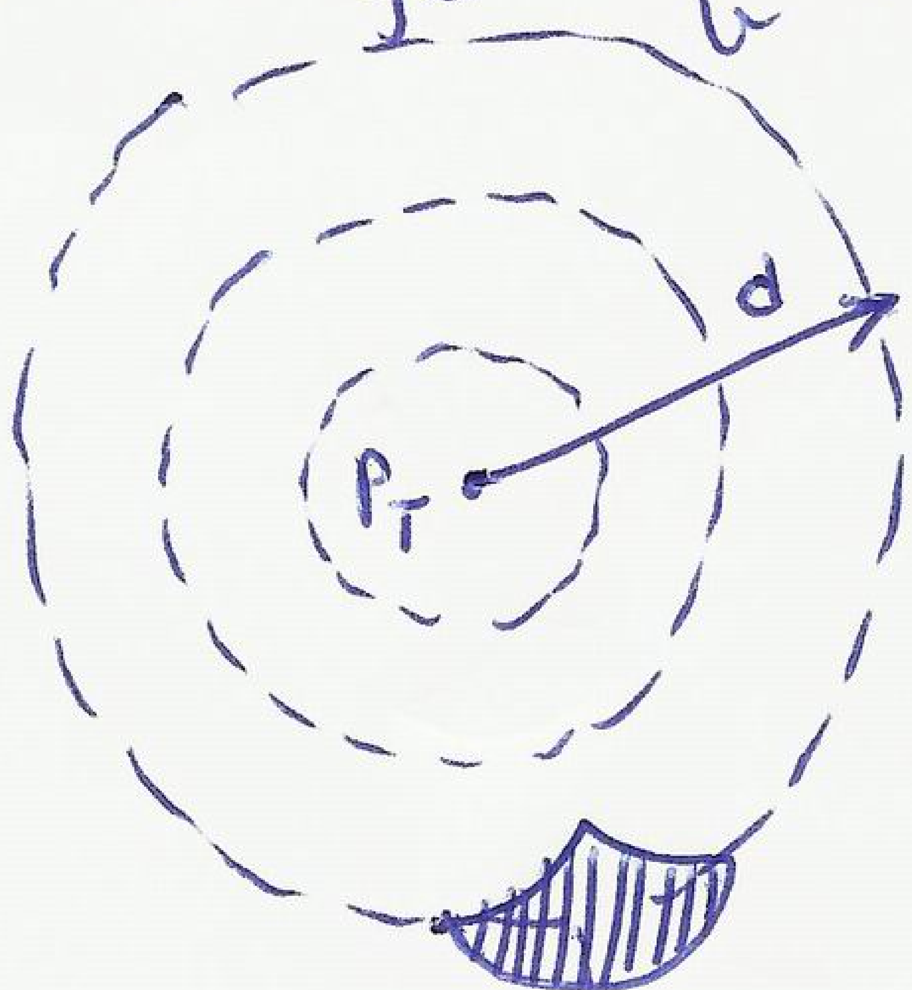
ECE645 Digital Communications

Lecture 13 10/20/2011

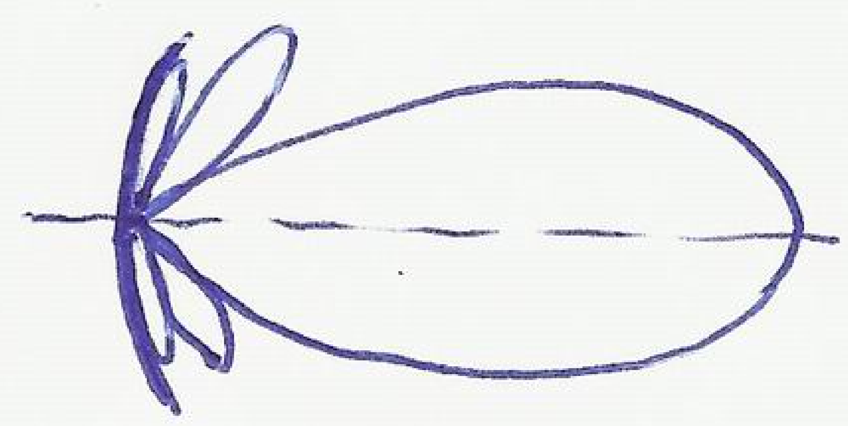
VI. Communication Link Analysis

1. Range Equation



$$p(d) = \frac{P_T}{4\pi d^2}$$

$$\Downarrow$$



$$p(d) = \frac{P_T G_T}{4\pi d^2} \longleftarrow \text{antenna gain}$$

$P_T G_T \triangleq$ EIRP (effective istropic radiated power)

