CHAPTER V

CONCLUSIONS

An analysis of the literature indicates that the study of acceptable real time processors that estimate f_a and/or B is incomplete. The classical solutions involve complicated equipment and, for a time limited measurement, are difficult to use in the case of B. Bello's estimators (3) are easy to implement. But, the lack of a statistical analysis of his results brings the value of these results into question. Therefore, this estimator requires further work. Although the system suggested by Ciciora (5) is shown to be statistically valid, it involves analog filters which are difficult to synthesize accurately. Hence, in all cases the proposed real time processor has some drawbacks.

Chapter II introduces and develops the connection between Bello's estimators, which use the complex envelope as data for $\hat{f}_a$ and $\hat{B}$, and a classical approach, which uses the periodogram as an estimator for the PSD. The importance of this result derives from the fact that it allows a statistical analysis of the classical estimators to be applied to these direct parameter estimates. A second form of each of the estimators is presented. The latter derivations utilize the instantaneous amplitude and instantaneous frequency of the process as its data. This polar form allows some additional intuitive understanding of the quantities being measured, as well as providing a second choice for the implementation of each estimator.

The statistical analysis of Chapter III proves that the direct estimators are asymptotically unbiased and have reasonable mean square errors which tend to zero with order T^{-1}. Another result of interest arises from consideration of the mean square errors:

$$\epsilon(\hat{f}_a) = (P^2T)^{-1} \int_f f^2 S_\xi^2(f + f_a)df$$

$$\epsilon(\hat{B}^2) = (P^2T)^{-1} \int_f (f^2 - B^2)^2 S_\xi^2(f + f_a)df.$$

Both of these quantities are a function of the bandwidth of $S_\xi(f)$. But, the bandwidth of $S_\xi(f)$ is

$$B_\xi^2 = B^2 + B_\rho^2$$

where B_ρ^2 is the bandwidth of the spectrum of the auxiliary window function that is employed. Therefore, the mean square error of $\hat{f}_a$ is minimized only in the case of a rectangular window function. This result for $\hat{f}_a$ is reasonable.

This investigation has demonstrated the validity of two implementations of real time estimators for each of the spectral parameters discussed. Since both implementations yield equivalent results, the choice of which implementation to use may be left to the design engineer. The statistical analysis gives a measure of the reliability of this instrumentation and can be used to determine the length of time required to make the measurement. The magnitude of the RMS error for an estimator ($\hat{f}_a$ or $\hat{B}$) is $k[B/T]^{1/2}$, where k is dependent on the estimator and on the shape of the input spectrum. In Chapter IV it is seen that in the case of $\hat{f}_a$, k ranges from 0.37 for a gaussian shaped spectrum to a worst case of 0.71 for a spectrum consisting of two δ-functions of equal weight. In the case of $\hat{B}$, k ranges from a best case of zero for two δ-functions of equal weight to 0.29 for the rectangular spectrum. It should be noted that, although no claim has been made that these estimators are optimum under the constraints given, no other estimators for these quantities have been shown to have a smaller mean square error.