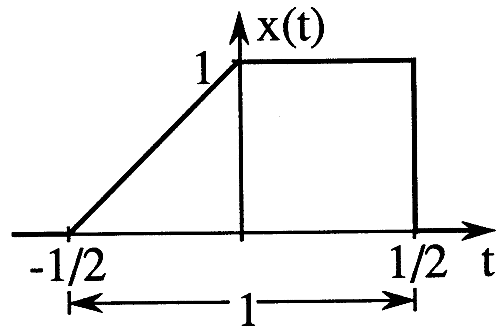
1.1.3 SIGNAL TRANSFORMATIONS

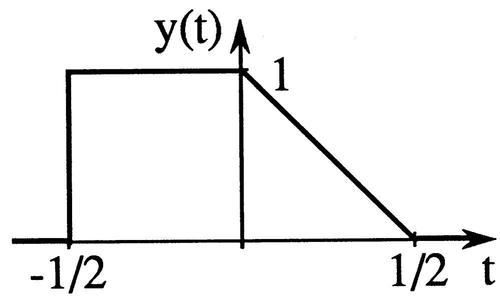
Continuous-Time

Consider



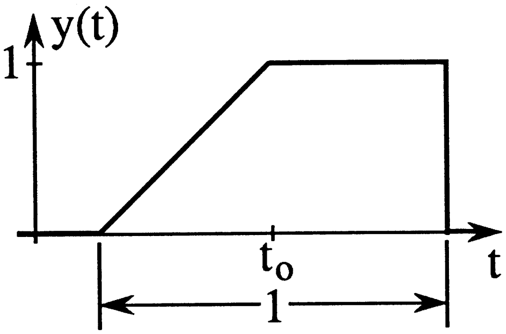
1. Reflection

$$y(t) = x(-t)$$



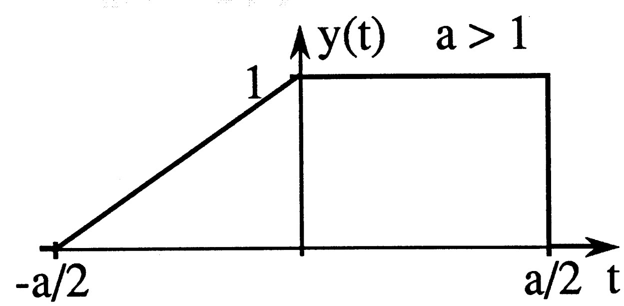
2. Shifting

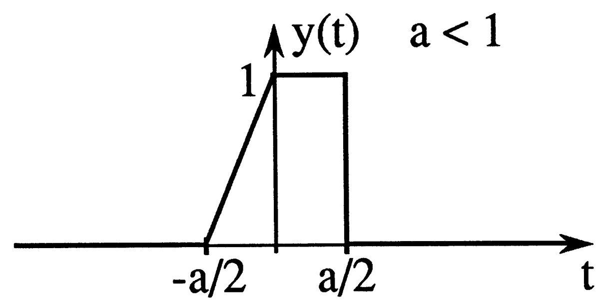
$$y(t) = x(t - t_0)$$



3. Scaling

$$y(t) = x(t/a)$$



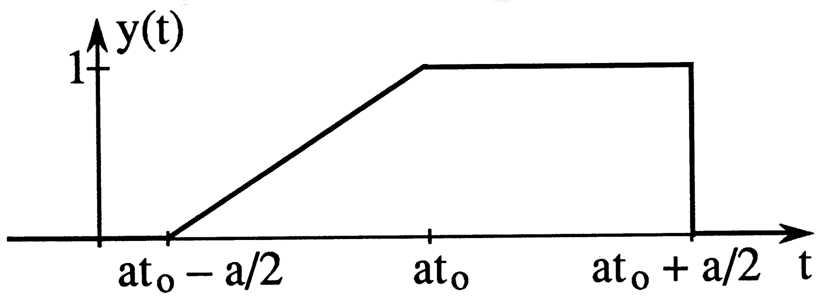


4. Scaling and Shifting

$$y(t) = x(t/a - t_0)$$

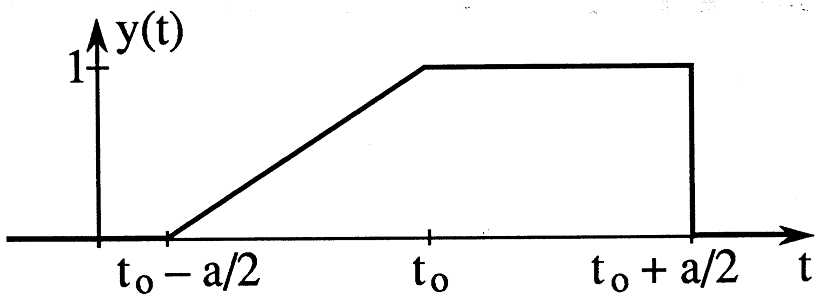
What does this look like?

