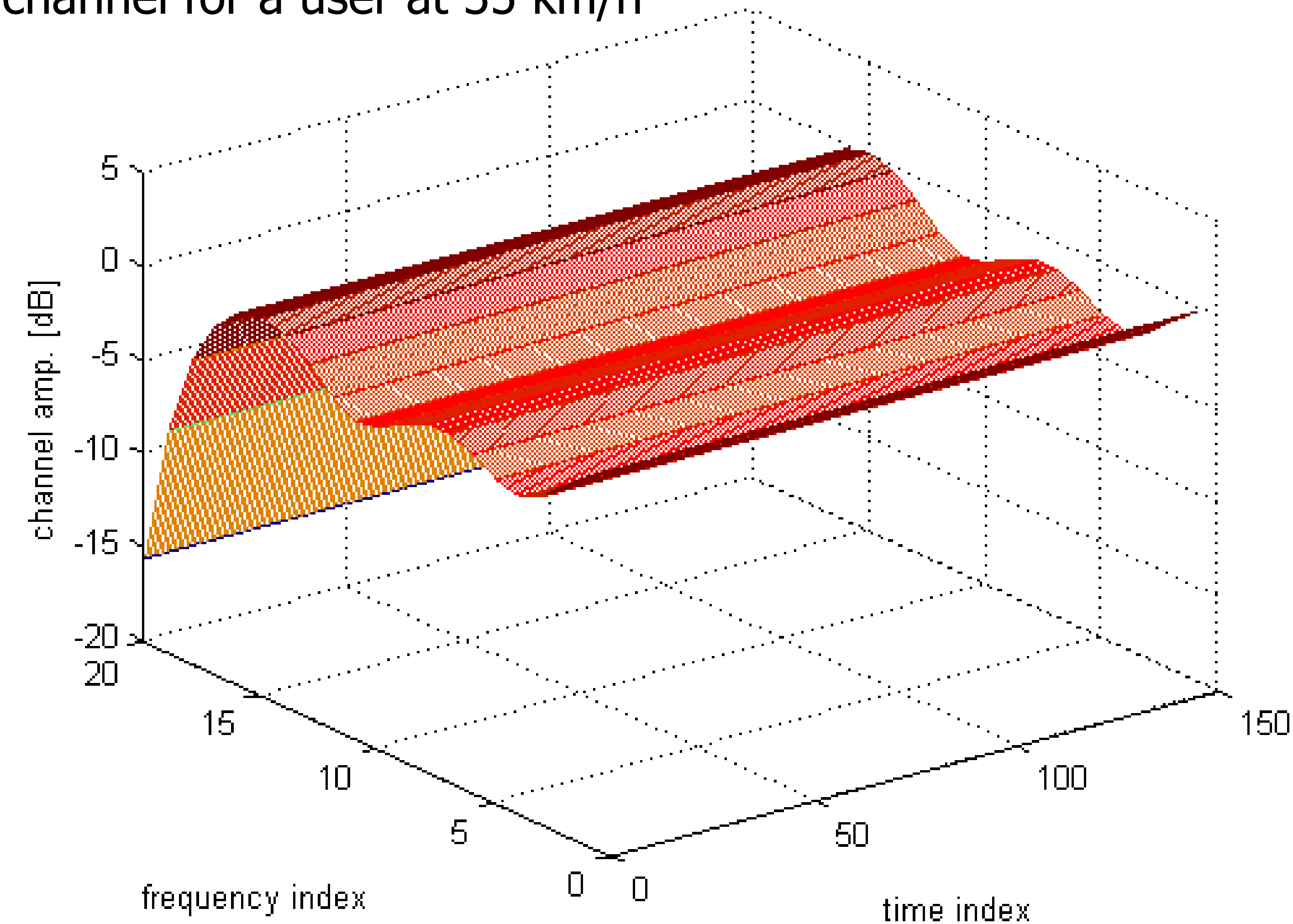
- ISI due to filtering effect of the communications channel (e.g. wireless channels)
  - Channels behave like band-limited filters

