RESISTIVE CIRCUITS

Here we introduce the basic concepts and laws that are fundamental to circuit analysis

LEARNING GOALS

- OHM'S LAW - DEFINES THE SIMPLEST PASSIVE ELEMENT: THE RESISTOR

- KIRCHHOFF'S LAWS - THE FUNDAMENTAL CIRCUIT CONSERVATION LAWS- KIRCHHOFF CURRENT (KCL) AND KIRCHHOFF VOLTAGE (KVL)

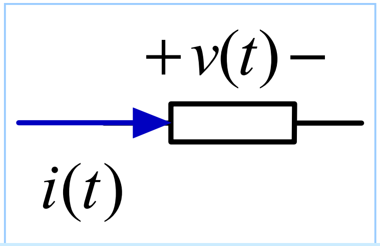
- LEARN TO ANALYZE THE SIMPLEST CIRCUITS
- SINGLE LOOP - THE VOLTAGE DIVIDER
- SINGLE NODE-PAIR - THE CURRENT DIVIDER

- SERIES/PARALLEL RESISTOR COMBINATIONS - A TECHNIQUE TO REDUCE THE COMPLEXITY OF SOME CIRCUITS

- WYE - DELTA TRANSFORMATION - A TECHNIQUE TO REDUCE COMMON RESISTOR CONNECTIONS THAT ARE NEITHER SERIES NOR PARALLEL

- CIRCUITS WITH DEPENDENT SOURCES - (NOTHING VERY SPECIAL)

RESISTORS



A resistor is a passive element characterized by an algebraic relation between the voltage across its terminals and the current through it

$v(t) = F(i(t))$ General Model for a Resistor

A linear resistor obeys OHM's Law

$$v(t) = Ri(t)$$

The constant, R, is called the resistance of the component and is measured in units of Ohm (Ω)

From a dimensional point of view Ohms is a derived unit of Volt/Amp

Since the equation is algebraic the time dependence can be omitted

Standard Multiples of Ohm

$M\Omega$ Mega Ohm$(10^6\,\Omega)$

$k\Omega$ Kilo Ohm$(10^3\,\Omega)$

A common occurrence is $\frac{\text{Volt}}{\text{mA}}$ resulting in resistance in kΩ

Conductance

If instead of expressing voltage as a function of current one expresses current in terms of voltage, OHM's law can be written

$$i = \frac{1}{R}v$$

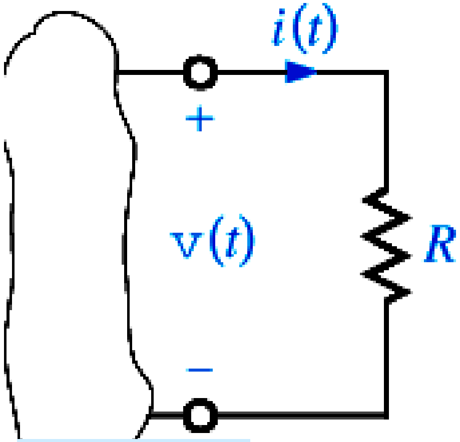
We define $G = \frac{1}{R}$ as Conductance of the component and write

