APPENDIX A

GENERAL FORMULATION OF THE PROBABILISTIC ANALYSIS

Let M_k be the kth moment of the periodogram $\hat{S}(f)$ for the process $\xi(t)$. The quantities of interest are:

$$E(M_0),$$

$$E(M_1),$$

$$E(M_2 - \hat{P}B_T^2),$$

$$E(M_0^2),$$

$$E(M_1^2),$$

$$E[(M_2 - \hat{P}B_T^2)^2],$$

$$E(M_0 M_1),$$

$$E[M_0(M_2 - \hat{P}B_T^2)],$$

$$E[M_1(M_2 - \hat{P}B_T^2)]. \qquad (A.1)$$

From the definition,

$$M_k = T^{-1} \int_f f^k |Z_T(2\pi f)|^2 df \qquad (A.2)$$

But,

$$Z_T(2\pi f) = \int_t \xi(t) G_T(t) e^{-j2\pi ft} dt \qquad (A.3)$$

Therefore,

$$M_k = T^{-1} \int_f f^k [\int_t \xi(t) G_T(t) e^{-j2\pi f t} dt]$$

$$\cdot [\int_\tau \xi^*(\tau) G_T(\tau) e^{j2\pi f \tau} d\tau] df. \qquad (A.4)$$

Changing the order of integration yields

$$M_k = (j2\pi)^{-k} T^{-1} \int_t \int_\tau \xi(t) \xi^*(\tau) G_T(t) G_T(\tau)$$
$$\cdot \int_f (j2\pi f)^k e^{j2\pi f(\tau - t)} df\, d\tau dt. \qquad (A.5)$$

The integral over f yields the kth derivative of the Dirac delta function. Therefore,

$$M_k = (j2\pi)^{-k} T^{-1} \int_t \int_\tau \xi(t) \xi^*(\tau) G_T(t) G_T(\tau) \delta^{(k)}(\tau - t) d\tau dt. \qquad (A.6)$$

The desired expected values can be calculated from this result.