$$H_c(f) = |H_c(f)| e^{j\theta_c(f)}$$



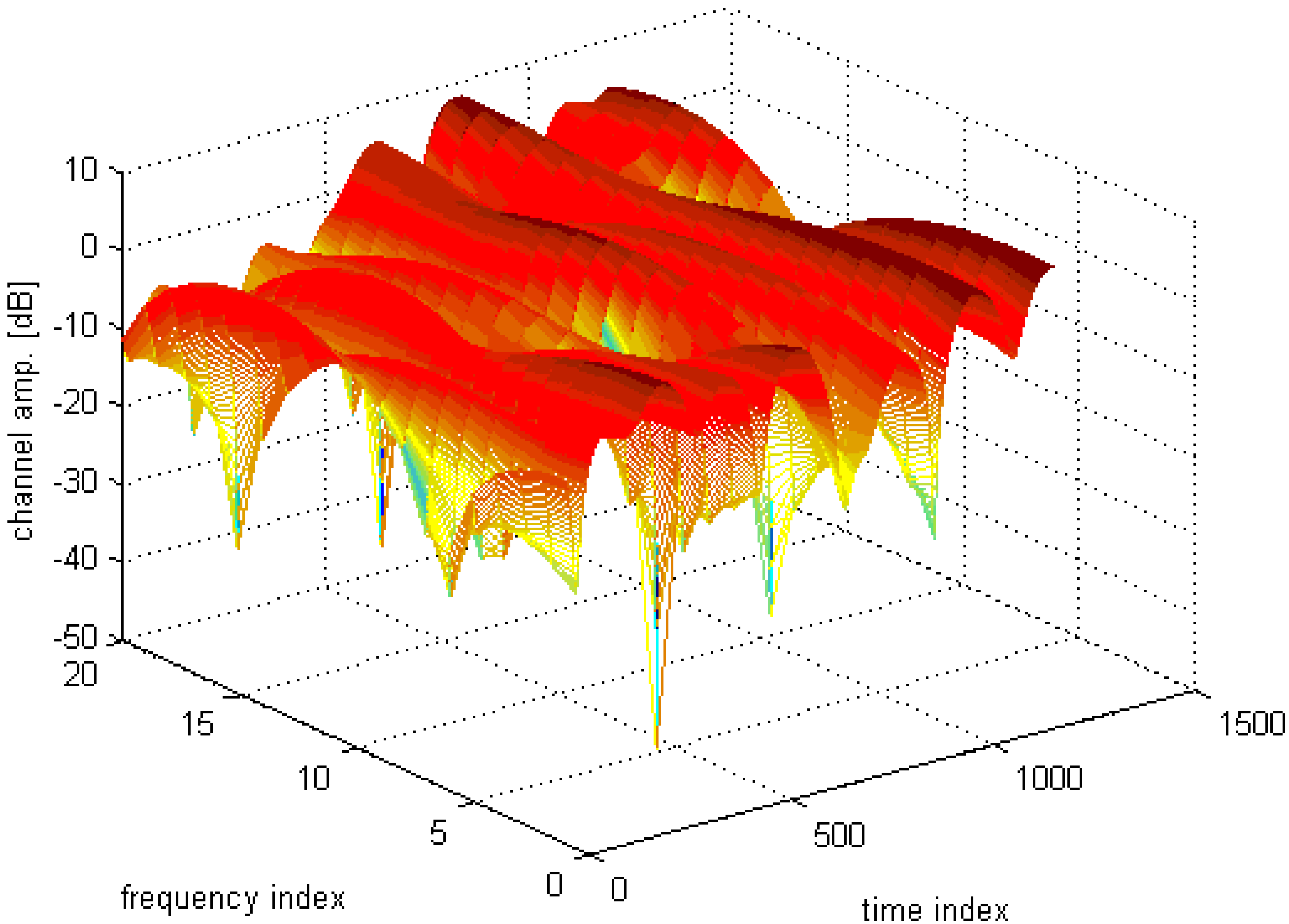
Equalization: Channel examples

- Example of a frequency selective, slowly changing (slow fading) channel for a user at 35 km/h



