Alexander-Sadiku
Fundamentals of Electric Circuits

Chapter 6
Capacitors and Inductors

Capacitors and Inductors
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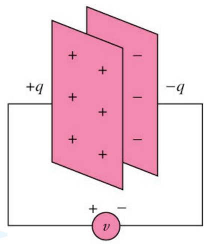
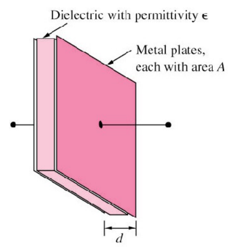
6.4 Series and Parallel Inductors

6.1 Capacitors (1)

A capacitor is a passive element designed to **store energy** in its **electric field.**

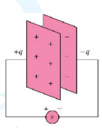




A **capacitor** consists of two conducting plates separated by an insulator (or dielectric).

6.1 Capacitors (2)

Capacitance *C* is the ratio of the charge *q* on one plate of a capacitor to the voltage difference *v* between the two plates, measured in farads (F).



$$q = C\,v \quad \text{and} \quad C = \frac{\varepsilon\, A}{d}$$

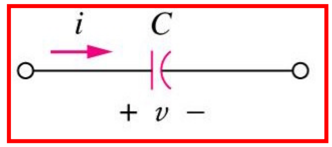
Where <u>ε</u> is the permittivity of the dielectric material between the plates, <u>A</u> is the surface area of each plate, <u>d</u> is the distance between the plates.

Unit: F, pF (10 ^{12}), nF (10 ^{9}), and **<u>μF (10 ^{6})</u>**

6.1 Capacitors (3)

If *i* is flowing into the +ve terminal of C

- Charging => *i* is +ve
- Discharging => *i* is ve



The current-voltage relationship of capacitor according to above convention is

$$i = C\frac{dv}{dt} \quad \text{and} \quad v = \frac{1}{C}\int_{t_0}^{t} i\,dt + v(t_0)$$

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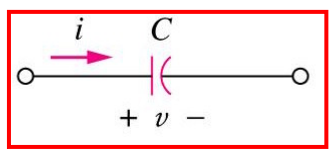
6.1 Capacitors (4)

The energy, *w*, stored in the capacitor is

$$w = \frac{1}{2}Cv^2$$



A capacitor is

- an **open circuit** to dc (dv/dt = 0).
- its voltage **cannot change abruptly.**

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6.1 Capacitors (5)

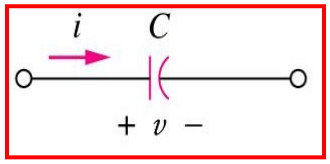
Example 1

The current through a 100-μF capacitor is

$$i(t) = 50 \sin(120\,\pi t) \text{ mA.}$$

Calculate the voltage across it at t =1 ms and t = 5 ms.

Take $v(0)$ =0.



Answer:

$v(1ms)$ = **93.14mV**

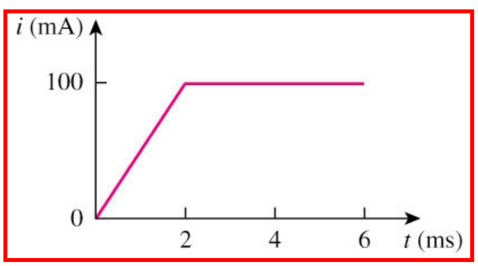
$v(5ms)$ = **1.7361V**

6.1 Capacitors (6)

Example 2

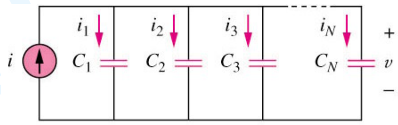
An initially uncharged 1-mF capacitor has the current shown below across it.

Calculate the voltage across it at t = 2 ms and t = 5 ms.



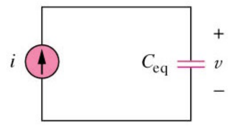
6.2 Series and Parallel Capacitors (1)

The equivalent capacitance of *N* **parallel-connected** capacitors is the sum of the individual capacitances.



(a)

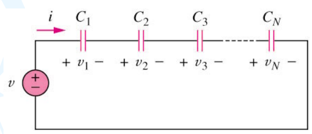
$$C_{eq} = C_1 + C_2 + ... + C_N$$



(b)

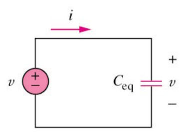
6.2 Series and Parallel Capacitors (2)

The equivalent capacitance of *N* **series-connected** capacitors is the reciprocal of the sum of the reciprocals of the individual capacitances.



(a)

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + ... + \frac{1}{C_N}$$

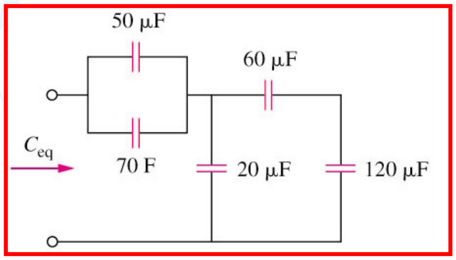


(b)

6.2 Series and Parallel Capacitors (3)

Example 3

Find the equivalent capacitance seen at the terminals of the circuit in the circuit shown below:



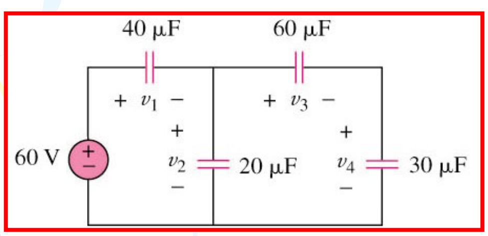
Answer:

C_{eq} = **40μF**

6.2 Series and Parallel Capacitors (4)

Example 4

Find the voltage across each of the capacitors in the circuit shown below:



Answer:

v_1 = 30V

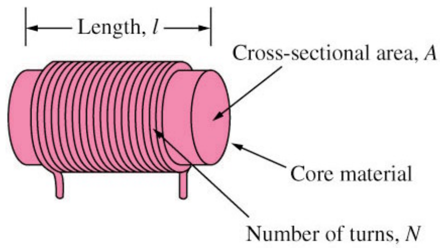
v_2 = 30V

v_3 = 10V

v_4 = 20V

6.3 Inductors (1)

An inductor is a passive element designed to store energy in its magnetic field.



