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Lecture 10: Power system representation and equations

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One-line (single-line) diagrams

Almost all modern power systems are three-phase systems with the phases of equal amplitude and shifted by 120° . Since phases are similar, it is customary to sketch power systems in a simple form with a single line representing all three phases of the real system.

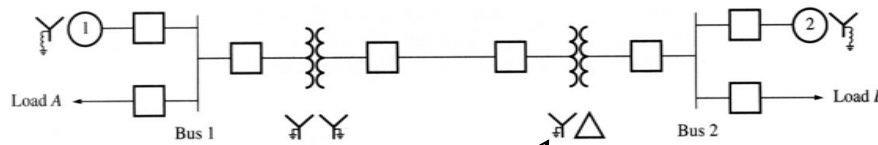
Combined with a standard set of symbols for electrical components, such one-line diagrams provide a compact way to represent information.

Rotating machine		Oil (or liquid) circuit breaker	
Two-winding power transformer		Air circuit breaker	
Three-winding power transformer		Delta connection	
Fuse		Wye connection, neutral ungrounded	
Current transformer		Wye connection, neutral grounded	
Potential transformer			



One-line (single-line) diagrams

Example 10.1: a power system containing two synchronous machines, two loads, two busses, two transformers, and a transmission line to connect busses together.



All devices are protected by oil circuit breakers (OCBs). We notice that the diagram indicates the type of connection for each machine and transformer, and also the points in the system connected to the ground.

The ground connections are important since they affect the current flowing in nonsymmetrical faults. These connection can be direct or through a resistor or inductor (they help reducing the fault current that flows in unsymmetrical faults, while having no impact on the steady-state operation of the system since the current through them will be zero). Machine ratings, impedances, and/or consumed (or supplied) powers are usually included in the diagrams.

Per-phase, per-unit equivalent circuits

As we have learned, the easiest way to analyze a balanced three-phase circuit is by a per-phase equivalent circuit with all Δ connections converted in their equivalent Y connections. The solution obtained can be extended to three phases knowing that the voltages and currents in other two phases would be the same except for the 120° phase shift.

An advantage of per-unit representation is that circuits containing transformers can be easily analyzed.

Real power systems are convenient to analyze using their **per-phase** (since the system is three-phase) **per-unit** (since there are many transformers) equivalent circuits. The per-phase base voltage, current, apparent power, and impedance are

$$I_{base} = \frac{S_{1\phi,base}}{V_{LN,base}} \quad (10.4.1)$$

$$Z_{base} = \frac{V_{LN,base}}{I_{base}} = \frac{(V_{LN,base})^2}{S_{1\phi,base}} \quad (10.4.2)$$

Per-phase, per-unit equivalent circuits

Where $V_{LN,base}$ is the line-to-neutral base voltage in the three-phase circuit (same as the base phase voltage in a Y-connected circuit) $S_{1\phi,base}$ is the base apparent power of a single phase in the circuit.

The base current and impedance in a per-unit system can also be expressed in terms of the three-phase apparent power (which is 3 times the apparent power of a single phase) and line-to-line voltages (which is $\sqrt{3}$ times the line-to-neutral voltage):

$$I_{base} = \frac{S_{3\phi,base}}{\sqrt{3}V_{LL,base}} \quad (10.5.1)$$

$$Z_{base} = \frac{V_{LL,base}}{\sqrt{3}I_{base}} = \frac{(V_{LL,base})^2}{S_{3\phi,base}} \quad (10.5.2)$$

In the per-unit system, all quantities are represented as a fraction of the base value:

$$\text{Quantity in per-unit} = \frac{\text{actual value}}{\text{base value of quantity}} \quad (10.5.3)$$



Per-phase, per-unit equivalent circuits

If any two of the four base quantities are specified, the other base values can be calculated. Usually, base apparent power and base voltage are specified at a point in the circuit, and the other values are calculated from them. The base voltage varies by the voltage ratio of each transformer in the circuit but the base apparent power stays the same through the circuit.