Equalization: Channel examples …

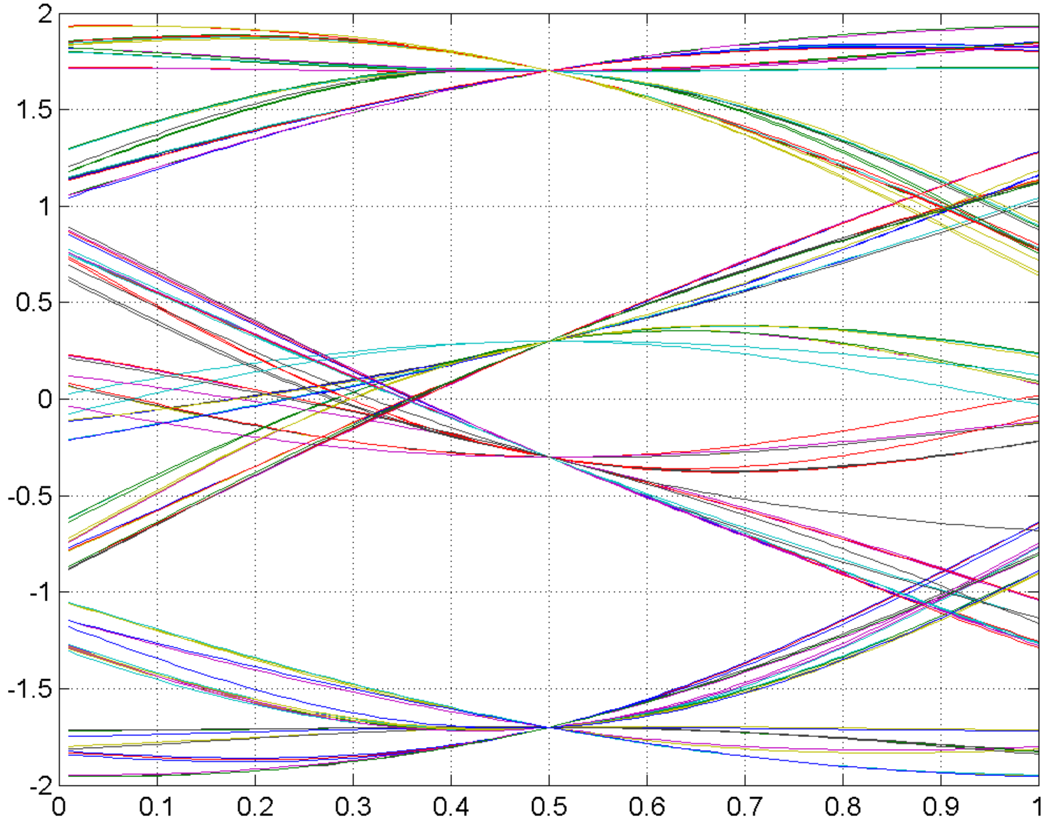
- Example of a frequency selective, fast changing (fast fading)
  (



Example of eye pattern with ISI: Binary-PAM, SRRQ pulse

- Non-ideal channel and no noise

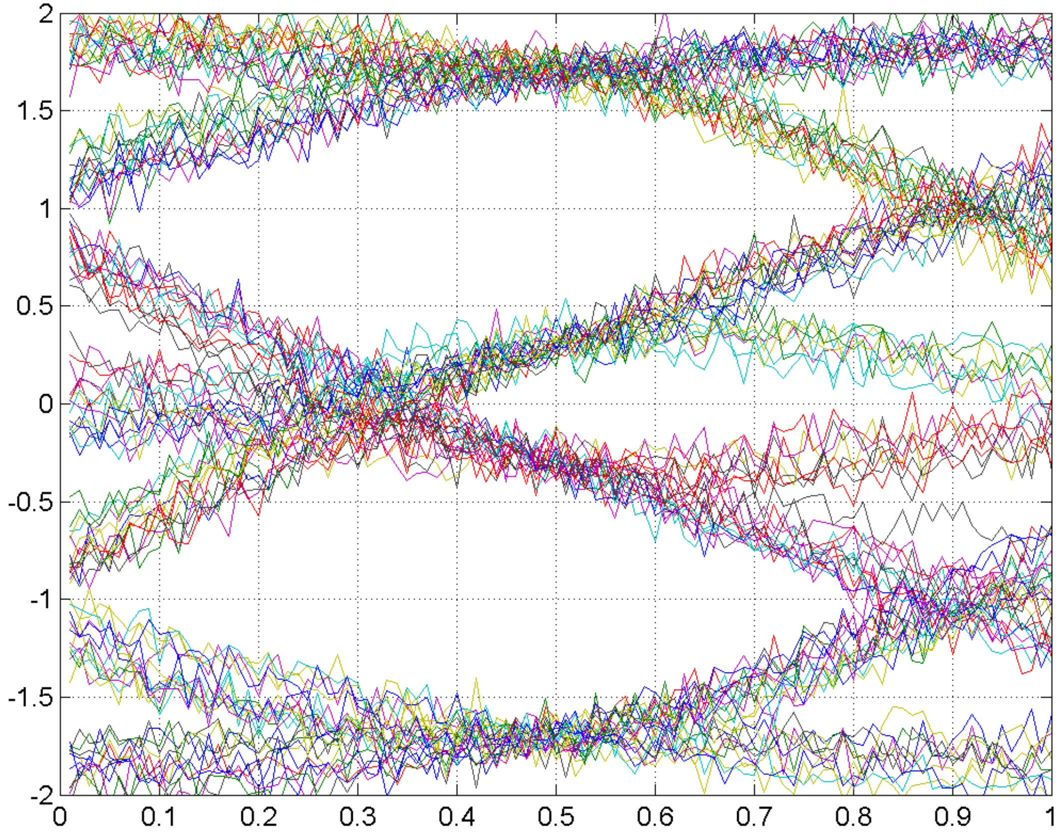
$$h_c(t) = \delta(t) + 0.7\delta(t - T)$$