An inductor consists of a coil of conducting wire.

6.3 Inductors (2)

Inductance is the property whereby an inductor exhibits opposition to the change of current flowing through it, measured in henrys (H).

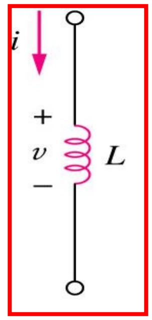
$$v = L\frac{di}{dt} \quad \text{and} \quad L = \frac{N^2 \mu A}{l}$$

The unit of inductors is Henry (H), mH (10^{3}) and μH (10^{6}).

6.3 Inductors (3)

The current-voltage relationship of an inductor:

$$i = \frac{1}{L}\int_{t_0}^{t} v(t)\,dt + i(t_0)$$

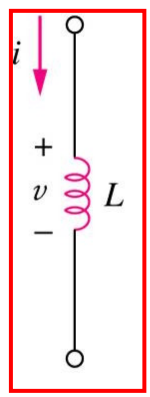


The power stored by an inductor:

$$w = \frac{1}{2} L\, i^2$$

An inductor acts like a short circuit to dc (di/dt = 0) and its current cannot change abruptly.

6.3 Inductors (4)



Example 5

The terminal voltage of a 2-H inductor is

v = 10(1-t) V

Find the current flowing through it at t = 4 s and the energy stored in it within 0 < t < 4 s.

Assume i(0) = 2 A.

Answer:

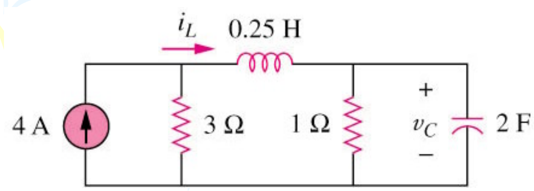
i(4s) **= -18V**

w(4s) **= 320J**

6.3 Inductors (5)

Example 6

Determine v_c, i_L, and the energy stored in the capacitor and inductor in the circuit of circuit shown below under dc conditions.



Answer:

$i_L = 3A$

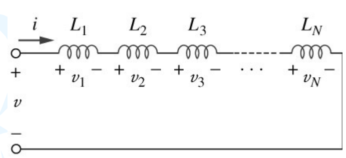
$v_C = 3V$

w_L **= 1.125J**

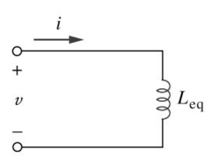
w_C **= 9J**

6.4 Series and Parallel Inductors (1)

The equivalent inductance of **series-connected** inductors is the sum of the individual inductances.



(a)

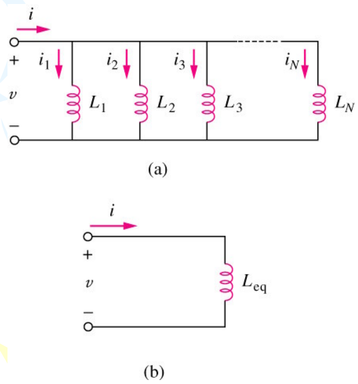


(b)

$$L_{eq} = L_1 + L_2 + ... + L_N$$

6.4 Series and Parallel Inductors (2)

The equivalent capacitance of **parallel** inductors is the reciprocal of the sum of the reciprocals of the individual inductances.

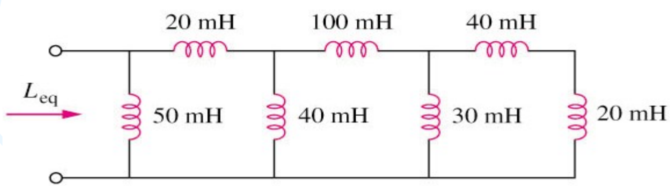


$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \ldots + \frac{1}{L_N}$$

6.4 Series and Parallel Capacitors (3)

Example 7

Calculate the equivalent inductance for the inductive ladder network in the circuit shown below:

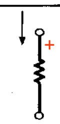
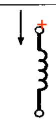
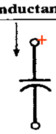


Answer:

L_{eq} = 25mH

6.4 Series and Parallel Capacitors (4)

Current and voltage relationship for R, L, C

Circuit element	Units	Voltage	Current	Power
 Resistance	ohms (Ω)	$v = Ri$ (Ohm's law)	$i = \frac{v}{R}$	$p = vi = i^2 R$
 Inductance	henries (H)	$v = L\frac{di}{dt}$	$i = \frac{1}{L}\int v\,dt + k_1$	$p = vi = Li\frac{di}{dt}$
 Capacitance	farads (F)	$v = \frac{1}{C}\int i\,dt + k_2$	$i = C\frac{dv}{dt}$	$p = vi = Cv\frac{dv}{dt}$