EE126: Probability and Random Processes **SP'07**

Problem Set 9 — Due May, 3

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Problem 9.1. *You are visiting the rainforest, but unfortunately your insect repellent has run out. As a result, at each second, a mosquito lands on your neck with probability 0.5. If one lands, with probability 0.2 it bites you, and with probability 0.8 it never bothers you, independently of other mosquitoes. What is the expected time between successive bites? What is the variance of the time between successive bites?*

Problem 9.2. *We repeatedly toss a coin independent. Let N be the first time in which we have a success immediately following a previous success. (That is, N is the first i for which $X_{i-1} = X_i = 1$.)*
Compute $E[N]$.
What is the probability $P(X_{N+1} = X_{N+2} = 0)$ that there are no successes in the two trials that follow?

Problem 9.3. *Gambler ruin problem.*
A gambler wins \$1 at each play round, with probability p and loses \$1 with probability $1-p$. Different rounds are assumed to be independent. The gambler keeps playing until he either accumulate \$$m$ or loses all his money. What is the probability of eventually accumulating the target amount.
Assume that the gambler starts with $\frac{m}{2}$

Problem 9.4. *Let X_1 and X_2 be independent exponential random variables, each having rate μ. Let*

$$X_m = \min(X_1, X_2) \qquad X_M = \max(X_1, X_2)$$

Find

1. $E[X_m]$ *and* $var[X_m]$.
2. $E[X_M]$ *and* $var[X_M]$.

Problem 9.5. *Suppose that people immigrate into a territory at a Poisson rate $\lambda = 1$ per day.*

1. *What is the expected time until the tenth immigrant arrives?*
2. *What is the probability that the elapsed time between the tenth and the eleventh arrival exceeds 3 days?*

Problem 9.6. *A train bridge is constructed across a wide river. Trains arrive at the bridge according to a Poisson process of rate $\lambda = 3$ per day.*

1. *If a train arrives on day 0, find the probability that there will be no trains on days 1, 2, and 3.*
2. *Find the probability that the first train to arrive, after the train on day 0, takes more than 3 days to arrive.*
3. *Find the probability that no trains arrive in the first 2 days, but 4 trains arrive on the 4^{th} day.*
4. *Find the probability that it takes more than 2 days for the 5^{th} train to arrive at the bridge.*

Problem 9.7. *Cars pass a certain street location according to a Poisson process with rate λ. A woman who wants to cross the street at that location waits until she can see that no cars will come by the next T time units.*

1. *Find the probability that her waiting time is zero.*
2. *Find her expected waiting time.*