EE 3025 Recitation 5 — Chapter 1,2,3 Review Problems for Exam 1

- **Directions**: Your instructor will spend the entire 50 minute recitation period working out some of these review problems. The solutions to all these problems will be posted at the Exams web page by evening of Tuesday, Feb. 20.

1. Events A_1, A_2, A_3, A_4 are independent and each have probability 0.90. Let E, F, G be the events:

$$\begin{aligned} E &= A_1 \cap A_2 \cap A_3 \\ F &= A_1 \cap A_2 \cap A_4 \\ G &= A_2 \cap A_3 \cap A_4 \end{aligned}$$

(a) Compute the prob that at least one of the events E,F,G occurs.

(b) Given that at least one of the events E,F,G occurs, compute the prob that all of these events occur.

2. Let events A_1, A_2, A_3 partition a sample space S, and let events B_1, B_2, B_3 partition S, too. Let

$$P(A_1) = 0.25, \quad P(A_2) = 0.40.$$

Assume each "forward conditional probability" $P(B_j|A_i)$ is of the form

$$P(B_j|A_i) = C_i(|i - j| + 1),$$

where C_i is a constant that can depend on i but not on j.

(a) Compute C_1, C_2, C_3.

(b) Which of the events B_1, B_2, B_3 is most likely to occur?

(c) Given that B_3 occurs, which of the events A_1, A_2, A_3 is most likely to occur?

3. Five guests go to Bill's house for dinner, and they each put their coat in Bill's hallway closet. (There are no other coats in Bill's closet.) When each guest leaves, Bill hands them a coat randomly from the hallway closet, and, without complaining (out of politeness) the guest goes home with whatever coat Bill hands them. Answer the following questions by first devising a sample space and a probability model for this space.

(a) Compute the probability that exactly two of the 5 guests go home with the same coat they came with.

(b) Re-compute the prob in (a) given that at least one guest goes home with the same coat they came with.

4. A RV X has moment generating function

$$\phi_X(s) = \frac{2\exp(2s)}{3 - \exp(3s)}.$$

Compute

(a) Compute $P(X \leq 15)$.

(b) Compute $P(X \leq 5 | X \leq 15)$.

(c) Compute μ_X.

5. A discrete RV X satisfies

$$\begin{aligned} p_X(1) &= 0.1 \\ p_X(2) &= 0.3 \\ p_X(3) &= 0.2 \\ p_X(4) &= 0.1 \\ p_X(5) &= 0.3 \end{aligned}$$

(a) Plot the CDF.

(b) Compute the mean and variance of X.

(c) Plot the conditional PMF of X given $2 \leq X \leq 4$.

(d) Compute $E[X|2 \leq X \leq 4]$ given the cond PMF you found in (c).

6. A continuous RV has PDF $f_X(x)$ of the form $Cx(4-x)$ for $1 \le x \le 4$ (and equal to zero elsewhere), where C is a positive constant.

(a) Compute C.

(b) Compute $P(2 \le X \le 3)$.

(c) Compute $P(X > 2.5 | 2 \le X \le 3)$.

(d) Compute the mean and variance of X.

(e) Compute $E[X | 2 \le X \le 3]$.

7. Generate Pascal's triangle down through the row whose first 2 elements are $1, 8$. Then answer the following:

(a) Let X be the number of heads in flipping a fair coin 8 times. Compute $P(X = \mu_X)$.

(b) Let X be the number of heads in flipping an unfair coin 8 times, where $P[H] = 0.625$. Compute $P(X = \mu_X)$.

8. Message packets arrive at a server. The number of packets arriving in any finite time interval is modeled as a Poisson RV. Assume that the probability of getting exactly one message packet in a one millisecond time interval is twice the probability of getting exactly two message packets in a two millisecond time interval. What is the probability of getting exactly three message packets in a three millisecond time interval?

