EE-387 Probability for Electrical and Computer Engineers
Assignment 3 (due on Thursday, July 28, 2005 before lecture)

Problem 1: (Problem 3.2.1 from Yates and Goodman) The random variable X has PDF

$$f_X(x) = \begin{cases} cx & 0 \leq x \leq 2, \\ 0 & \text{otherwise.} \end{cases}$$

Use the PDF to find (a) the constant c. (b) $P[0 \leq X \leq 1]$. (c) $P[-1/2 \leq X \leq 1/2]$. (d) the CDF $F_X(x)$.

Problem 2: (Problem 3.3.5 from Yates and Goodman) The CDF of a random variable Y is

$$F_Y(y) = \begin{cases} 0 & y < -1, \\ (y+1)/2 & -1 \leq y \leq 1, \\ 1 & y > 1, \end{cases}$$

(a) What is $E[Y]$? (b) What is Var[Y]?

Problem 3: In mobile communications, Rayleigh distribution is often used to model the random fading amplitude. The CDF of a Rayleigh random variable is given by

$$F_X(x) = \begin{cases} 1 - e^{-x^2/2\sigma^2} & x \geq 0 \\ 0 & x < 0 \end{cases}$$

(a) Find the PDF of X, $f_X(x)$. (b) Find $E[X]$, the mean and Var$[X]$, the variance.

Problem 4: The average received radio signal power over a large distance fluctuates randomly and this phenomenon is called *shadowing*. The lognormal PDF is commonly used to model the statistic of shadowing. The PDF of a lognormal-distributed random variable X is given by

$$f_X(x) = \begin{cases} \frac{\exp\{-[\ln(x-b)-a_X]^2/2\sigma_X^2}{\sqrt{2\pi}\sigma_X(x-b)}\} & x \geq b \\ 0 & x < b \end{cases}$$

where $\sigma_X > 0$, $-\infty < a_X < \infty$, and $-\infty < b < \infty$. Show that the corresponding CDF is

$$F_X(x) = \begin{cases} \Phi\left[\frac{\ln(x-b)-a_X}{\sigma_X}\right] & x \geq b, \\ 0 & x < b \end{cases}$$

where the function $\Phi(x)$ is defined as

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-u^2/2} du.$$

Problem 5: The Cauchy random variable has the probability density function

$$f_X(x) = \frac{b/\pi}{b^2 + (x-a)^2}$$

for real numbers $b > 0$ and $-\infty < a < \infty$. (a) Show that the CDF of X is

$$F_X(x) = \frac{1}{2} + \frac{1}{\pi} \tan^{-1}\left(\frac{x-a}{b}\right)$$

(b) Determine mean of the Cauchy random variable. (c) What can you say about the variance of the Cauchy random variable?

Problem 6: (Problem 3.5.1 from Yates and Goodman) The peak temperature T, as measured in degrees Fahrenheit, on a July day in New Jersey is the Gaussian $(85, 10)$ random variable. What is $P[T > 100]$, $P[T < 60]$, and $P[70 \leq T \leq 100]$?

Problem 7: (Problem 3.7.4 from Yates and Goodman) Random variable X has CDF

$$F_X(x) = \begin{cases} 0 & x < -1, \\ x/3 + 1/3 & -1 \leq x < 0, \\ x/3 + 2/3 & 0 \leq x < 1, \\ 1 & 1 \leq x. \end{cases}$$

If $Y = g(X)$ where

$$g(X) = \begin{cases} 0 & X < 0, \\ 100 & X \geq 0. \end{cases}$$

(a) What is $F_Y(y)$? (b) What is $f_Y(y)$? (c) What is $E[Y]$?

Problem 8: (Problem 3.7.11 from Yates and Goodman) The input voltage to a rectifier is a random variable U with uniform distribution on $[-1,1]$. The rectifier output is a random variable W defined by

$$W = g(U) = \begin{cases} 0 & U < 0, \\ U & U \geq 0. \end{cases}$$

Find the CDF $F_W(w)$ and the expected value $E[W]$.

Problem 9: (Problem 3.8.5 from Yates and Goodman) The time between telephone calls at a telephone switch is an exponential random variable T with expected value 0.01. Given $T > 0.02$,

(a) What is $E[T|T > 0.02]$, the conditional expected value of T?

(b) What is $\text{Var}[T|T > 0.02]$, the conditional variance of T?

Problem 10: For the quantizer example given in the lecture, the difference $Z = X - Y$ is the quantization error or quantization "noise." As in the quantizer example, we assume that X has a uniform $(-r/2, r/2)$ PDF.

(a) Given event B_i that $Y = y_i = \Delta/2 + i\Delta$ and X is in the ith quantization interval, find the conditional PDF of Z.

(b) Show that Z is a uniform random variable. Find the PDF, the expected value, and the variance of Z.