$$\lambda = \frac{c}{f}$$

$$P_R = p(d)\, A_R = \frac{P_T G_T A_R}{4\pi d^2} = \frac{P_T G_T G_R}{\left(\frac{4\pi d}{\lambda}\right)^2}$$

$\uparrow$ effective area $= \dfrac{G_R \lambda^2}{4\pi}$

$$\left(\frac{4\pi \times 1\,m}{\frac{3\times 10^8\, m/s}{1\times 10^9\, Hz}}\right)^2 = \left(\frac{4\pi \times 10}{3}\right)^2 = 1754.6 = 32.44\, dB$$

$$L_S \triangleq \left(\frac{4\pi d}{\lambda}\right)^2 = \text{free space path loss}$$

$$P_R = \frac{P_T G_T G_R}{L_S}$$

$\Downarrow$ additional loss, L_a

$$\boxed{P_R = \frac{P_T G_T G_R}{L_S L_a}}$$

dBW ?
dBm ?

Usually in dB:

$$P_{R\,(dBW)} = P_{T\,(dBW)} + G_{T\,(dB)} + G_{R\,(dB)} - L_{S\,(dB)} - L_{a\,(dB)}$$

2. Thermal Noise Power

$$N_0 = k\,T$$

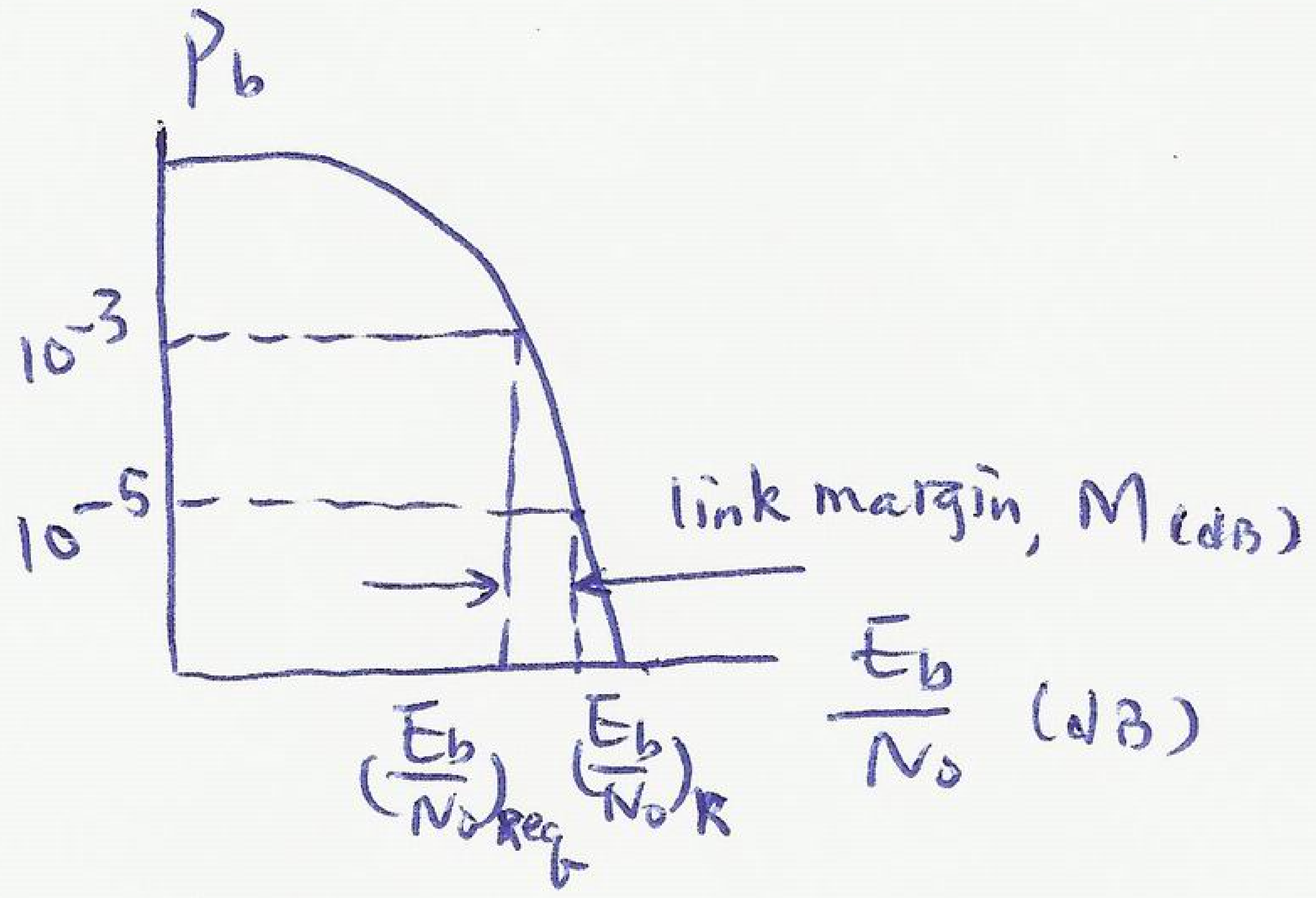
T → temperature in K

k → Boltzmann's constant, 1.38×10^{-23} J/K } J, or $\frac{W}{Hz}$

total noise power $N = N_0 W$

W → signal bandwidth

$$\frac{E_b}{N_0} = \frac{T_b P_R}{N_0} = \frac{1}{R}\frac{P_R}{N_0}$$