9. Determine which of the following three events is most likely to occur and which one is least likely to occur:

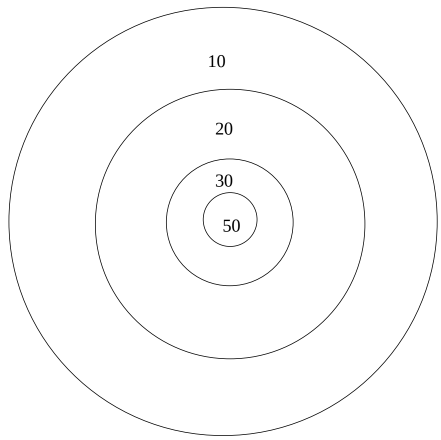
- The event that a Uniform RV exceeds its mean by more than 0.5 times the standard deviation.
- The event that a Gaussian RV exceeds its mean by more than 0.5 times the standard deviation.
- The event that an exponential RV exceeds its mean by more than 0.5 times the standard deviation.

10. Given the following information about a random variable X:

$$\begin{aligned} P(X > 0) &= 2/3 \\ E[X|X > 0] &= 3 \\ Var[X|X > 0] &= 4 \\ E[X|X \leq 0] &= -3 \\ Var[X|X \leq 0] &= 4 \end{aligned}$$

Compute the mean and variance of X.

11. Suppose you are guaranteed to hit the target below on each throw.



Suppose the 50 point region has radius 1 inch, the 30 point region has outer radius 2 inches, the 20 point region has outer radius 5 inches, and the 10 point region has outer radius 8 inches. Let X denote the random score obtained on a trial of this experiment (throw the dart one time).

(a) Find the PMF of X.

(b) Compute the mean of X.

12. Let discrete RV X be equidistributed over the set of integers $\{0, 1, 2, 3, 4, 5\}$. Let Y be the discrete RV

$$Y = \sin(\pi X/2).$$

(a) Plot the PMF of Y.

(b) Compute the mean of Y.

(c) Compute the variance of Y.

13. Suppose three events A, B, C satisfy

$$P(A) = P(B) = P(C) = 0.5, \quad P(A \cap B \cap C) = 0.18,$$

and suppose also that any two of the events are independent. Let random variable X be the number of events A, B, C which occur on the performance of the experiment.

(a) Find the PMF of X.

(b) Find the mean of X.

(c) Find the second moment of X.

14. An urn contains one green ball, two white balls, and three blue balls. Mr. Green and Mr. White play the following game. Balls are drawn randomly from the urn one after the other until the first nonblue ball is obtained. If this first nonblue ball is green, Mr. Green wins the game. If it is white, Mr. White wins the game.

- **(a)** What is the prob Mr. White wins if the balls are drawn with replacement?
- **(b)** What is the prob Mr. White wins if the balls are drawn without replacement?

15. A mixed RV X has density

$$f_X(x) = a\delta(x-1) + (1/3)\exp(-2x)u(x) + (4/3)\exp(-4x)u(x),$$

where a is a positive parameter.

(a) Find the value of a.

(b) Using decomposition ideas, find the mean of X in a simple manner.

(c) Write down a one-line expression for the CDF of X.

16. You are given a Gaussian RV X with mean 3 and variance 9.

(a) What linear change of variable should you make on X in order to obtain a Gaussian RV Y with mean -2 and variance 16? (In other words, $Y = aX + b$; what are the constants a, b?)

(b) What is the probability that Y will be between -3 and 2.5? (Use page 123 of textbook.)

(c) Let p be the probability you found in (b). Find the constant C such that $P(X \geq C) = p$. (Use page 123 of textbook.)

17. A, B are independent events with $P(A) = 0.75$ and $P(B) = 0.5$. Let X_A be the RV which is equal to 1 if A occurs and equal to 0 if A does not occur. Let X_B be the RV which is equal to 1 if B occurs and equal to 0 if B does not occur. Let Y be the discrete RV

$$Y = |X_A - X_B|.$$

(a) Plot the PMF of Y.

(b) Compute the mean of Y.

(c) Compute the variance of Y.

18. Work Problems 1.10.1 and 1.10.4 in the textbook.