$$i = Gv$$

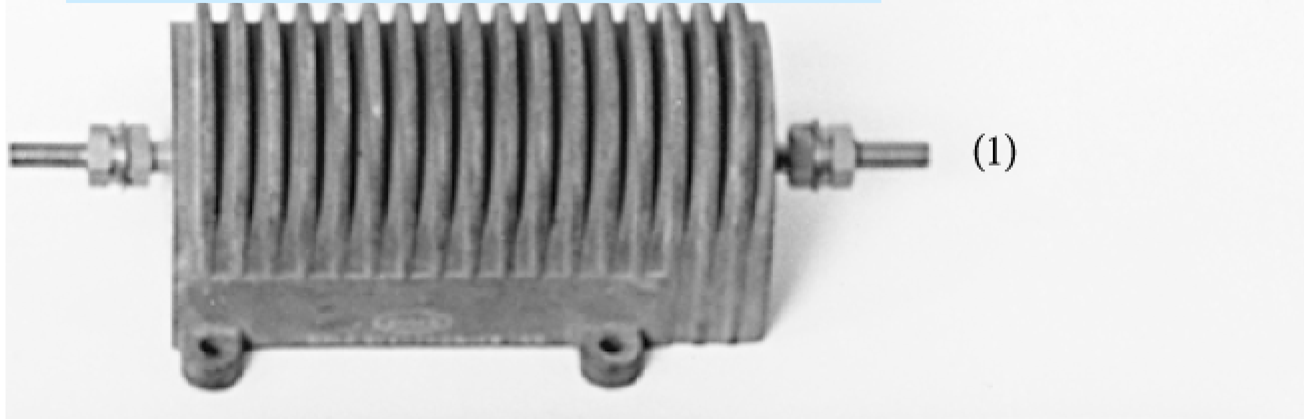
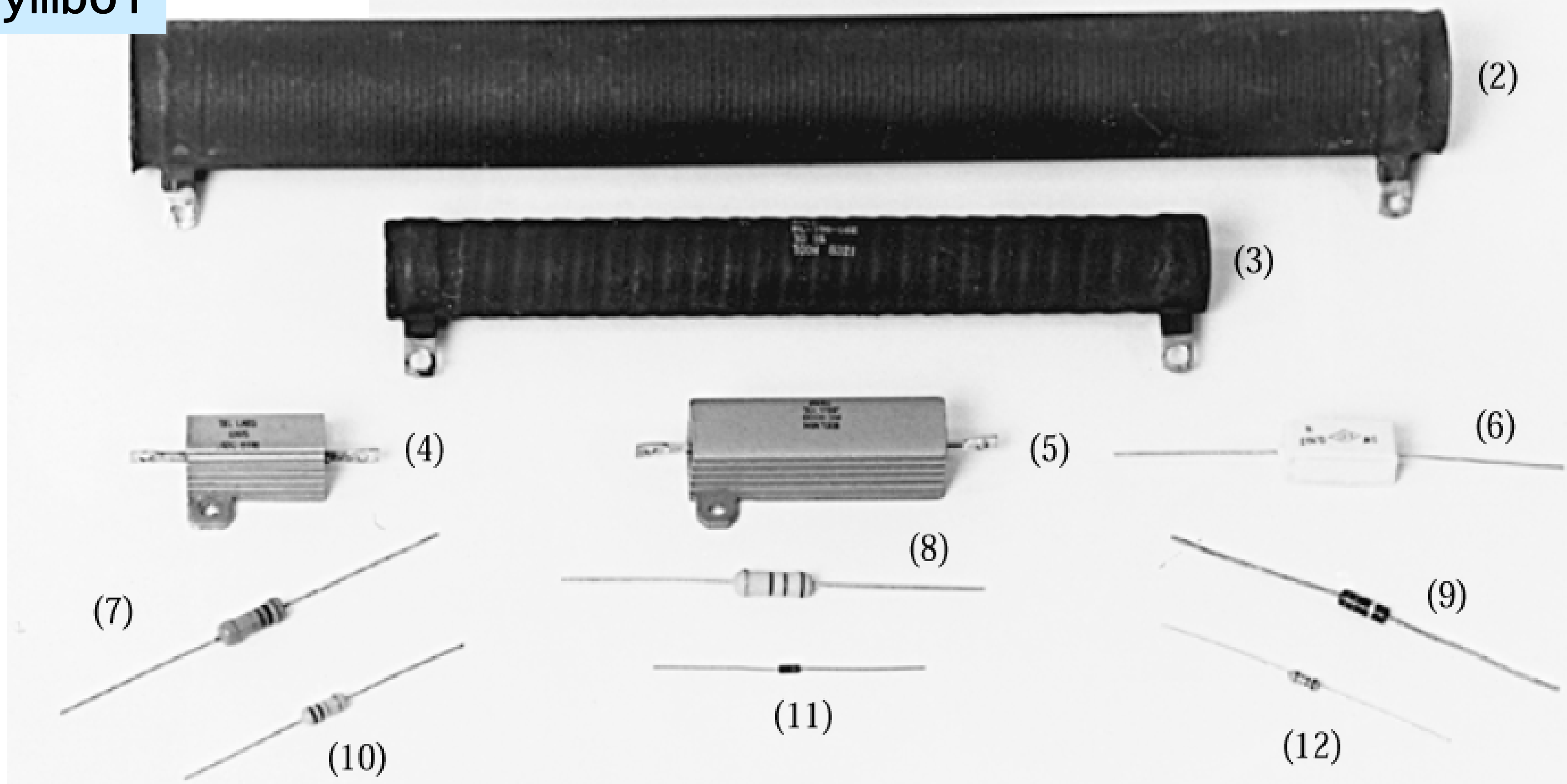
The unit of conductance is Siemens