The per-unit impedance may be transformed from one base to another as:

$$\text{Per-unit } Z_{new} = \text{per-unit } Z_{old} \left(\frac{V_{old}}{V_{new}} \right)^2 \left(\frac{S_{new}}{S_{old}} \right) \quad (10.6.1)$$

Example 10.2: a power system consists of one synchronous generator and one synchronous motor connected by two transformers and a transmission line. Create a per-phase, per-unit equivalent circuit of this power system using a base apparent power of 100 MVA and a base line voltage of the generator G_1 of 13.8 kV. Given that:

G_1 ratings: 100 MVA, 13.8 kV, $R = 0.1$ pu, $X_s = 0.9$ pu;

T_1 ratings: 100 MVA, 13.8/110 kV, $R = 0.01$ pu, $X_s = 0.05$ pu;

T_2 ratings: 50 MVA, 120/14.4 kV, $R = 0.01$ pu, $X_s = 0.05$ pu;

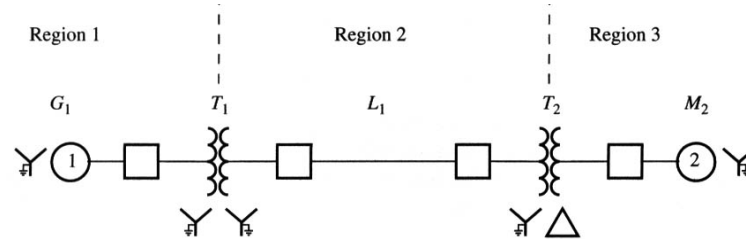
M ratings: 50 MVA, 13.8 kV, $R = 0.1$ pu, $X_s = 1.1$ pu;

L_1 impedance: $R = 15 \Omega$, $X = 75 \Omega$.



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Per-phase, per-unit equivalent circuits



To create a per-phase, per-unit equivalent circuit, we need first to calculate the impedances of each component in the power system in per-unit to the system base. The system base apparent power is $S_{base} = 100 \text{ MVA}$ everywhere in the power system. The base voltage in the three regions will vary as the voltage ratios of the transformers that delineate the regions. These base voltages are:

$$V_{base,1} = 13.8 \text{ kV} \quad \text{Region 1}$$

$$V_{base,2} = V_{base,1} \frac{110}{13.8} = 110 \text{ kV} \quad \text{Region 2 (10.7.1)}$$

$$V_{base,3} = V_{base,2} \frac{14.4}{120} = 13.2 \text{ kV} \quad \text{Region 2}$$



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Per-phase, per-unit equivalent circuits

The corresponding base impedances in each region are:

$$Z_{base,1} = \frac{V_{LL,base}^2}{S_{3\phi,base}} = \frac{(13.8 \text{ kV})^2}{100 \text{ MVA}} = 1.904 \Omega \quad \text{Region 1}$$

$$Z_{base,2} = \frac{V_{LL,base}^2}{S_{3\phi,base}} = \frac{(110 \text{ kV})^2}{100 \text{ MVA}} = 121 \Omega \quad \text{Region 1 (10.8.1)}$$

$$Z_{base,3} = \frac{V_{LL,base}^2}{S_{3\phi,base}} = \frac{(13.2 \text{ kV})^2}{100 \text{ MVA}} = 1.743 \Omega \quad \text{Region 1}$$

The impedances of G_1 and T_1 are specified in per-unit on a base of 13.8 kV and 100 MVA, which is the same as the system base in Region 1. Therefore, the per-unit resistances and reactances of these components on the system base are unchanged:

$$\begin{aligned} R_{G1,pu} &= 0.1 \text{ per unit} \\ X_{G1,pu} &= 0.9 \text{ per unit} \\ R_{T1,pu} &= 0.01 \text{ per unit} \\ X_{T1,pu} &= 0.05 \text{ per unit} \end{aligned}$$



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Per-phase, per-unit equivalent circuits