Link margin $M_{(dB)} = \left(\frac{E_b}{N_0}\right)_R (dB) - \left(\frac{E_b}{N_0}\right)_{Req} (dB)$

$$M_{(dB)} = \left(\frac{P_R}{N_0}\right)_{(dB\text{-}Hz)} - R_{(dB\text{-}bps)} - \left(\frac{E_b}{N_0}\right)_{Req}(dB)$$

$$= P_{T\,(dBW)} + G_{T\,(dB)} + G_{R\,(dB)} - L_{s\,(dB)} - L_{a\,(dB)}$$

$$- N_0\ dBW/Hz - R_{(dB\text{-}bps)} - \left(\frac{E_b}{N_0}\right)_{Req}(dB)$$

$$M = \frac{P_T G_T G_R}{L_s L_a R N_0 \left(\frac{E_b}{N_0}\right)_{Req}}$$

The link can be closed: $M > 1$ or $M_{(dB)} > 0$ dB

3. Noise Figure

$$P_i = S_i + N_i \longrightarrow \boxed{\text{noisy network}} \longrightarrow P_o = S_o + N_o$$

$$F = \frac{(SNR)_{in}}{(SNR)_{out}} = \frac{S_i / N_i}{G S_i / G(N_i + N_{ai})}$$

network noise referred to the input port

$$F = \frac{N_i + N_{ai}}{N_i} = 1 + \frac{N_{ai}}{N_i}$$

$kT_o W$

$$T_o = 290\,K \Rightarrow kT_o = 1.38 \times 10^{-23} \times 290 = 4 \times 10^{-21}\ W/Hz$$

$$N_{ai} = (F-1) N_i$$

$$kT_e W = (F-1) k T_o W$$

$$T_e = (F-1) T_o$$

effective noise temperature

<u>passive lossy component</u>

$$T_e = (L-1) T$$

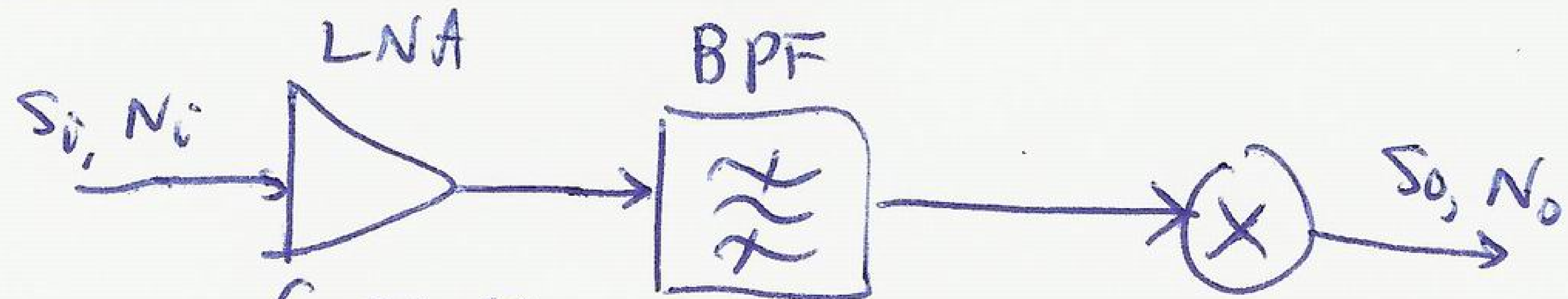
$$F = 1 + (L-1)\frac{T}{T_o} \xrightarrow{T=T_o} L$$

Cascaded system:

$$T_{cas} = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1 G_2} + \cdots + \frac{T_{en}}{G_1 G_2 \cdots G_{n-1}}$$

$$F_{cas} = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \cdots + \frac{F_n - 1}{G_1 G_2 \cdots G_{n-1}}$$

e.g.:



LNA	BPF	Mixer
$G_a = 10\,dB$	$L_f = 1\,dB$	$L_m = 3\,dB$
$F_a = 2\,dB$		$F_m = 4\,dB$
$\binom{10}{1.58}$	$\binom{0.79}{1.26}$	$\binom{0.5}{2.51}$

$$F = F_a + \frac{F_f - 1}{G_a} + \frac{F_m - 1}{G_a G_f}$$

$$= 1.58 + \frac{1.26 - 1}{10} + \frac{2.51 - 1}{10 \cdot 0.79}$$

$$= 1.80 \quad = 2.55\,dB$$

$$T_e = (F-1)\,T_0 = (1.80 - 1) \cdot 290 = 232\,K$$

Say $N_i = k T_A W$

$W \to 10\,MHz$

$T_A \to 150\,K$

Then $N_o = Gk(T_A + T_e)W$

$$= (10 \times 0.79 \times 0.5) \times (1.38 \times 10^{-23}) \times (150 + 232) \times (10 \times 10^6)$$

$$= 2.08 \times 10^{-13}\,W = -96.8\,dBm$$

$G = 10 \times 0.79 \times 0.5 = 3.95$

Suppose a minimum SNR of 20dB at the output of the receiver is required, then

$$S_i = \frac{S_o}{G} = \frac{S_o}{N_o}\frac{N_o}{G} \geq 10^{20/10} \cdot \frac{2.08\times10^{-13}}{3.95} = 5.27\times10^{-12}\ \text{W}$$

$$= -82.8\ \text{dBm}$$

For a 50Ω system, this corresponding to an input signal voltage of

$$V_i = \sqrt{Z_o S_i} = \sqrt{50\times5.27\times10^{-12}} = 1.62\times10^{-5}\ \text{V} = 16.2\ \mu\text{V (rms)}$$

Q: is it true?

