Checklist for EECS126 - Spring 2007

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Abstract

This note lists the topics that you are supposed to know for the final of EECS126

1 Probability Space

- Sample Space, Events, Probability

You know that the collection of events is closed under (countable) set operations; also, the empty set $\emptyset$ and the whole set of outcomes Ω are events. You can provide examples and counterexamples.

The probability is countably additive.

2 Conditional Probability and Independence

- Conditional Probability
- Bayes' Rule
- Pairwise Independence
- Mutual Independence

3 Random Variable

- Definition of Random Variable; Discrete, Continuous, Mixed
- Definitions: p.m.f.; c.p.d.f.; p.d.f.
- Examples: Bernoulli, Binomial, Geometric, Poisson, Uniform, Exponential, Gaussian
- Function of a random variable
- c.p.d.f. and p.d.f. of $g(X)$

4 Multiple Random Variables

- Definitions: Joint p.m.f., joint c.p.d.f., joint p.d.f.
- Pairwise and mutual independence
- If X, Y are independent, then so are $g(X)$ and $h(Y)$
- Joint density of $AX + b$ when X is a vector of r.v.'s and A a matrix.

5 Expectation

- Definition of $E(X)$
- Formula for $E(h(X))$
- Variance, moments, Markov inequality, Chebychev inequality
- Characteristic function
- Examples for the familiar distributions
- If X, Y are independent, then $E(XY) = ...$
- If $X, Y, Z, ...$ are independent and exponentially distributed, then $\min\{X, Y, Z, ...\}$ is ...

6 Conditional Expectation

- Definition of $E[X|Y]$
- Properties of conditional expectation
- Familiarity with calculations: Either using conditional density, or properties, or symmetry.

7 Gaussian Random Variables

- Definition of $N(\mu, \sigma^2)$: Density, Characteristic function
- Moments of $N(0, 1)$
- Definition of Jointly Gaussian, Density, Characteristic Function
- If J.G., then uncorrelated implies independent
- $E[X|Y]$ for J.G. (vector case)
- If X, Y are independent $N(0, 1)$, then $X^2 + Y^2$ is ...

8 Estimation

- $E[X|Y]$ is the MMSE of X given Y
- $L[X|Y]$ is the LLSE of X given Y. Formula for $L[X|Y]$ (vector case).
- Understand why in general $L[X|Y] \neq E[X|Y]$.
- Understand how to find $g(Y)$ that minimizes $E(C(X, g(Y)))$.

9 Detection

- Understand the Baysian and non-Bayesian formulations
- Know the definition and how to calculate $MAP[X|Y]$

- Know the definition and how to calculate $MLE[X|Y]$
- Know how to find $\hat{X}$ from Y so as to maximize $P[\hat{X} = 1|X = 1]$ subject to $P[\hat{X} = 1|X = 0] \leq \beta$.
- Know how to find Z from Y so as to maximize $\sup_{a \in A} P[Z = 1|X = a]$ subject to $\sup_{b \in B} P[\hat{X} = 1|X = b] \leq \beta$.

10 Limits of Random Variables

- Know the Strong Law of Large Numbers
- Know the Central Limit Theorem

11 Random Processes

- Understand the definition of discrete time Markov chain
- Remember the classification theorem: If irreducible and finite, then there is one invariant distribution and the strong law of large number holds. If in addition it is aperiodic, then the distribution converges to the invariant distribution. We do not expect you to know the infinite case (e.g., recurrence or transience).
- Know how to solve simple balance equations $\pi = \pi P$.
- Understand how to use the first step equations to calculate probabilities and mean hitting times.
- Understand the memoryless property of the exponential distribution.
- Understand the construction of a continuous time Markov chain (holding times, jumps, transition rates).
- Understand the Poisson process (how many jump in t seconds?)
- Know how to calculate simple balance equations $\pi Q = 0$ for a continuous time Markov chain.