EEL 5544 Lecture 7

# RANDOM VARIABLES (RVS)

- What is a random variable?

- We define a random variable is defined on a probability space $(\Omega, \mathcal{F}, P)$ as a ______________ from ___ to ____

**Examples**

- Recall that we cannot define probabilities for all possible subsets of a continuous sample space $\Omega$
- Thus, in defining a random variable $X$ as a function on $\Omega$, we must ensure that any region of $X$ for which we wish to assign probability must map to an *event* in the event class $\mathcal{F}$
- We will only assign probabilities to Borel sets of the real line

**DEFN** A *real random variable* $X(\omega)$ defined on a probability space $(\Omega, \mathcal{F}, P)$ is a real-valued function on $\Omega$ that satisfies the following:
(i) For every Borel set of real numbers $B \in \mathcal{B}$, the set $E_B \triangleq \{\xi \in \Omega, X(\xi) \in B\}$ is an event and
(ii) $P[X = -\infty] = 0$ and $P[X = +\infty] = 0$.

- The Borel field on $\mathcal{R}$ contains all sets that can be formed from countable unions, intersections, and complements of sets of the form $\{x | x \in (-\infty, x]\}$
- Thus, it is convenient to define a function that assigns probabilities to sets of this form:

**DEFN** If $(\Omega, \mathcal{F}, P)$ is a prob space with $X(\omega)$ a real RV on $\Omega$, the ______________________________________________
(____________), denoted ____________ is

- $F_X(x)$ is a prob. measure
- Properties of $F_X$:

1. $0 \leq F_X(x) \leq 1$

2. $F_X(-\infty) = 0$ and $F_X(\infty) = 1$

3. $F_X(x)$ is monotonically nondecreasing,
   i.e., $F_X(a) \leq F_X(b)$ iff $a \leq b$

4. $P(a < X \leq b) = F_X(b) - F_X(a)$

5. $F_X(x)$ is continuous on the right,
   i.e., $F_X(b) = \lim_{h \to 0} F_X(b + h) = F_X(b)$

   *(The value at a jump discontinuity is the value after the jump.)*

   Pf omitted.

- If $F_X(x)$ is continuous function of $x$, then
  $F(x) = F(x^-)$

- If $F_X(s)$ is not a continuous function of $x$, then from above,

$$\begin{aligned} F_X(x) - F_X(x^-) &= P[x^- < X \leq x] \\ &= \lim_{\epsilon \to 0} P[x - \epsilon < X \leq x] \\ &= P[X = x] \end{aligned}$$

**Example**