Example of eye pattern with ISI: Binary-PAM, SRRQ pulse …

- AWGN (*Eb/N0=20* dB) and ISI

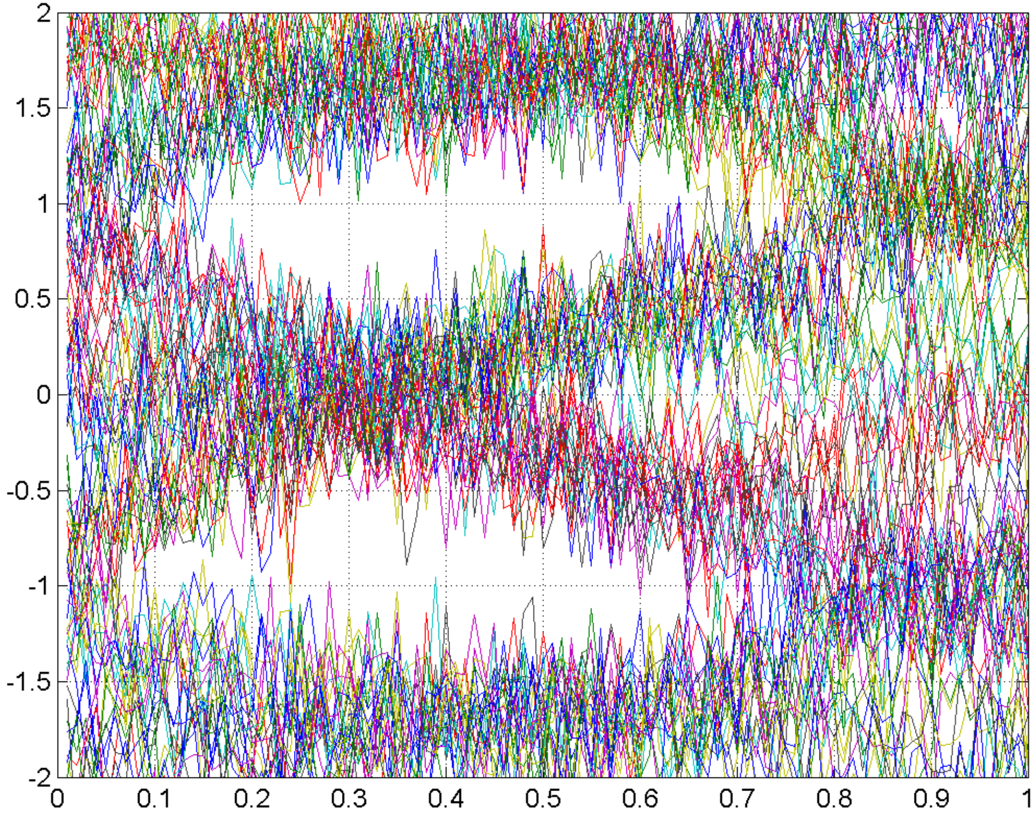
$$h_c(t) = \delta(t) + 0.7\delta(t - T)$$



