Probability Theory

We are dealing here with mass phenomena and their "average" behavior. The physical system that creates the OBSERVABLE phenomena is usually assumed to be CONSISTENT and the observation {either over time (a sequence) or among a simultaneous population (a SET, an ENSEMBLE)} will have statistics that are also consistent. We need a theory that describes and predicts these statistics.

Approaches

- Physical
  - Experiment (frequency ratio, apostiori)
  - Classical Definition (# of trials large)
- Conceptual - Axiomatic
  - Axioms and Reasoning (heavy math)
- Prediction
  - Symmetry (thought experiment, apriori)
  - The Sample Space

Sample Space

- S is the collection or set of all possible outcomes of an experiment. It is the universal set for this experiment.
  - Discrete Sample Space: Finite or countably infinite set. (e.g. faces of a die, the positive integers, students in this class)
  - Continuous Sample Space: not countable. (e.g. the real line, battery voltage in a car)

The Probability Function

- Definition

  To each outcome in S we associate a non-negative number $P_n = P(x_n)$ such that :

  $0<P_n<1$ and $\text{Sum}[P(x_n)] = 1$

  – x_n is in S
  – Sum is over all elements of S
  – $\{x_n\}$ is collectively exhaustive and mutually exclusive

Truths

- P(A) is greater than or equal to 0
- P(S) = 1
- P(A + B) = P(A) + P(B) - P(A*B)
- P({}) = 0
- Mutually Exclusive Events:
  P(A + B) = P(A) + P(B)
- The three axioms of probability