GEAUX

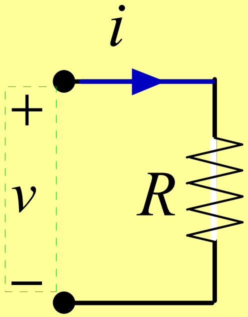
Some practical resistors



Symbol

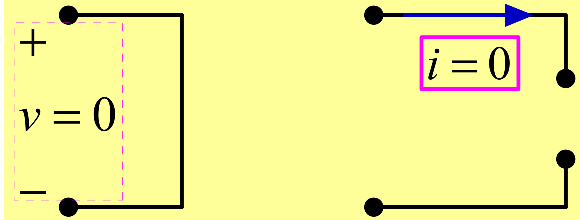
GEAUX



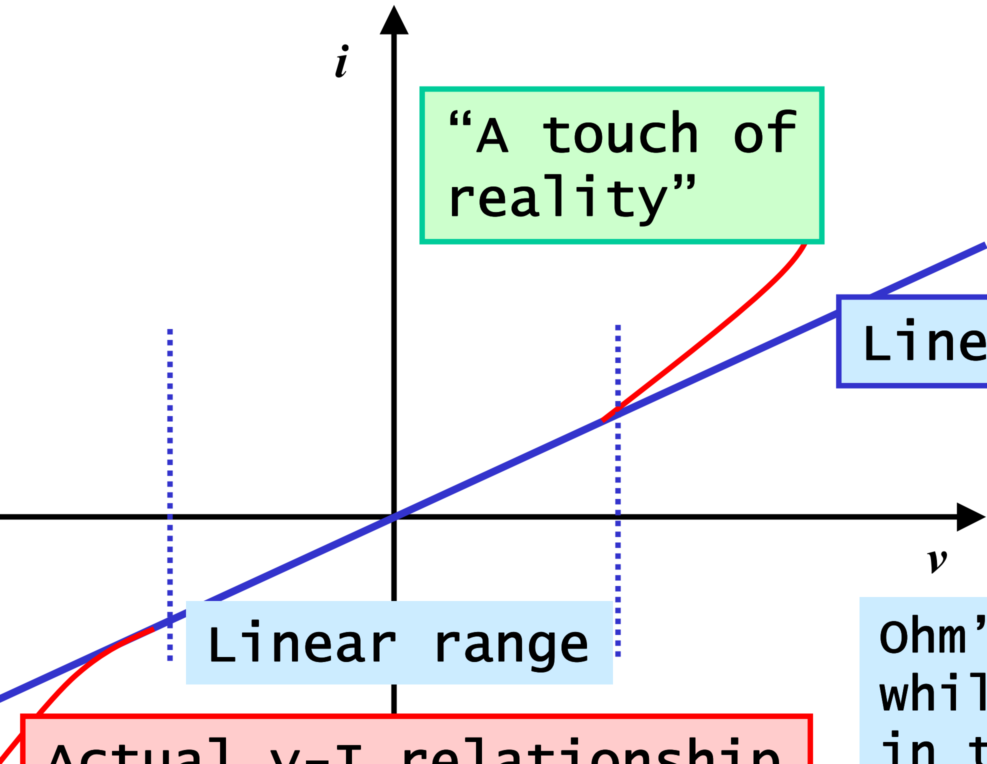
Circuit Representation

Notice passive sign convention

Two special resistor values



Short Circuit	Open Circuit
$R = 0$	$R = \infty$
$G = \infty$	$G = 0$



Ohm's Law is an approximation valid while voltages and currents remain in the Linear Range

