EEL 5544 Noise in Linear Systems Lecture 22

EXPECTED VALUE OF A RV

DEFN The *expected value* or *mean* of a random variable X, when defined, is

$$\mu_X = E\left[X\right] \quad =$$

- To determine if $E[X]$ is defined, let

$$\begin{aligned} E[X^+] &= \int_0^\infty f_X(x)dx, \text{ and} \\ E[X^-] &= \int_0^\infty f_X(-x)dx, \text{ and} \end{aligned}$$

- Then $E[X]$ is defined if either $E[X^+] \neq \infty$ or $E[X^-] \neq \infty$, and

$$E[X] = E[X^+] - E[X^-]$$

- Note that $E[X]$ may be infinite

  Examples

- If X is a discrete RV, the expected value can be computed from the pmf as

$$E\left[X\right] \quad =$$

  Examples

- If X is a nonnegative RV, then

$$E\left[X\right] \quad =$$

  (from partial integration of original expression).

EXPECTED VALUE OF A FUNCTION OF A RV

- If $Y = g(X)$, it is not necessary to compute the pdf or cdf of Y to find its expected value:

$$E[Y] =$$

- This is sometimes known as the ______________________________________

  Proof sketch.

- Properties of Expected value:

  1. Expected value of a constant is _________________________:

  $$E[c] =$$

  $$=$$

  2. Expected value is a _______________ operator:

  $$E[ag(X) + bh(X)] =$$

- Some common moments (expected values):

  – nth moment of X:

  $$E[X^n] =$$

  – nth central moment of X:

  $$E[(X - \mu_X)^n] =$$

  where $\mu_X = E[X]$.

- Variance of X is 2nd central moment:

$$\begin{aligned}\text{Var}[X] &= E[(X-\mu_X)^2] \\ &= \\ &= \\ &= \end{aligned}$$

(this latter formula is usually a more convenient way to find the variance.)

Examples on blackboard.

Properties of variance:

1. $\text{Var}[c] =$
2. $\text{Var}[X+c] =$
3. $\text{Var}[cX] =$