$$N_o = N_i F \frac{S_o}{S_i} = N_i F G = kT_A W F G$$

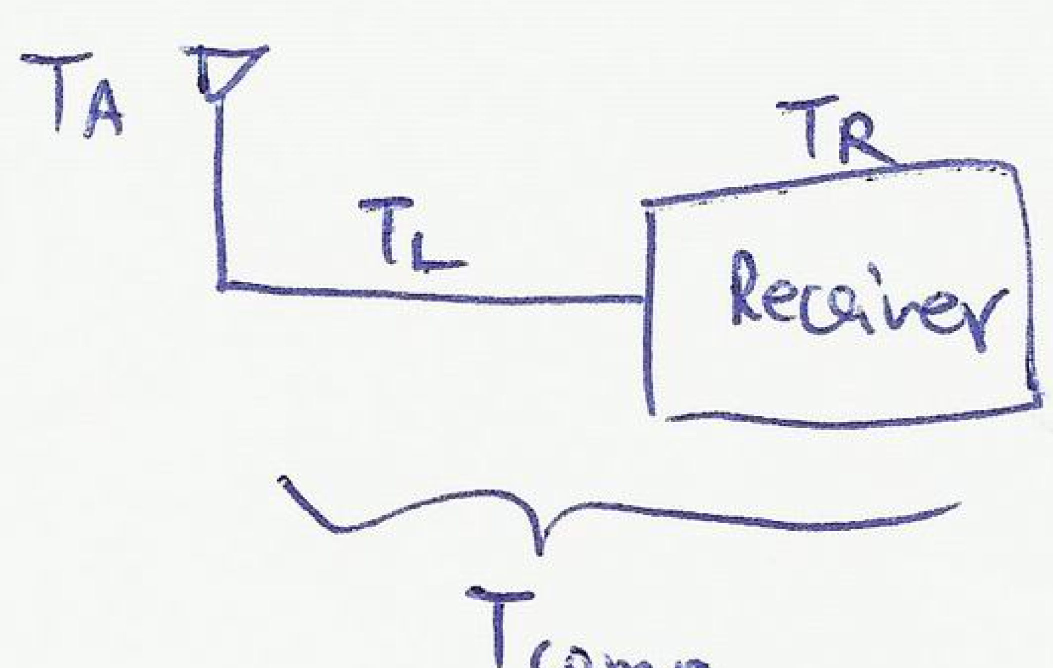
$$= (1.38\times10^{-23})\times(150)\times(10\times10^{6})\times(1.8)\times(3.95)$$

$$= 1.47\times10^{-13}\ \text{W}$$

why?