There is a transmission line in Region 2 of the power system. The impedance of the line is specified in ohms, and the base impedance in that region is 121 Ω . Therefore, the per-unit resistance and reactance of the transmission line are:

$$R_{line,system} = \frac{15}{121} = 0.124 \text{ per unit} \quad (10.9.1)$$

$$X_{line,system} = \frac{75}{121} = 0.620 \text{ per unit}$$

The impedance of T_2 is specified in per-unit on a base of 14.4 kV and 50 MVA in Region 3. Therefore, the per-unit resistances and reactances of this component on the system base are:

$$\text{per-unit } Z_{new} = \text{per-unit } Z_{given} \left(\frac{V_{given}}{V_{new}} \right)^2 \left(\frac{S_{new}}{S_{given}} \right) \quad (10.9.2)$$

$$R_{T2,pu} = 0.01(14.4/13.2)^2 (100/50) = 0.238 \text{ per unit} \quad (10.9.3)$$

$$X_{T2,pu} = 0.05(14.4/13.2)^2 (100/50) = 0.119 \text{ per unit}$$



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Per-phase, per-unit equivalent circuits

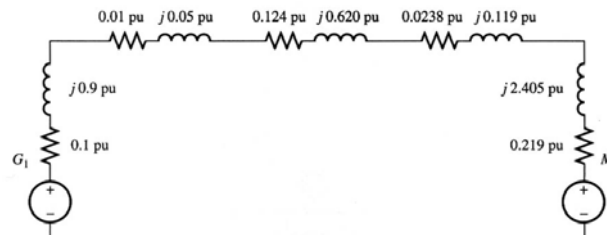
The impedance of M_2 is specified in per-unit on a base of 13.8 kV and 50 MVA in Region 3. Therefore, the per-unit resistances and reactances of this component on the system base are:

$$\text{per-unit } Z_{new} = \text{per-unit } Z_{given} \left(\frac{V_{given}}{V_{new}} \right)^2 \left(\frac{S_{new}}{S_{given}} \right) \quad (10.10.1)$$

$$R_{M2,pu} = 0.1(14.8/13.2)^2 (100/50) = 0.219 \text{ per unit}$$

$$X_{M2,pu} = 1.1(14.8/13.2)^2 (100/50) = 2.405 \text{ per unit}$$

Therefore, the per-phase, per-unit equivalent circuit of this power system is shown:

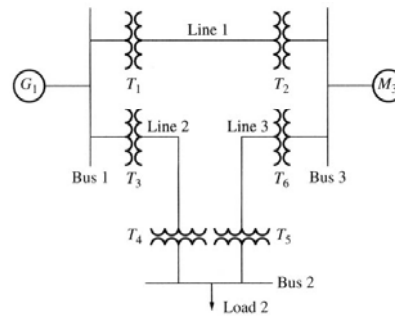


Writing node equations for equivalent circuits

Once the per-phase, per-unit equivalent circuit of a power system is created, it can be used to find the voltages, currents, and powers present at various points in a power system. The most common technique used to solve such circuits is nodal analysis.

In nodal analysis, we use Kirchhoff's current law equations to determine the voltages at each node (each bus) in the power system, and then use the resulting voltages to calculate the currents and power flows at various points in the system.

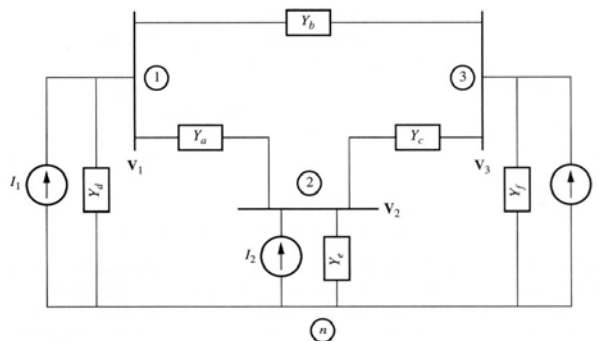
A simple three-phase power system with three busses connected by three transmission lines. The system also includes a generator connected to bus 1, a load connected to bus 2, and a motor connected to bus 3.



Writing node equations for equivalent circuits

The per-phase, per-unit equivalent circuit of this power system:

The busses are labeled as nodes (1), (2), and (3), while the neutral is labeled as node (n).



Note that the per-unit series impedances of the transformers and the transmission lines between each pair of busses have been added up, and the resulting impedances were expressed as admittances ($Y=1/Z$) to simplify nodal analysis.

Shunt admittance at each bus is shown between the bus and the neutral. This admittance can include the shunt admittance of the line models and shunt admittance associated with any generators or loads on a bus.

