

**EE-387 Probability for Electrical and Computer Engineers**  
**Midterm Examination**  
**July 22, 2005**  
**Department of Electrical and Computer Engineering**  
**University of Alberta, Edmonton, AB, Canada**

**Student Name:**

**Student ID:**

**Section:** B1

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**Instructions:**

- This is a closed book examination.
- A calculator is allowed.
- You have 90 minutes to complete the examination.
- There are 7 pages in total, including this cover page.
- If necessary, use back of the pages.
- There are total of 30 points.
- Read each question carefully.
- Good luck!

**Question 1** (5 points total) Unlike the city of Nirvana, NY where 911 is the all-purpose number for emergencies, in Xond, Matf you dial 01 for a fire emergency, 02 for the police, and 03 for an ambulance. It is estimated that emergency calls in Matf have the same frequency distribution as in Nirvana, namely 60 percent are for the police, 25 percent are for ambulance service, and 15 percent are for the fire department. Assume that 10 calls are monitored and that none of the calls overlap in time.

(a) What is the probability that the ten dialed numbers create the sequence

02030202030102030202.

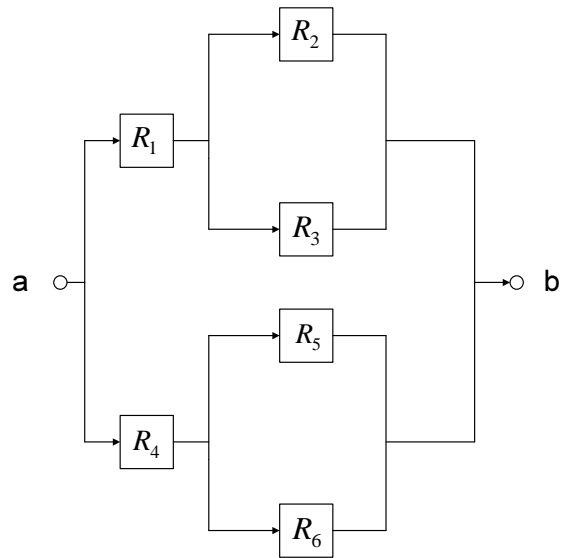
(b) How many distinguishable sequences exist that involve six calls to the police, three for an ambulance, and one to the fire department?

**Question 2** (5 points total) A smuggler, trying to pass himself off as a glass-bead importer, attempts to smuggle diamonds by mixing diamonds beads among glass beads in the proportion of one diamond bead per 1000 beads. A customs inspector examines a sample of 100 beads. What is the probability that the smuggler will be caught?

**Question 3** (5 points total) A binary communication system has digits “0” and “1” transmitted and digits “0” and “1” received. Denote by  $A_1$  and  $A_2$  the events that symbols “0” and “1” are received, respectively. Denote by  $B_1$  and  $B_2$  the events that symbols “0” and “1” are transmitted, respectively. Suppose the channel is very noisy and we have the channel transition probabilities  $P[A_1|B_1] = 0.2$ ,  $P[A_2|B_1] = 0.8$ ,  $P[A_1|B_2] = 0.7$ , and  $P[A_2|B_2] = 0.3$ . Assume that the *a priori* symbol transmission probabilities are  $P[B_1] = 0.6$  and  $P[B_2] = 0.4$ .

- (a) Find the received symbol probabilities  $P[A_1]$  and  $P[A_2]$ .
- (b) Find the *a posteriori* probabilities  $P[B_1|A_1]$ ,  $P[B_2|A_1]$ ,  $P[B_1|A_2]$ , and  $P[B_2|A_2]$ .
- (c) Determine the decision rule used by a maximum *a posteriori* (MAP) receiver.

**Question 4** (5 points total) In a communication system the signal sent from point  $a$  to point  $b$  passing a series repeaters shown in the following figure. The probabilities of the repeaters failing (independently) are  $p_1 = P[R_1] = 0.005$ ,  $p_2 = P[R_2] = P[R_3] = P[R_4] = 0.01$ , and  $p_3 = P[R_5] = P[R_6] = 0.05$ . Find the probability that the signal will not arrive at point  $b$ .



**Question 5** (5 points total) A biased coin with  $P[H] = 1/4$  is tossed 3 times. Let  $X$  be the number of heads that appear. (a) Find the probability mass function (PMF) of  $X$ . (b) Find the cumulative distribution function (CDF) of  $X$ .

**Question 6** (5 points total) A discrete random variable  $X$  has the following probability mass function (PMF)

$$P_X(x) = \begin{cases} 0.2 & x = -2 \\ 0.1 & x = -1 \\ 0.1 & x = 0 \\ 0.3 & x = 1 \\ 0.3 & x = 4 \\ 0 & \text{otherwise.} \end{cases}$$

Let  $Y = X^2$ . (a) Find  $P_Y(y)$ . (b) Find  $E[Y]$  and  $\text{Var}[Y]$ .