EE-387 Probability for Electrical and Computer Engineers
Assignment 2 (due on Thursday, July 21, 2005 before lecture)

Problem 1: (Problem 2.2.3 from Yates and Goodman) The random variable V has PMF

$$P_V(v) = \begin{cases} cv^2 & v = 1,2,3,4 \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the value of the constant c.

(b) Find $P[V \in \{u^2 | u = 1,2,3,\ldots\}]$.

(c) Find the probability that V is an even number.

(d) Find $P[V > 2]$.

Problem 2: (Problem 2.3.11 from Yates and Goodman) In a packet voice communications system, a source transmits packets containing digitized speech to a receiver. Because transmission errors occasionally occur, an acknowledgment (ACK) or a nonacknolwedgement (NAK) is transmitted back to the source to indicate the status of each received packet. When the transmitter gets a NAK, the packet is retransmitted. Voice packets are delay sensitive and a packet can be transmitted a maximum d times. If a packet transmission is an independent Bernoulli trial with success probability p, what is the PMF of T, the number of times a packet is transmitted?

Problem 3: (Problem 2.4.2 from Yates and Goodman) The random variable X has CDF

$$F_X(x) = \begin{cases} 0 & x < -1 \\ 0.2 & -1 \le x < 0 \\ 0.7 & 0 \le x < 1 \\ 1 & x \ge 1 \end{cases}$$

(a) Draw a graph of the CDF.

(b) Write $P_X(x)$, the PMF of X. Be sure to write the value of $P_X(x)$ for all x from $-\infty$ and ∞.

Problem 4: (Problem 2.5.10 from Yates and Goodman) Let binomial random variable X_n denote the number of successes in n Bernoulli trials with success probability p. Prove that $E[X_n] = np$.

Hint: Use the fact that $\sum_{x=0}^{n-1} P_{X_{n-1}}(x) = 1$.

Problem 5: (Problem 2.6.2 from Yates and Goodman) Given the random variable X is Problem 3, let $V = g(X) = |X|$. (a) Find $P_V(v)$. (b) Find $F_V(v)$. (c) Find $E[V]$.

Problem 6: (Problem 2.6.5 from Yates and Goodman) A source wishes to transmit data packets to a receiver over a radio link. The receiver uses error detection to identify packets that have been corrupted by radio noise. When a packet is received error-free, the receiver sends an acknowledgment (ACK) back to the source. When the receiver gets a packet with errors, a negative acknowledgment (NAK) message is sent back to the source. Each time the source receives a NAK, the packet is retransmitted. We assume that each packet transmission is independently corrupted by errors with probability q.

(a) Find the PMF of X, the number of times that a packet is transmitted by the source.

(b) Suppose each packet takes 1 millisecond to transmit and that the source waits an additional millisecond to receive the acknowledgment message (ACK or NAK) before retransmitting. Let T equal the time required until the packet is successfully received. What is the relationship between T and X? What is the PMF of T?

Problem 7: (Problem 2.8.5 from Yates and Goodman) Let X have the binomial PMF

$$P_X(x) = \binom{4}{x}(1/2)^4.$$

(a) Find the standard deviation of the random variable X. (b) What is $P[\mu_X - \sigma_X \leq X \leq \mu_X + \sigma_X]$, the probability that X is within one standard deviation of the expected value?

Problem 8: (Problem 2.8.5 from Yates and Goodman) In packet data transmission, the time between successfully received packets is called the interarrival time, and randomness in packet interarrival times is called *jitter*. In real-time packet data communications, jitter is undesirable. One measure of jitter is the standard deviation of the packet arrival time. From Problem 6, calculate the jitter σ_T. How large can q be such that the jitter is less than 2 milliseconds?

Problem 9: (Problem 2.8.10 from Yates and Goodman) Let random variable X have PMF $P_X(x)$. We wish to guess the value of X before performing the actual experiment. If we call our guess $\hat{x}$, the expected square of the error in our guess is

$$e(\hat{x}) = E[(X - \hat{x})^2].$$

Show that $e(\hat{x})$ is minimized by $\hat{x} = E[X]$.

Problem 10: (Problem 2.9.2 from Yates and Goodman) In Problem 3, find $P_{X|B}(x)$, where the condition $B = \{|X| > 0\}$. What are $E[X|B]$ and $\text{Var}[X|B]$?