LECTURE 17

The Bernoulli process

- **Readings:** Section 5.1

Lecture outline

- Definition of Bernoulli process

- Random processes

- Basic properties of Bernoulli process

- Distribution of interarrival times

- The time of the kth success

The Bernoulli process

- A sequence of independent Bernoulli trials

- At each trial:
  - $\mathbf{P}(\text{success}) = \mathbf{P}(X = 1) = p$
  - $\mathbf{P}(\text{failure}) = P(X = 0) = 1 - p$

- Examples:
  - Sequence of ups and downs of the Dow Jones
  - Sequence of lottery wins/losses
  - Arrivals (each second) to a bank

Random processes

- First view: sequence of random variables $X_1, X_2, \ldots$

- $\mathbf{E}[X_t] =$

- $\text{Var}(X_t) =$

- Second view: what is the right sample space?

- $\mathbf{P}(X_t = 1 \text{ for all } t) =$

Number of successes S in n time slots

- $\mathbf{P}(S = k) =$

- $\mathbf{E}[S] =$

- $\text{Var}(S) =$

Interarrival times

- T_1: number of trials until first success

  - $\mathbf{P}(T_1 = t) =$

  - Memoryless property

  - $\mathbf{E}[T_1] =$

  - $\text{Var}(T_1) =$

- If you buy a lottery ticket every day, what is the distribution of the length of the first string of losing days?

Time of the kth arrival

- Y_k: number of trials to kth success

  - $\mathbf{E}[Y_k] =$

  - $\text{Var}(Y_k) =$

  - $\mathbf{P}(Y_k = t) =$