Writing node equations for equivalent circuits

The voltages between each bus and neutral are represented by single subscripts (V_1, V_2) in the equivalent circuit, while the voltages between any two busses are indicated by double subscripts (V_{12}).

The generators and loads are represented by current sources injecting currents into the specific nodes. Conventionally, current sources always flow into a node meaning that the power flow of generators will be positive, while the power flow for motors will be negative.

According to Kirchhoff's current flow law (KCL), **the sum of all currents entering any node equals to the sum of all currents leaving the node**. KCL can be used to establish and solve a system of simultaneous equations with the unknown node voltages.

Assuming that the current from the current sources are entering each node, and that all other currents are leaving the node, applying the KCL to the node (1) yields:

$$(V_1 - V_2)Y_a + (V_1 - V_3)Y_b + V_1Y_d = I_1 \quad (10.13.1)$$



Writing node equations for equivalent circuits

Similarly, for the nodes (2) and (3):

$$(V_2 - V_1)Y_a + (V_2 - V_3)Y_c + V_2Y_e = I_2 \quad (10.14.1)$$

$$(V_3 - V_1)Y_b + (V_3 - V_2)Y_c + V_3Y_f = I_3 \quad (10.14.2)$$

Rearranging these equations, we arrive at:

$$\begin{aligned} (Y_a + Y_b + Y_d)V_1 - Y_aV_2 - Y_bV_3 &= I_1 \\ -Y_aV_1 + (Y_a + Y_c + Y_e)V_2 - Y_cV_3 &= I_2 \end{aligned} \quad (10.14.3)$$

$$-Y_bV_1 - Y_cV_2 + (Y_b + Y_c + Y_f)V_3 = I_3$$

In matrix form:

$$\begin{bmatrix} Y_a + Y_b + Y_d & -Y_a & -Y_b \\ -Y_a & Y_a + Y_c + Y_e & -Y_c \\ -Y_b & -Y_c & Y_b + Y_c + Y_f \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \end{bmatrix} = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} \quad (10.14.4)$$



Writing node equations for equivalent circuits

Which is an equation of the form:

$$Y_{bus} V = I \quad (10.15.1)$$

where Y_{bus} is the bus admittance matrix of a system, which has the form:

$$Y_{bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} \\ Y_{21} & Y_{22} & Y_{23} \\ Y_{31} & Y_{32} & Y_{33} \end{bmatrix} \quad (10.15.2)$$

Y_{bus} has a regular form that is easy to calculate:

- 1) The diagonal elements Y_{ii} equal the sum of all admittances connected to node i .
- 2) Other elements Y_{ij} equal to the negative admittances connected to nodes i and j .

The diagonal elements of Y_{bus} are called the self-admittance or driving-point admittances of the nodes; the off-diagonal elements are called the mutual admittances or transfer admittances of the nodes.



Writing node equations for equivalent circuits

Inverting the bus admittance matrix Y_{bus} yields the bus impedance matrix:

$$Z_{bus} = Y_{bus}^{-1} \quad (10.16.1)$$

Simple technique for constructing Y_{bus} is only applicable for components that are not mutually coupled. The technique applicable to mutually coupled components can be found elsewhere.

Once Y_{bus} is calculated, the solution to (10.15.1) is

$$V = Y_{bus}^{-1} I \quad (10.16.2)$$

or

$$V = Z_{bus} I \quad (10.16.3)$$



Solving power system node equation with MATLAB™

A number of techniques can be used to solve systems of simultaneous linear equations, such as substitution, Gaussian elimination, LU factorization, etc. MATLAB has built-in system solvers that can be used efficiently.

A system of n linear equations in n unknowns

$$Ax = b \quad (10.17.1)$$

where A is an $n \times n$ matrix and b is an n -element column vector; the solution will be

$$x = A^{-1}b \quad (10.17.2)$$

where A^{-1} is the $n \times n$ matrix inverse of A .

Using MATLAB, the solution to (10.17.1) can be evaluated, for instance, by direct evaluation of inverse as in (10.17.2), or via the left division (`\`).