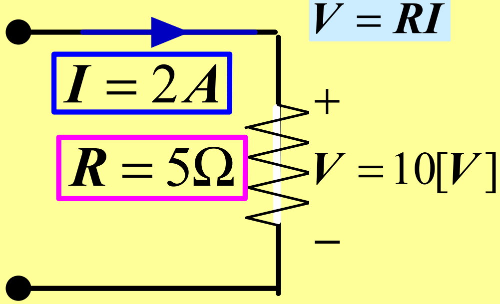
OHM'S LAW PROBLEM SOLVING TIP

$v = Ri$ $\qquad i = Gv$ OHM's Law

One equation and three variables.
Given ANY two the third can be found

Given current and resistance
Find the voltage

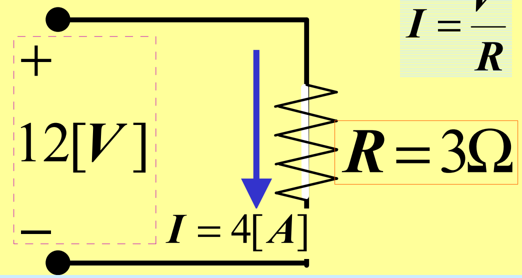
$V = RI$



Notice use of passive sign convention

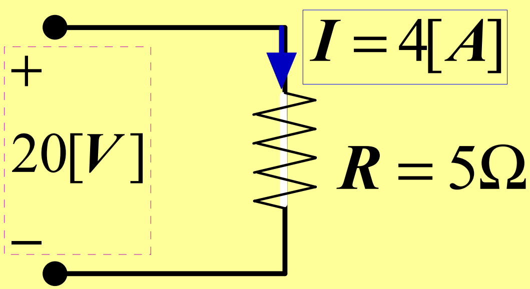
Given Voltage and Resistance
Compute Current

$I = \frac{V}{R}$



Determine direction of the current using passive sign convention

Given Current and Voltage
Find Resistance

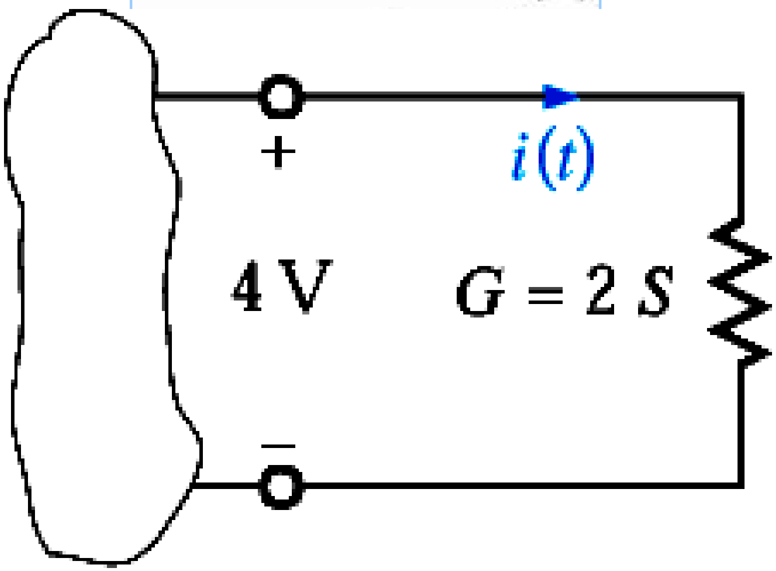


$R = \frac{V}{I}$

Table 1 Keeping Units Straight

Voltage	Current	Resistance
Volts	Amps	Ohms
Volts	mA	$k\Omega$
mV	A	$m\Omega$
mV	mA	Ω

