EEL 5544 Noise in Linear Systems Lecture 11

MULTIPLE RANDOM VARIABLES

DEFN Given a probability space $(\Omega, \mathcal{F}, P)$, a *real vector random variable* $\underline{X}$ is a function that assigns a vector of real numbers to each outcome ω in Ω.

- Random vectors with n elements are also known as n-dimensional random variables.
- A common special case is that a random vector $\underline{Z}$ consists of two real random variables, $\underline{Z} = (X, Y)$.

  Example

- We typically define the distribution and density functions for $\underline{Z}$ in terms of X and Y:

  DEFN The __________________________________ of random variables X and Y is

  $$F_{X,Y}(x, y) \quad =$$

- More commonly, we use short-hand notation to write:

  $$F_{X,Y}(x, y) \quad =$$

  Example

Properties

1. $0 \leq F_{X,Y}(x, y) \leq 1$

   Pf:

2. $F_{X,Y}(-\infty, y) = F_{X,Y}(x, -\infty) = F_{X,Y}(-\infty, -\infty) = 0$

   PF: $F_{X,Y}(-\infty, y) \quad =$

3. $F_{X,Y}(x,y)$ is a montonically nondecreasing function of both x and y

   Pf: Consider $F_{X,Y}(x+\Delta x, y)$, where $\Delta x > 0$

4.

$$\begin{aligned} F_{X,Y}(x,\infty) &= F_X(x) \\ F_{X,Y}(\infty,y) &= F_Y(y) \end{aligned}$$

In this case, F_X and F_Y are known as ________________________ for X and Y, respectively

PF: $F_{X,Y}(x,\infty) \quad =$

5.

$$\begin{aligned} P(a_1 < \quad X \quad &\leq a_2, b_1 < Y \leq b_2) = \\ &F_{X,Y}(a_2,b_2) - F_{X,Y}(a_1,b_2) \\ &\ - F_{X,Y}(a_2,b_1) + F_{X,Y}(a_1,b_1) \end{aligned}$$

Pf: On separate sheets.

THE JOINT PROBABILITY DENSITY FUNCTION

DEFN The ______________________________ of X and Y is

$$f_{X,Y}(x,y) =$$

Properties

1. $f_{X,Y}(x,y) \geq 0$, for $-\infty < x < \infty$ and $-\infty < y < \infty$

   Pf:

2. $$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{X,Y}(x,y)dxdy = 1$$

   Pf:

3. $$\begin{aligned} f_X(x) &= \int_{-\infty}^{\infty} f_{X,Y}(x,y)dy \\ f_Y(y) &= \int_{-\infty}^{\infty} f_{X,Y}(x,y)dx \end{aligned}$$

   $f_X(x)$ and $f_Y(y)$ are known as ____________________

   Pf:

4. $$P(a_1 \leq X \leq a_2, b_1 \leq Y \leq b_2) = \int_{a_1}^{a_2}\int_{b_1}^{b_2} f_{X,Y}(x,y)dydx$$

   Pf: