UC Berkeley
Department of Electrical Engineering and Computer Science

EE 126: Probablity and Random Processes

Problem Set 3
Spring 2007

Issued: Tuesday, Janauary 30, 2007 **Due:** Thursday, February 8, 2007

Problem 3.1

Suppose you roll two fair dice.

a) What is the a priori probability that at least one lands on a six?

b) What is the conditional probability of the above event, given that they the two dice land on different numbers?

Problem 3.2

Alice and Bob love to challenge each other to coin tossing contests. On one particular day, Alice brings $2n + 1$ fair coins, and lets Bob toss $n + 1$ coins, while she tosses the remaining n coins. Show that the probability that after all the coins have been tossed Bob will have gotten more heads that Alice is 1/2.

Problem 3.3

Let A and B be independent events with $0 < kP(B) = P(A) < 1$, where $k \geq 1$. Let $C = (A \cap B^c) \cup (A^c \cap B)$ be the event that exactly one of A and B occurred. Show that

$$P(A \mid C) \geq \frac{k}{k+1}.$$

Problem 3.4

A new test has been developed to determine whether a given student is overly stressed. This test gives the correct diagnosis 95% of the time if the student is not overly stressed, but only 85% of the time if the student is in fact overly stressed. It is known that 99.5% of all students are overly stressed. Given that a particular student tests negative for stress, what is the probability that the test result is correct, and that this student is not overly stressed?

Problem 3.5

A magnetic tape storing information in binary form has been corrupted. Imagine that you are to make an effort to save as much information as possible. Due to the damage on the tape, you know that there will be errors in the reading. You know that if there was a 0, the probability that you correctly detect it is .9. The probability that you correctly detect a

1 is .85. Given that the each digit is a 1 or a 0 with equal probability, and given that you read in a 1, what is the probability that this is a correct reading?

Problem 3.6

Consider a communication channel model where the transmitter can send a binary input of either "0" or "1", and where the output of the channel is also binary. Due to noise, the probability of receiving a "0" given that a "0" was transmitted is equal to $(1 - \epsilon)$, and that of receiving a "1" given that "1" was transmitted is equal $(1 - \delta)$. We assume that ϵ and δ are less than half.

1. What is the probability of receiving "1" when "0" is transmitted? What is the probability of receiving "0" when "1" is transmitted?

2. Assume that the transmitter is three times more likely to transmit "1" than "0". What is the probability of reception error?

3. Instead of directly transmitting a bit, the transmitter decides to "encode" his message by repeatedly transmitting $(2n + 1)$ times the bit he needs to communicate to the receiver. Accordingly, the receiver uses a majority rule to decide what bit was transmitted (among $(2n + 1)$ channel outputs, if more "1"'s are received than "0"'s, then the receiver decides that the transmitter message was "1" and vice versa). Using this strategy, and assuming that channel errors are independent, compute the message probability of error when the assumption of part (b) still holds.

4. When $\delta = \epsilon = 0.1$, how many times does the transmitter need to repeat his message in order to guarantee a probability of error less than 10^{-6}?

Problem 3.7

Imno Nerd, an MIT Freshman, makes one to five new friends every week, with equal probability. The number of friends she makes during each week is independent from all other weeks. We are concerned with two consecutive weeks.

Let event A be "Imno made a total of 10 friends during the two weeks". Let event B be "Imno made more than 5 friends during the two weeks."

1. Are events A and B independent?

2. Let C be the event "Imno made exactly 5 friends during the first week". Are A and B independent, conditioned on C?

3. Is A independent of C? Is B independent of C?

4. Given that Imno made a total of 6 friends in two weeks, what is the probability that she made exactly 2 friends in the first week? How about 3 friends in the first week?

Problem 3.8

Anne, Betty, Chloe and Daisy were all friends in school. Subsequently each of the six subpairs meet up once; at each of the six meetings, the pair quarrels with some fixed probability p and otherwise the pair retains a firm friendship. Quarrels take place independently of each other.

In the future, if any one of the four hears a rumour, she tells it to her firm friends only. Supposing that Anne hears a rumour, what is the probability that:

1. Daisy hears it?
2. Daisy hears it if Anne and Betty have quarrelled?
3. Daisy hears it if Betty and Chloe have quarrelled?
4. Daisy hears it if she has quarrelled with Anne?

Problem 3.9

Find a sample space Ω, two probability laws P and Q, and three events A, B and C (subsets of Ω) such that *all* of the following are satisfied:

1. A and B are independent for P,
2. A and B are not independent for Q,
3. A and B are conditionally (given C) independent for Q.