Solving power system node equation with MATLAB™

For example, the system

$$\begin{aligned} 1.0x_1 + 0.5x_2 - 0.5x_3 &= 1.0 \\ 0.5x_1 + 1.0x_2 + 0.25x_3 &= 2.0 \\ -0.5x_1 + 0.25x_2 + 1.0x_3 &= 1.0 \end{aligned}$$

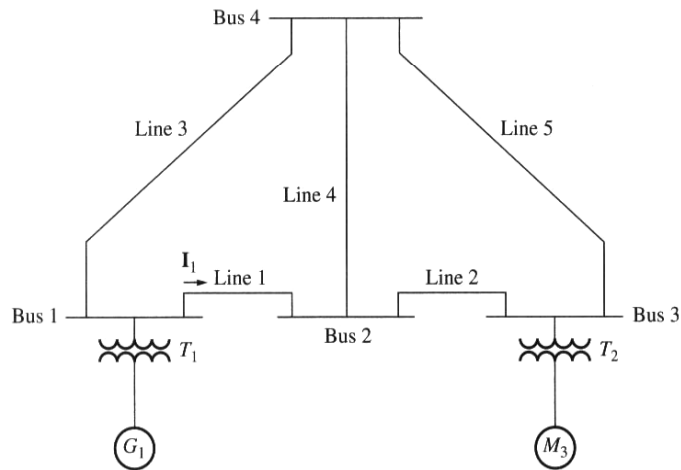
Can be solved by the following MATLAB code:

```
>> A = [1, 0.5, -0.5; 0.5, 1, 0.25; -0.5, 0.25, 1];
>> b = [1; 2; 1];
>> x = inv(A) * b;
or
>> x = A\b;
```



Solving power system node equation with MATLAB™

Example 10.3: a power system consists of four busses interconnected by five transmission lines. It includes one generator attached to bus 1 and one synchronous motor connected to bus 3.

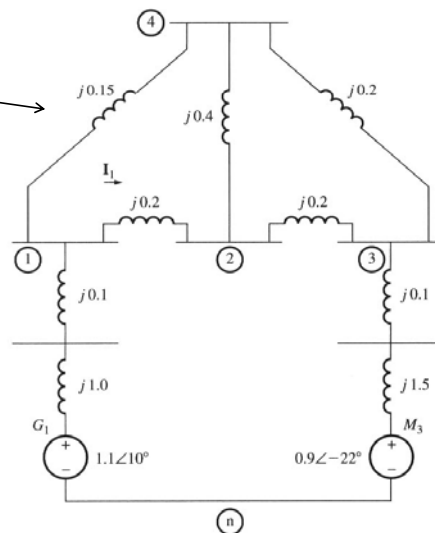


Solving power system node equation with MATLAB™

The per-phase, per-unit equivalent circuit is shown.

We observe that all impedances are considered as pure reactances to simplify the case since reactance is much larger than resistance in typical transformers, synchronous machines, and overhead transmission lines.

Find the per-unit voltage at each bus in the power system and the per-unit current flow in line 1.

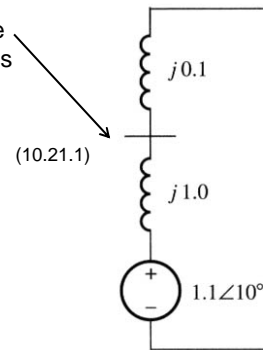
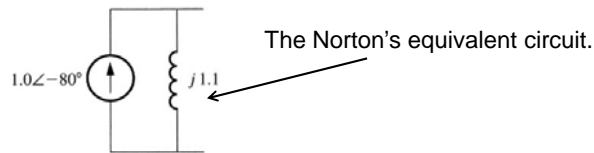


Solving power system node equation with MATLAB™

The first step in solving for bus voltages is to convert the voltage sources into the equivalent current sources by using the Norton's theorem. Next, we need to convert all of the impedance values into admittances and form the admittance matrix Y_{bus} then use it to solve for the bus voltages, and finally use voltages on buses 1 and 2 to find the current in line 1.