Determine $i(t)$



GIVEN VOLTAGE AND CONDUCTANCE

REFERENCE DIRECTIONS SATISFY
PASSIVE SIGN CONVENTION

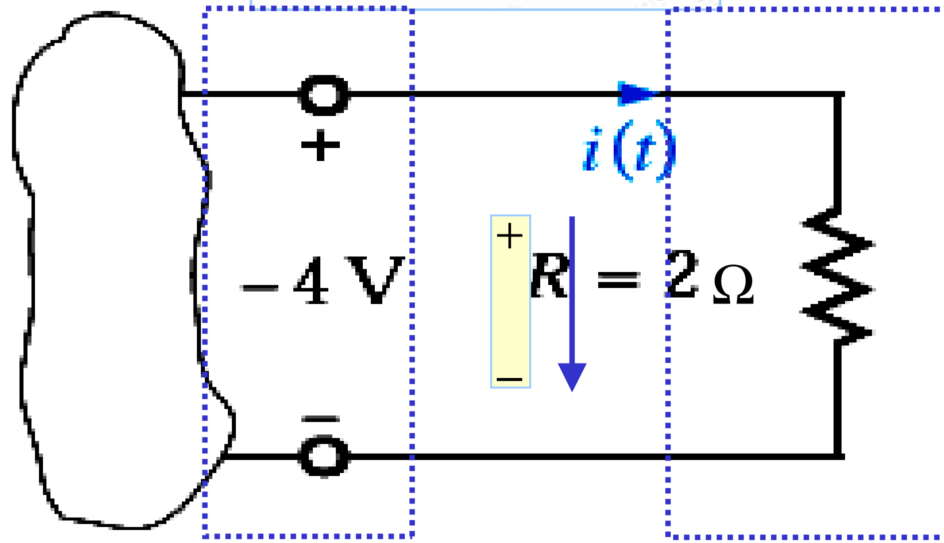
$i(t) = Gv(t)$ OHM'S LAW

UNITS?

CONDUCTANCE IN SIEMENS, VOLTAGE
IN VOLTS. HENCE CURRENT IN AMPERES

$i(t) = 8[A]$

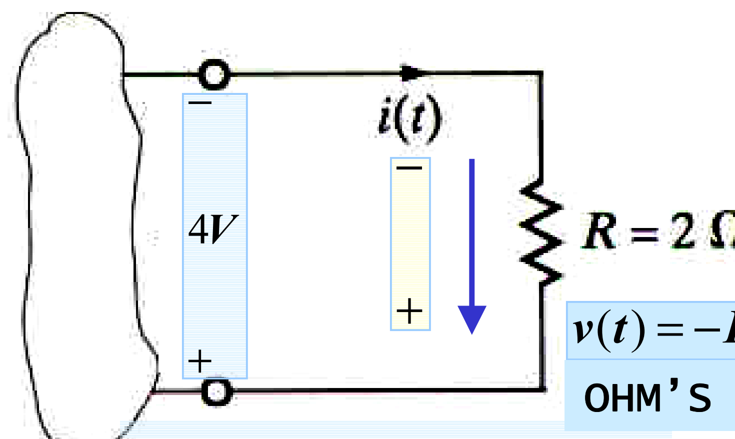
Determine $i(t)$



OHM'S LAW $v(t) = Ri(t)$

UNITS?

$-4[V] = (2\Omega)i(t) \Rightarrow i(t) = -2[A]$



$v(t) = -Ri(t)$

OHM'S LAW

THE EXAMPLE COULD BE
GIVEN LIKE THIS

GEAUX

RESISTORS AND ELECTRIC POWER

Resistors are passive components that can only absorb energy. Combining Ohm's law and the expressions for power we can derive several useful expressions

$P = vi$ (Power)

$v = Ri$, or $i = Gv$ (Ohm's Law)

<u>Problem solving tip</u>: There are four variables (P,v,i,R) and two equations. Given any two variables one can find the other two.

Given P, i

$$v = \frac{P}{i}, R = \frac{v}{i}$$

Given v, R

$$i = \frac{v}{R}, P = vi = \frac{v^2}{R}$$

Given i, R

$$v = Ri, P = vi = Ri^2$$

Given P, R

$$i = \sqrt{\frac{P}{R}}, v = Ri = \sqrt{PR}$$

If not given, the reference direction for voltage or current can be chosen and the other is given by the passive sign convention

A MATTER OF UNITS

Working with SI units Volt, Ampere Watt, Ohm, there is never a problem. One must be careful when using multiples or sub multiples.

EXAMPLE: $R = 40\ k\Omega, i = 2mA$

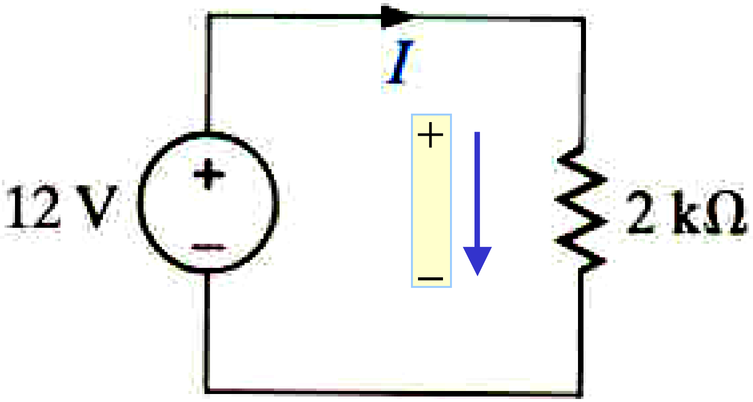
The basic strategy is to express all given variables in SI units

