1.5.3 ZT PROPERTIES AND PAIRS

Transform Relations

1. linearity

$$a_1 x_1(n) + a_2 x_2(n) \quad \overset{ZT}{\longleftrightarrow} \quad a_1 X_1(z) + a_2 X_2(z)$$

2. shifting

$$x(n - n_0) \quad \overset{ZT}{\longleftrightarrow} \quad z^{-n_0} X(z)$$

3. modulation

$$z_0^n \, x(n) \overset{ZT}{\longleftrightarrow} X(z/z_0)$$

4. multiplication by time index

$$n \, x(n) \overset{ZT}{\longleftrightarrow} -z \frac{d}{dz} [X(z)]$$

5. convolution

$$x(n) * y(n) \overset{ZT}{\longleftrightarrow} X(z) \, Y(z)$$

6. relation to DTFT

   If $X_{ZT}(z)$ converges for $|z| = 1$, then the

   DTFT of x(n) exists and

   $$X_{DTFT}(e^{j\omega}) = X_{ZT}(z)\,\big|_{z=e^{j\omega}}$$

Important Transform Pairs

1. $\delta(n) \overset{ZT}{\longleftrightarrow} 1,$ all z

2. $a^n u(n) \overset{ZT}{\longleftrightarrow} \dfrac{1}{1 - az^{-1}},$ $|z| > a$

3. $-a^n u(-n-1) \overset{ZT}{\longleftrightarrow} \dfrac{1}{1 - az^{-1}},$ $|z| < a$

4. $na^n u(n) \overset{ZT}{\longleftrightarrow} \dfrac{az^{-1}}{(1 - az^{-1})^2},$ $\qquad |z| > a$

5. $-na^n u(-n-1) \overset{ZT}{\longleftrightarrow} \dfrac{az^{-1}}{(1 - az^{-1})^2},$ $\quad |z| < a$