First, we need to find the Norton equivalent circuits for the combination of G_1 and T_1 . The Thevenin impedance of this combination is $Z_{TH} = j1.1$, and the short-circuit current is

$$I_{sc} = \frac{V_{oc}}{Z_{TH}} = \frac{1.1 \angle 10^\circ}{j1.1} = 1.0 \angle -80^\circ \quad (10.21.1)$$



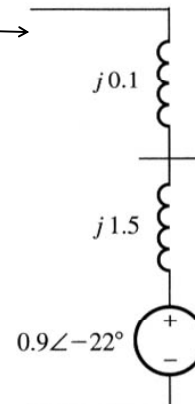
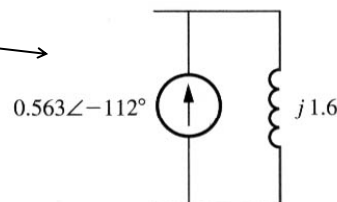
Solving power system node equation with MATLAB™

The combination of M_3 and T_2 is shown.

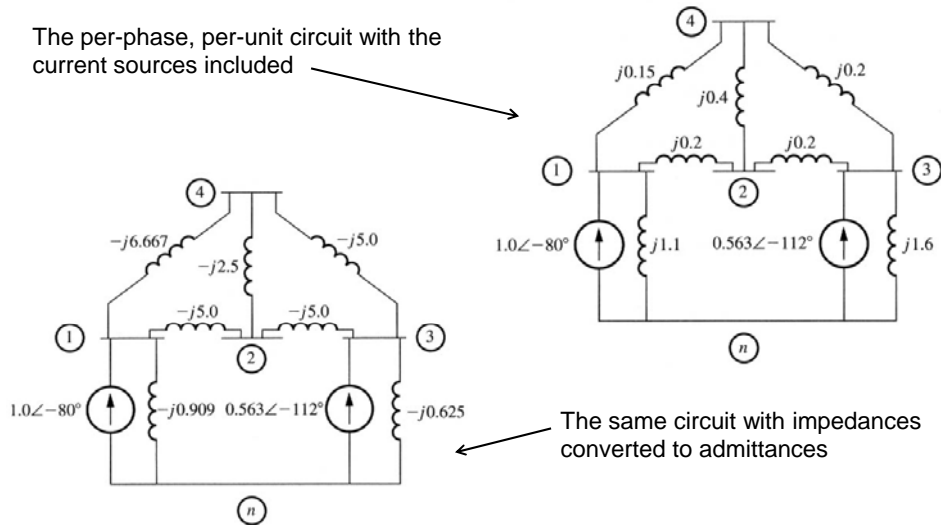
The Thevenin impedance of this combination is $Z_{TH} = j1.6$, and the short-circuit current is

$$I_{sc} = \frac{V_{oc}}{Z_{TH}} = \frac{0.9 \angle -22^\circ}{j1.6} = 0.563 \angle -112^\circ \quad (10.22.1)$$

The Norton's equivalent circuit.



Solving power system node equation with MATLAB™



Solving power system node equation with MATLAB™

The resulting admittance matrix is:

$$Y_{bus} = \begin{bmatrix} -j12.576 & j5.0 & 0 & j6.667 \\ j5.0 & -j12.5 & j5.0 & j2.5 \\ 0 & j5.0 & -10.625 & j5.0 \\ j6.667 & j2.5 & j5.0 & -j14.167 \end{bmatrix} \quad (10.24.1)$$

The current vector for this circuit is:

$$I = \begin{bmatrix} 1.0\angle -80^\circ \\ 0 \\ 0.563\angle -112^\circ \\ 0 \end{bmatrix} \quad (10.24.2)$$

Solving power system node equation with MATLAB™

The solution to the system of equations will be

$$V = Y_{bus}^{-1}I = \begin{bmatrix} 0.989\angle -0.60^\circ \\ 0.981\angle -1.58^\circ \\ 0.974\angle -2.62^\circ \\ 0.982\angle -1.48^\circ \end{bmatrix} V \quad (10.25.1)$$

The current in line 1 can be calculated from the equation:

$$\begin{aligned} I_1 &= (V_1 - V_2)Y_{line1} = (0.989\angle -0.60^\circ - 0.981\angle -1.58^\circ) \cdot (-j5.0) \\ &= 0.092\angle -25.16^\circ \end{aligned} \quad (10.25.2)$$