$$v = (40 * 10^3 \Omega) * (2 * 10^{-3} A) = 80[V]$$

$$P = Ri^2 = (40 * 10^3 \Omega) * (2 * 10^{-3} A)^2 = 160 * 10^{-3} [W]$$

GEAUX

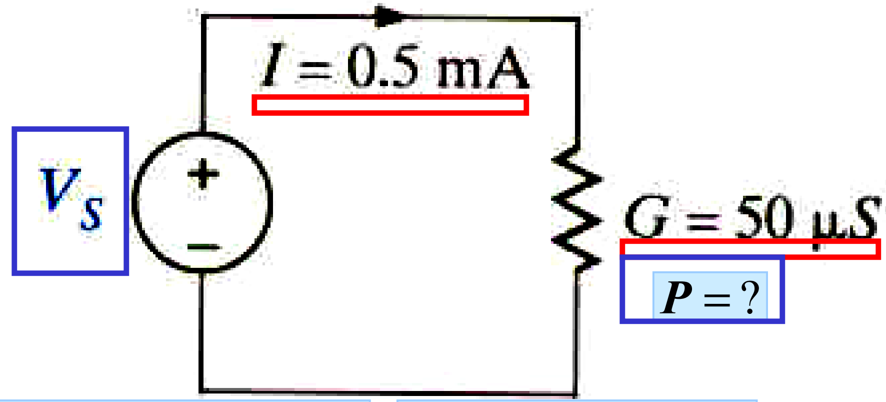
DETERMINE CURRENT AND POWER ABSORBED BY RESISTOR



$I = V/R = 12/2k = 6mA$

$P = VI = I^2R = \frac{V^2}{R}$

$P = (12[V])(6[mA]) = 72[mW]$



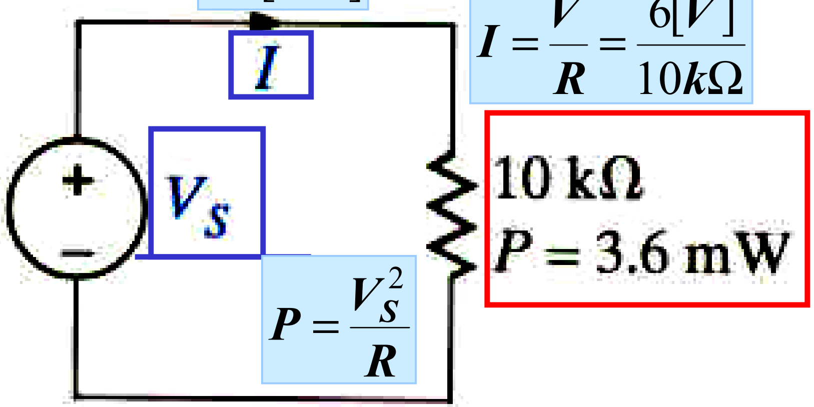
$V_S = IR \Rightarrow V_S = \frac{I}{G}$

