# CHAPTER I

# INTRODUCTION

This investigation develops and analyzes a system which provides real time estimates of two parameters of the power spectral density, PSD, of a narrow band random process. The PSD parameters of interest are the power mean frequency, $f_a$, and the RMS spectral bandwidth, B.

If the reflected signal's complex envelope, z(t), is referenced to the original transmitter carrier frequency and if S(f) is the PSD of this complex envelope, then by definition:

$$f_a \equiv \frac{\int_f f\,S(f)df}{\int_f S(f)df} \qquad (1.1)$$

$$B^2 \equiv \frac{\int_f (f - f_a)^2 S(f)df}{\int_f S(f)df}. \qquad (1.2)$$

It should be noted that the variance, $B^2$, of the PSD is a parameter which is equivalent to B.

Motivation for this investigation is found in the field of radar meteorology. Specifically, the illumination of a sector of the atmosphere by a coherent radar results in a reflection off of each particle in that sector. If a particle in that sector is in motion along the radar beam, the reflected wave is shifted in frequency. For a sector that contains a homogeneous group of particles, the percentage of the total power that is found in a given band of doppler frequencies, in the reflected signal, is related to that percentage of the sector's particles with velocities corresponding to the given band of frequencies. Hence, the reflected radar signal is a random process whose PSD shape parameters are related to the velocity distribution of the particles that are in motion along the radar beam.

The classical technique that is used in measuring PSD shape parameters first estimates the PSD and then uses the result to calculate the desired parameters. Two implementations of this procedure are traditional. The first approach utilizes a band of very narrow, band-pass filters. Each filter samples the power content of the random process as a function of frequency. An analog computer then processes this data to provide the shape parameters. The second, and more popular approach, digitizes time samples of the signal with an analog-to-digital, A-to-D, converter. Time windows of this digitized signal are then processed by the Fast Fourier Transform, FFT, on a high speed digital computer. The resulting frequency domain samples are then used to calculate the desired parameters. These two methods yield equivalent results when variations are made in the transfer function of the band-pass filters or changes are made in the shape of the window function. However, the high cost of real time processing limits the applicability of these methods.

Bello's study of the parameters of a fading communication channel (3)* contains different estimators for the PSD shape parameters. He first proposes the estimators

$$\hat{f}_a = \frac{\int_0^T [x\dot{y} - y\dot{x}]dt}{\int_0^T [x^2 + y^2]dt} \tag{1.3}$$

$$\hat{D}_1 \equiv 2\hat{B} = (\pi)^{-1} \left[ \frac{\int_0^T [\dot{x}^2 + \dot{y}^2]dt}{\int_0^T [x^2 + y^2]dt} - 4\pi^2(f_a)^2 \right]^{1/2} \tag{1.4}$$

where x and y are quadrature components of the band-pass process which are referenced to the transmitted frequency, the hat (^) over a variable denotes an estimate of that variable, and a dot over a variable denotes differentiation with respect to time. Only a long averaging time, T, and ergodicity of the process being measured are assumed.

Bellow also presents an independent derivation of another estimator, $\hat{D}_2$, for the doppler spread, D. His derivation assumes narrow band gaussian process and displays $\hat{D}_2$ in terms of the instantaneous amplitude of the process:

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* For all numbered references, see Bibliography.

$$\hat{D}_2 = (\pi\gamma)^{-1} \frac{\int_0^T [\dot{A}]^2 dt}{\int_0^T A^2 \, dt} \qquad (1.5)$$

Here, $\gamma$ is a real number dependent only on the type of detector that is employed to measure the instantaneous envelope, A. In the case of the linear detector, $\gamma = 1/\sqrt{2}$ (2). Again, this result is valid only for large T. Although Bello does not consider the consistency or the accuracy of these estimates, the simplicity of his results indicates an area for future work.

A proposal, based on the gaussian assumption, for the design of a simple system for the estimation of PSD shape parameters is made by Ciciora (5). This analog approach utilizes a two branch amplitude discriminator that is specialized to yield $f_a$ as the mean value of its output. The form of the discriminator is shown in Figure 1. $H_1(2\pi f)$ and $H_2(2\pi f)$ are chosen so that

$$|H_1(2\pi f)|^2 - |H_2(2\pi f)|^2 \sim (f - f_0)G_W(f - f_0) \qquad (1.6)$$

where

$$G_W(f) = \begin{cases} 1 & |f| < W/2 \\ -1/2 & |f| = W/2 \\ 0 & |f| > W/2. \end{cases} \qquad (1.7)$$