Example of eye pattern with ISI: Binary-PAM, SRRQ pulse …

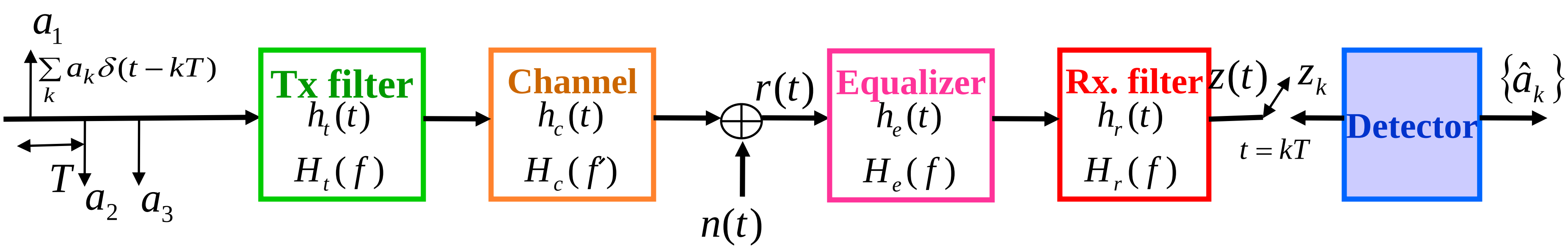
- AWGN (*Eb/N0=10* dB) and ISI

$$h_c(t) = \delta(t) + 0.7\delta(t - T)$$



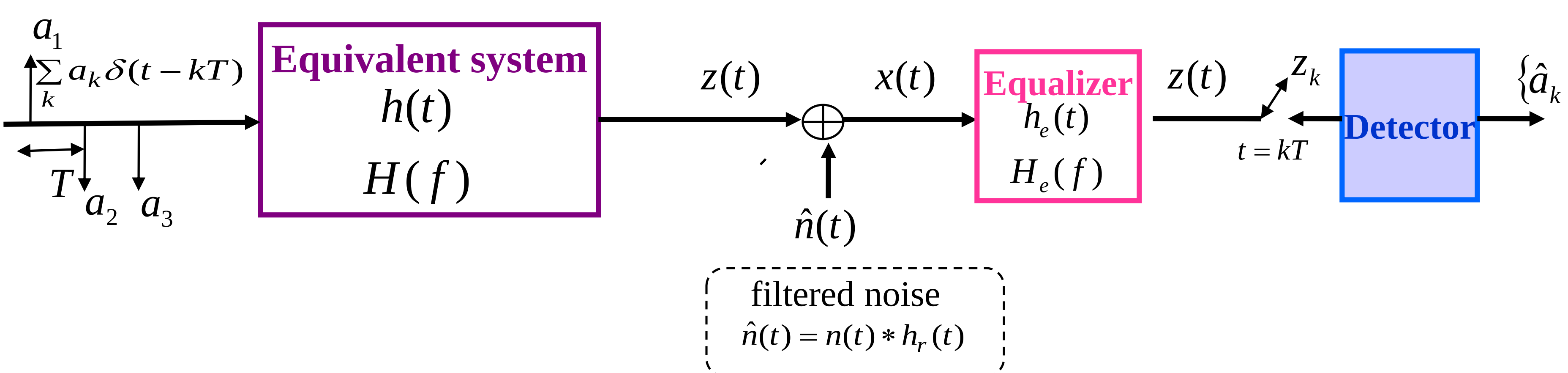
Equalizing filters …

- Baseband system model



- Equivalent model

$$H(f) = H_t(f) H_c(f) H_r(f)$$



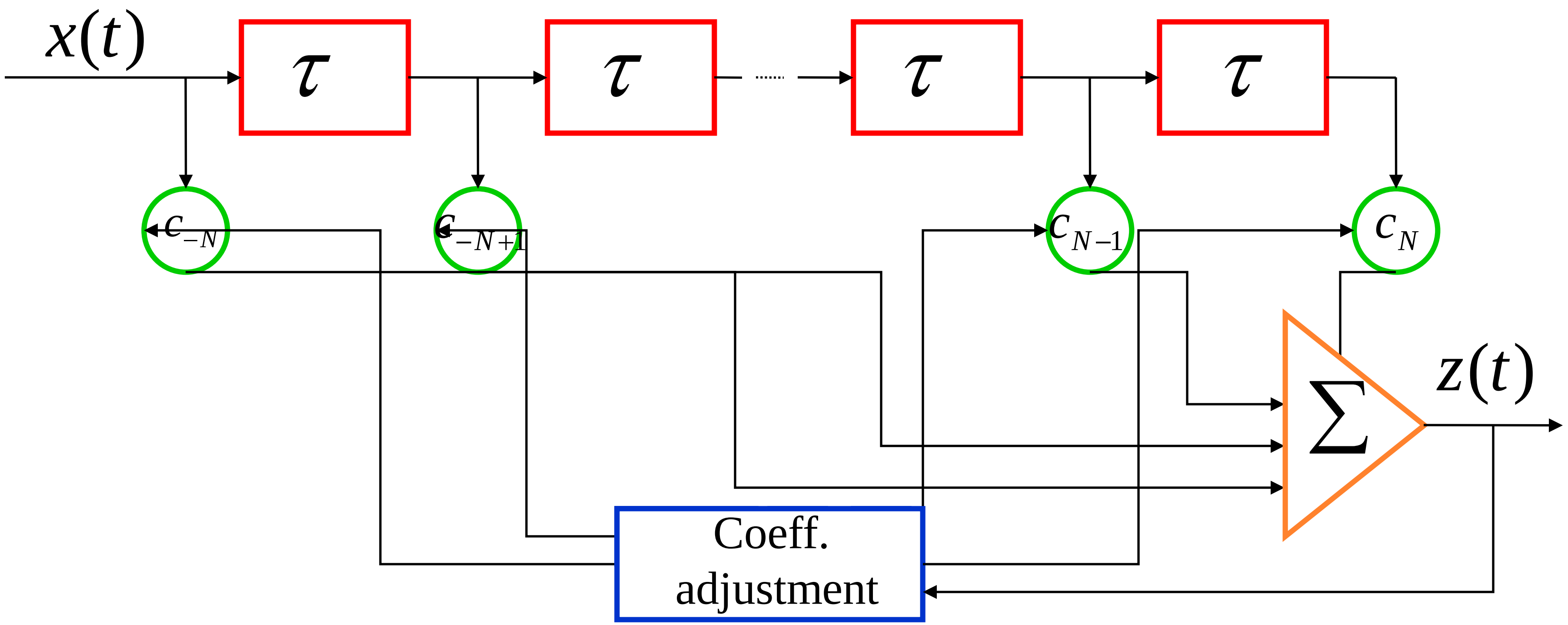
Equalization – cont'd

- Equalization using
  - MLSE (Maximum likelihood sequence estimation)
  - Filtering – See notes on z-Transform and Digital Filters
    - Transversal filtering
      - Zero-forcing equalizer
      - Minimum mean square error (MSE) equalizer
    - Decision feedback
      - Using the past decisions to remove the ISI contributed by them
    - Adaptive equalizer

Equalization by transversal filtering

- Transversal filter:
  - A weighted tap delayed line that reduces the effect of ISI by proper adjustment of the filter taps.

$$z(t) = \sum_{n=-N}^{N} c_n x(t - n\tau) \qquad n = -N, \ldots, N \quad k = -2N, \ldots, 2N$$



Transversal equalizing filter …

- Zero-forcing equalizer:
  - The filter taps are adjusted such that the equalizer output is forced to be zero at N sample points on each side:

Adjust $\{c_n\}_{n=-N}^{N}$ $\Rightarrow$ $z(k) = \begin{cases} 1 & k = 0 \\ 0 & k = \pm 1, \ldots, \pm N \end{cases}$

- Mean Square Error (MSE) equalizer:
  - The filter taps are adjusted such that the MSE of ISI and noise power at the equalizer output is minimized.

Adjust $\{c_n\}_{n=-N}^{N}$ $\Rightarrow$ $\min E\left[(z(kT) - a_k)^2\right]$

Example of equalizer

- 2-PAM with SRRQ
- Non-ideal channel

$$h_c(t) = \delta(t) + 0.3\delta(t-T)$$

- One-tap DFE

ISI-no noise,
No equalizer

ISI-no noise,
DFE equalizer

ISI- noise
No equalizer

ISI- noise
DFE equalizer