

**13.35** Solve the following differential equations using Laplace transforms.

(a)  $\frac{dx(t)}{dt} + 4x(t) = e^{-2t}, \quad x(0) = 1$

(b)  $\frac{dx(t)}{dt} + 6x(t) = 4u(t), \quad x(0) = 2$

SOLUTION:

a)  $\frac{dx(t)}{dt} \Rightarrow sX(s) - x(0)$

so,  $sX(s) - 1 + 4X(s) = \frac{1}{s+2} \Rightarrow X(s)[s+4] = \frac{1}{s+2} + 1 = \frac{s+3}{s+2}$

$$X(s) = \frac{s+3}{(s+2)(s+4)} = \frac{K_1}{s+2} + \frac{K_2}{s+4} = \frac{1/2}{s+2} + \frac{1/2}{s+4}$$

$$x(t) = \frac{1}{2} [e^{-2t} + e^{-4t}] u(t)$$

b)  $sX(s) - x(0) + 6X(s) = \frac{4}{s} \Rightarrow X(s)[s+6] = \frac{4}{s} + 2 = \frac{(s+2)2}{s}$

$$X(s) = \frac{2(s+2)}{s(s+6)} = \frac{K_1}{s} + \frac{K_2}{s+6} = \frac{2/3}{s} + \frac{4/3}{s+6}$$

$$x(t) = \frac{2}{3} [1 + 2e^{-6t}] u(t)$$