center of pulse	$t/a - t_0 = 0$	$\Rightarrow t = at_0$
left edge of pulse	$t/a - t_0 = -1/2$	$\Rightarrow t = a(t_0 - 1/2)$
right edge of pulse	$t/a - t_0 = 1/2$	$\Rightarrow t = a(t_0 + 1/2)$



$$y(t) = x\left(\frac{t - t_0}{a}\right)$$

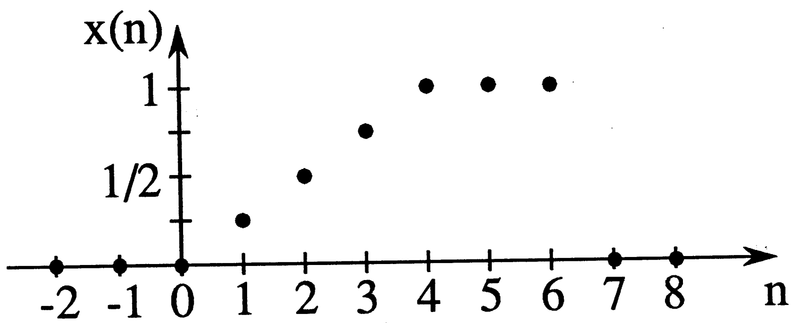
center	$(t - t_0)/a = 0$	$\Rightarrow t = t_0$
left edge	$(t - t_0)/a = -1/2$	$\Rightarrow t = t_0 - a/2$
right edge	$(t - t_0)/a = 1/2$	$\Rightarrow t = t_0 + a/2$



This is preferred form because it is easily recognizable.

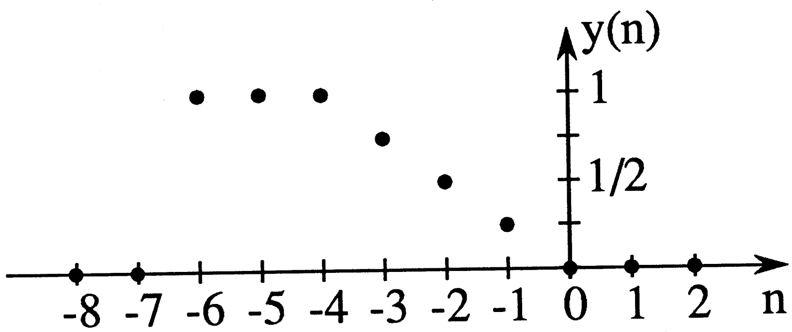
Discrete-Time

Consider



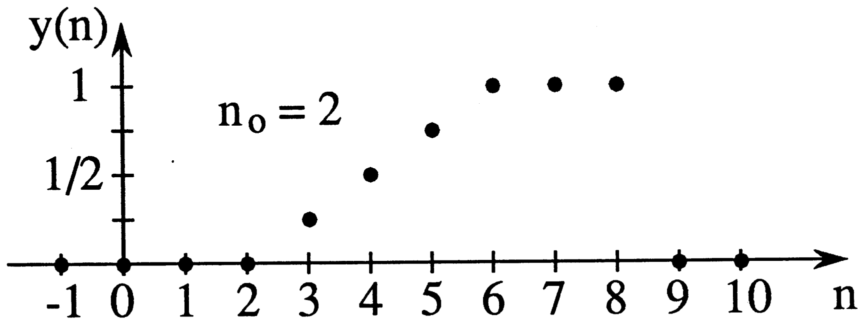
1. Reflection (same as CT)

$$y(n) = x(-n)$$



2. Shifting

$$y(n) = x(n - n_0)$$

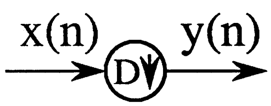


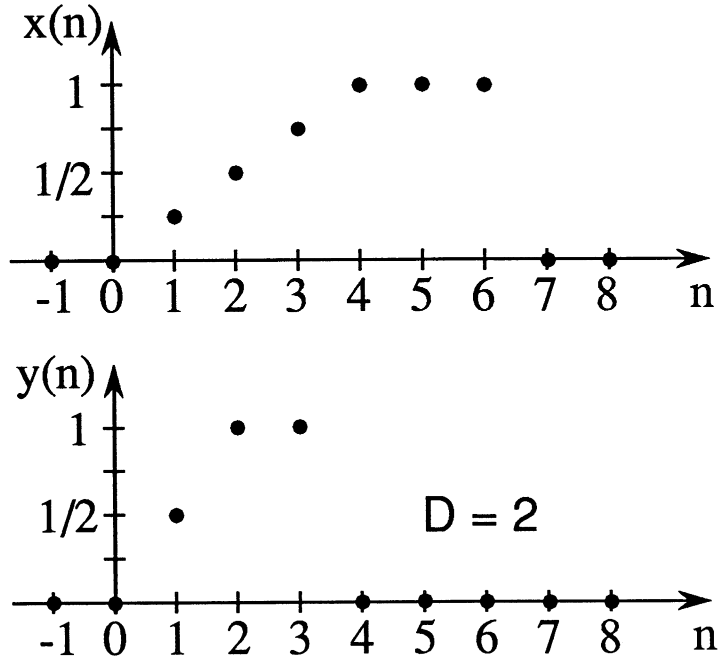
Note that n_0 must be an integer. Non-integer delays can only be defined in the context of a CT signal, and are implemented by interpolation.

3. Scaling

a. downsampler

$$y(n) = x(Dn)$$





b. upsampler

$$y(n) = \begin{cases} x(n/D) & , \quad \text{if } n/D \text{ is an integer} \\ 0 & , \quad \text{else} \end{cases}$$