EEL 5544 Noise in Linear Systems Lecture 34

CONDITIONAL DENSITIES

- We will use the notation $f_{X,n}(\cdot|\cdot)$ to denote a general conditional density, where $f_{X,n}(\cdot)$ is a joint density

EX: The conditional density for $X(t_1)$ given $X(t_2)$, $t_1 \neq t_2$ is

$$f_{X,2}(x_1; t_1|x_2; t_2) = \frac{f_{X,2}(x_1, x_2; t_1, t_2)}{f_{X,1}(x_2; t_2)}.$$

GAUSSIAN RPS

DEFN $X(t)$ is a *Gaussian random process* if for each $n \in \mathbf{Z}^+$, its n-dimensional distribution function is a Gaussian distribution.

- Note that each of the n-dimensional distribution functions is completely specified by the appropriate means and covariances.

  Alternate definition:

DEFN $X(t)$ is a *Gaussian random process* if:

- for each $n \in \mathbf{Z}^+$,
- any real coefficients $\{a_k : 1 \leq k \leq n\}$
- and any sampling times $\{t_k \in \mathbf{T} : 1 \leq k \leq n\}$,

the random variable
$a_1X(t_1) + a_2X(t_2) + \ldots + a_nX(t_n)$
is a Gaussian random variable.

MEAN, AUTOCORRELATION, AND AUTOCOVARIANCE FUNCTIONS

- It is generally not practical, and often unnecessary to determine n-dimensional distribution or density functions for RPs
- In many problems, it is sufficient to know about ____________ statistics and distribution functions

DEFN A random process $X(t)$ is a ____________________________ if $E\left\{[X(t)]^2\right\} < \infty$ for each t.

For second-order RPs, we define the following functions:

DEFN The ____________________ (or just ________) is given by

$$\mu_X(t) \quad =$$

DEFN The __________________________________ is given by

$$R_X(t_1, t_2) \quad =$$

DEFN The __________________________________ is given by

$$C_X(t_1, t_2) \quad =$$

Interpretation:

- At each t, $\mu_X(t)$ is the expected value of the random variable $X(t)$
  - $\mu_X(t)$ depends only on the one-dimensional density $f_{X,1}(x;t)$

- $R_X(t_1, t_2)$ and $C_X(t_1, t_2)$ measure the statistical "coupling" or "dependence" between $X(t_1)$ and $X(t_2)$
  - $R_X(t_1, t_2)$ and $C_X(t_1, t_2)$ depend only on the one- and two-dimensional densities
  - if $X(t_1)$ and $X(t_2)$ are statistically independent, then $C_X(t_1, t_2) = 0$

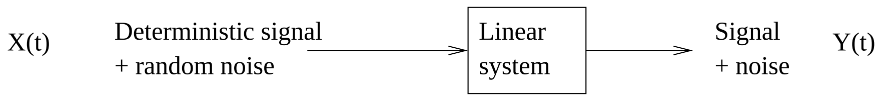
– if $C_X(t_1, t_2) = 0$, then $X(t_1)$ and $X(t_2)$ are uncorrelated

- Note that

$$C_X(t_1, t_2) = R_X(t_1, t_2) - \mu_X(t_1)\mu_X(t_2),$$

so generally only need to know one of C_X and R_X

Application preview: Noise in Linear Systems



What is the noise power at input and output?

- Input: Noise power = $E\left\{[X(t)]^2\right\} = R_X(t, t)$

- Output: Noise power = function of $R_X(t_1, t_2)$ and **not** just $R_X(t, t)$

Given $X(t)$ with finite power ($X(t)$ a second-order RP), are μ_X, R_X, and C_X all finite?

DEFN The *normalized autocovariance* of a random process $X(t)$ is given by

$$K_X(t_1, t_2) = \frac{C_X(t_1, t_2)}{\sigma_1 \sigma_2},$$

where $\sigma_i = \sqrt{\mathrm{Var}[X(t_i)]}$.

- Note that

$$-1 \le K_X(t_1, t_2) \le 1$$

Examples