

6.431 Fall 2002 Quiz 1

Wednesday 9th of October, 12:05-12:55pm

DO NOT TURN THIS QUIZ OVER UNTIL
YOU ARE TOLD TO DO SO

- You have 50 minutes to complete the quiz.
- Write your solutions in the exam booklet. We will not consider any work not in the exam booklet.
- This quiz has 2 problems that are not necessarily in order of difficulty.
- You may give an answer in the form of an arithmetic expression (sums, products, ratios, factorials) of numbers that could be evaluated using a calculator. Expressions like $\binom{8}{3}$ or $\sum_{k=0}^5 (1/2)^k$ are also fine.
- A correct answer does not guarantee full credit and a wrong answer does not guarantee loss of credit. You should concisely indicate your reasoning and show all relevant work. The grade on each problem is based on our judgment of your level of understanding as reflected by what you have written.
- This is a closed-book exam except for one single-sided, handwritten, 8.5 by 11 formula sheet plus a calculator.
- Be neat! If we can't read it, we can't grade it.
- At the end of the quiz, turn in your solutions along with this quiz (this piece of paper).

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Department of Electrical Engineering & Computer Science
6.431: Applied Probability
(Fall 2002)

Write your name and your TA's name on the front of the booklet. (2 points)

Problem 1: (52 points)

Bill has $n > 0$ different stocks in his portfolio. On any particular day, he believes that a stock in his portfolio will either go up with probability $p > 0$ or down with probability $(1 - p)$ independently of all other stocks (the probability of its staying unchanged is zero.)

- (a) (12 points) If the experiment is repeated independently every day, how often would the event “more of the stocks held by Bill are up than down” occur? Explain.
- (b) (12 points) The portfolio can be divided in three parts: A third of the stocks will either double in price or get divided by 2, a third will triple or get divided by 3, and a third would quadruple or get divided by 4 (Assume that n is a multiple of 6: $n = 6m$.)
What is the expected value of the money Bill holds after one day if one dollar is invested equally at the beginning of the day? (i.e. $1/n$ dollars are invested in each of the n stocks initially.)
- (c) (14 points) If $n/2$ of the stocks were up on a particular day, what is the probability that they included all the ones that get quadrupled?
- (d) (14 points) Assume now that the stock market changes in the following manner: For the first day of the year, all stocks have a probability p of going up and $(1 - p)$ of going down. If a stock is down one day, the probability that it will be up the following day is $2p$. If instead the stock was up then it will be up the following day still with probability p .

Bill decides the following: for the stocks that loose money on one day, he will invest twice as much (compared to gainers) on the second day.

On the third day, the stock of Energy Efficient Cooling Systems (EECS) went up. What is the probability that Bill invested more money in EECS on that day?

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Problem 2: (46 points)

Every day, Ramzi arrives first to work and parks his car at the edge of the parking lot that consists of n parking spaces in a row. Danielle arrives later and parks her car as close to Ramzi's as possible (Note that answers given as summations are acceptable.)

- (a) (16 points) If k people (excluding Ramzi) parked their cars randomly (in an uniform manner) in the available spots before Danielle arrived, let X be the random variable representing how many spots separate Danielle's and Ramzi's cars (X is zero when the cars are next to each other.) Find the PMF of X .
- (b) (15 points) Denote by p_i the PMF of X at i . If the same experiment is repeated independently every day, find the expected value of days between the times when two spots or less separate Danielle's and Ramzi's cars (Give your answer in terms of the p_i 's.)
- (c) (15 points) Let Y be the average number of spots separating the two cars over the period of m days, i.e.

$$Y = \frac{1}{m} \sum_{j=1}^m X_j,$$

where X_j is the random variable representing how many spots separate Danielle's and Ramzi's cars on day j .

Find the variance of Y (Give your answer in terms of the p_i 's.)