$V_S = \frac{0.5\times10^{-3}[A]}{50\times10^{-6}[S]} = 10[V]$

$P = I^2R = \frac{I^2}{G}$

$P = \frac{(0.5\times10^{-3}[A])^2}{50\times10^{-6}[S]} = 0.5\times10^{-2}[W]$

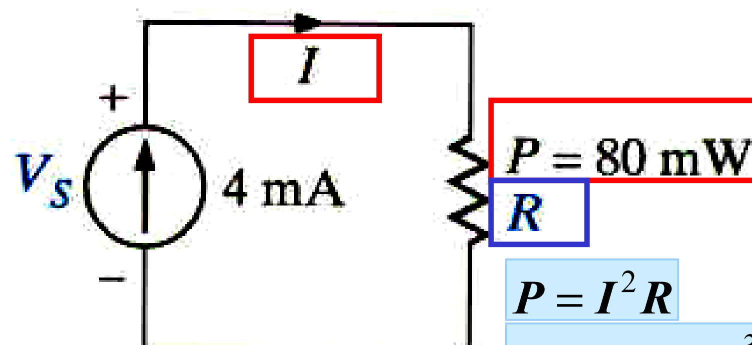
$5[mW]$



$I = \frac{V}{R} = \frac{6[V]}{10k\Omega}$

$P = \frac{V_S^2}{R}$

$V_S^2 = (10\times10^3\Omega)(3.6\times10^{-3}W)$ $V_S = 6[V]$



$P = I^2R$

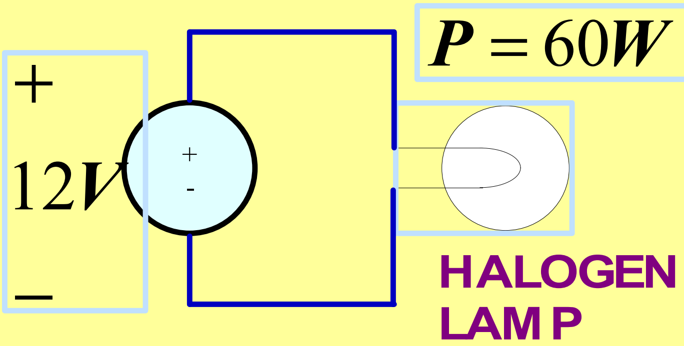
$R = \frac{80\times10^{-3}[W]}{(4\times10^{-3}A)^2}$

$R = 5k\Omega$

$P = V_S I$

$V_S = \frac{80[mW]}{4[mA]} = 5[V]$

GEAUX



Resistance of Lamp: R = V/I = 2.4 Ohms

Current through Lamp: I = P/V = 5A

$q = \int current$

Charge supplied by battery in 1min: Q=5*60[C]

SAMPLE PROBLEM

Recognizing the type of problem:
This is an application of Ohm's Law
We are given Power and Voltage.
We are asked for Resistance, Current and Charge

Possibly useful relationships

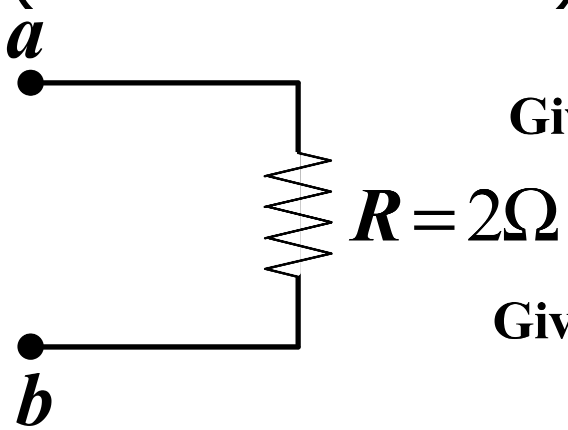
$$P = VI = \frac{V^2}{R} = I^2 R$$

$$V = IR$$

GEAUX

$q_a(t) = 10\cos(t)[mC]$ is the charge entering at a

(time is in seconds)



Given: charge. Required: current

$i_{ab}(1) =$

$i = \frac{dq}{dt} = -10\sin(t)[mA]$

$i(1) = -10\sin(1)$

Given: current. Required: voltage

$v_{ab}(t = \pi) =$

$V = Ri \quad -2 * 10\sin\pi = 0$

Given: current, resistor, voltage. Required: power

$p_R(t) =$

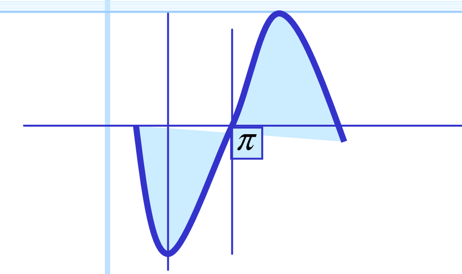
$p = Ri^2 = 2[\Omega] * (10^{-2})^2 * \sin^2(t)[A]^2$

$p = 200\sin^2(t)\mu W$

For $\frac{\pi}{2} \le t \le \pi$,

the current flows from___ to ___

point ___ has higher voltage than point ____



Sketch for -sin(t)

On the time interval, current from a to b is negative.

Current flows from b to a and point b has the higher voltage

SAMPLE QUESTION

STOP