However, this restriction on the filters makes their

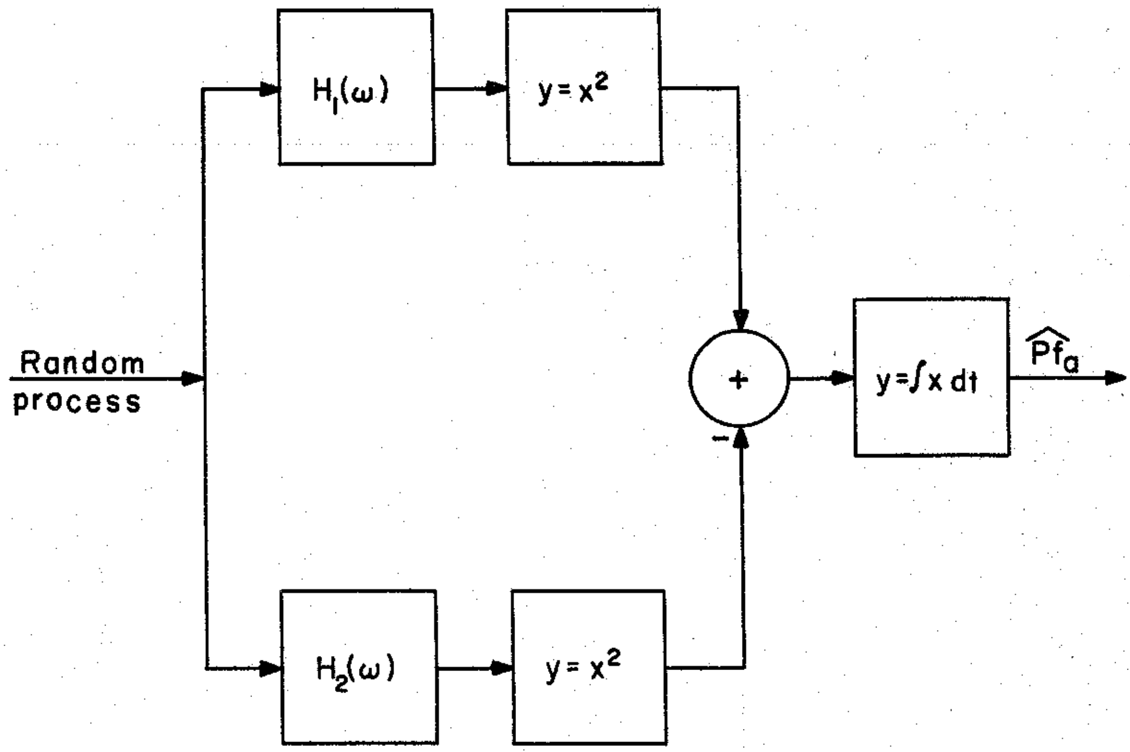


FIG. I The Specialized Discriminator

realization difficult.

A measure of the variance (or equivalently the expected error) is given for some special cases. An extension to higher moments is also obtained by a variation on the form of the filters $H_1(2\pi f)$, $H_2(2\pi f)$.

Srivastava and Carbone (10) consider the problem from a different viewpoint. Under the assumptions of a gaussian process and an exponential distribution of the power over frequency, a derivation of the joint probability density function, PDF, for the instantaneous envelope, A, and the instantaneous frequency, $\omega_1$, is presented:

$$\Psi(A^2,\omega_1) = \frac{A}{\sqrt{\pi}\ PB}\, e^{-A^2\{1/P + B^2[\omega_1 - \omega_a]^2\}} \qquad (1.8)$$

where P, the total power in the spectrum, is defined by

$$P \equiv \int_f S(f)df. \qquad (1.9)$$

On the basis of this result, the claim is made that the power mean frequency is the ensemble average of the instantaneous frequency. However, the infinite variance of $\omega_1$ brings the relevance of this result into question.

Miller and Rochwarger (7) investigate the same area. However, the presence of additive colored noise is included in their definition of the problem. The hypotheses of the derivation include the assumption of a gaussian process for the signal as well as for the noise. Some general results in the area of spectral moments are achieved that take into account the effect of a window function on the measurement and the effect of averaging over a large number of independent determinations of the estimate. These results give the estimators for the various moments as well as the variances of the power mean frequency and the bandwidth. It should be noted that this investigation is the only one which considers the whole estimator, including the normalization, with respect to the reflected signal's power.

At this writing, the latest work in this area is a study, by Serafin and Peach, of the calibration of doppler radars using meteorological echoes (9). Their results include a derivation of the power mean frequency estimator's variance when the estimator is based on the discrete Fourier transform. Here too, the assumptions of the derivation are a gaussian random process, large T, and a knowledge of the true reflected power.

The preliminary work of this investigation studies the problem of finding optimum estimators for $f_a$ and for $B^2$. It is found that the PSD, S(f), can be estimated and that the parameter estimates $\hat{f}_a$ and $\hat{B}^2$ can be calculated from this estimate of S(f). This method of deriving the PSD parameter estimates is a classical one. If $\sigma_s^2(f)$ is the mean square error of $\hat{S}(f)$, then the results are still optimum if $\hat{S}(f)$ is chosen to minimize

$$\epsilon(\hat{f}_a) \sim (1/T) \int_f f^2 \sigma_s^2(f + f_a) df$$

in the case of $\hat{f}_a$ and to minimize

$$\epsilon(\hat{B}^2) \sim (1/T) \int_f [f^2 - B^2]^2 \sigma_s^2(f + f_a) df$$

in the case of $\hat{B}^2$.

The remainder of this investigation is based on the periodogram as an estimator of S(f). The restriction of this research to the periodogram estimator is justified by considerations of mean square error. That is, the form of the periodogram's mean square error as a function of frequency results in a small mean square error in both $\hat{f}_a$ and $\hat{B}^2$. Formulas for the computation of $\hat{f}_a$ and $\hat{B}^2$ directly from the quadrature components of the complex envelope of the process are derived. These formulas are found to be the same as those suggested by Bello (3). Another form for each estimator is derived from data consisting of the instantaneous amplitude and instantaneous frequency of the process.

A statistical analysis of the parameter estimators' expected value and mean square error is presented as evidence of this approach's validity. This analysis is facilitated by the addition of the gaussian assumption to the definition (in terms of the periodogram) of each estimator.

Throughout this investigation, an attempt is made to give physical interpretations to each result. Whenever possible, the results of this investigation are compared to earlier results.