Because noise figure definition assumes an input noise level of kT_oW

System temperature

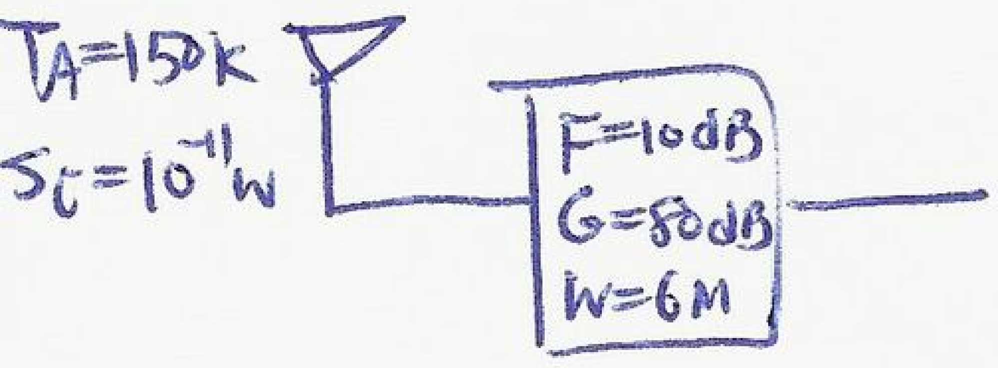


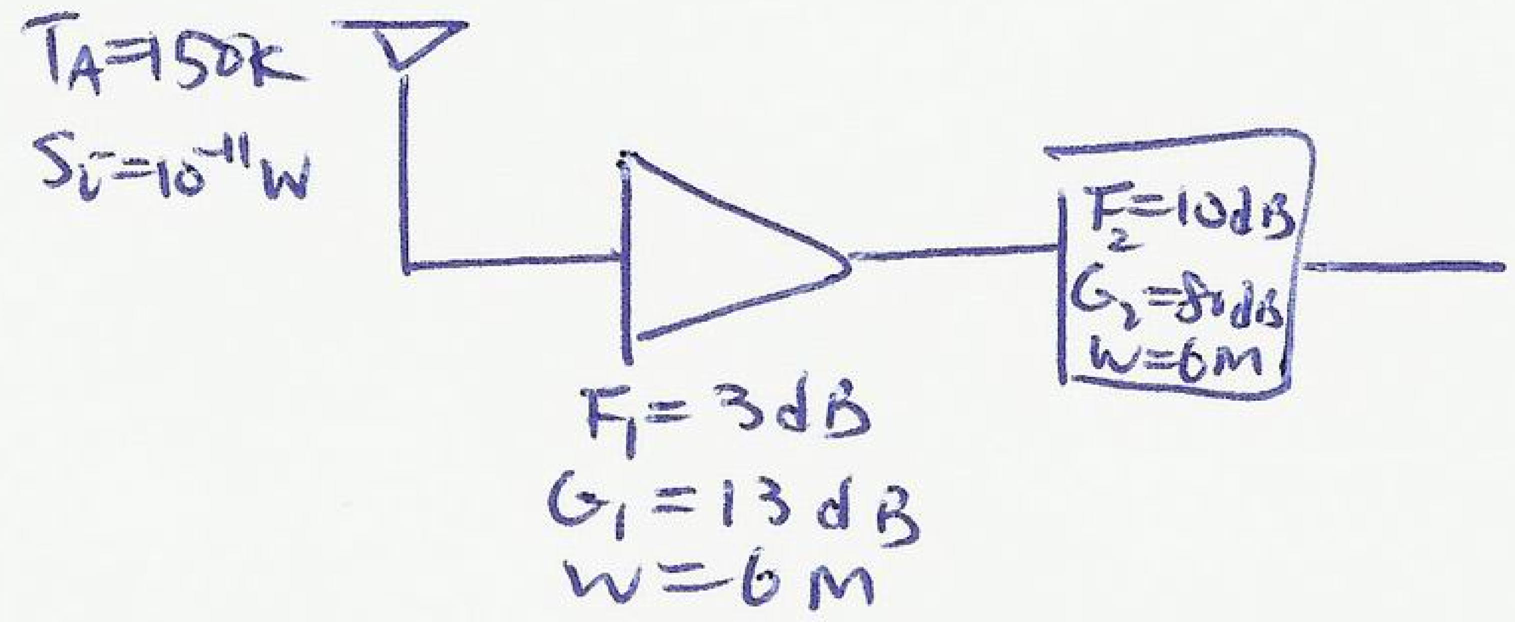
$$T_S = T_A + T_{comp}$$

$$= T_A + T_L + L T_R$$

$$= T_A + (L-1)T_o + L(F-1)T_o$$

$$= T_A + (LF-1)T_o$$





$T_R = (F-1)T_0 = 2610K$

$T_S = T_A + T_R = 2760K$

$N_{out} = G k T_A W + G k T_R W$

$= G k T_S W$

$= 22.8 \mu W$

$1.2 \mu W \quad 21.6 \mu W$

$S_{out} = G S_i$

$= 10^{-3} W$

$\left(\frac{S}{N}\right)_{out} = \frac{10^{-3}}{22.8 \times 10^{-6}} = 43.9$

$= 16.4 dB$

$T_{R_1} = (F_1 - 1) T_0 = 290K$

$T_{R_2} = (F_2 - 1) T_0 = 2610K$

$T_{comp} = T_{R_1} + \frac{T_{R_2}}{G_1} = 420.5K$

$T_S = T_A + T_{comp} = 570.5K$

$N_{out} = G k T_S W$

$= 94.4 \mu W$

$24.8 \mu W \quad 69.6 \mu W$

$S_{out} = G_1 G_2 S_i$

$= 2 \times 10^{-2} W$

$\left(\frac{S}{N}\right)_{out} = \frac{2 \times 10^{-2}}{94.4 \times 10^{-6}} = 212.0$

$= 23.3 dB$

6.9dB improvement!

If $T_A = 8000K$

$\left(\frac{S}{N}\right)_{out} = \frac{10^{-3}}{87.8 \times 10^{-6}} = 11.4$

$= 10.6 dB$

$\left(\frac{S}{N}\right)_{out} = \frac{2 \times 10^{-2}}{1.39 \times 10^{-3}} = 14.4$

$= 11.6 dB$

1.0 dB improvement!