Chapter 3

3.1 $X_a(j\Omega)=\int_{-\infty}^{\infty} x_a(t)e^{-j\Omega t}dt$. Now, $\int_{-\infty}^{\infty}|x_a(t)|dt<\infty$. Hence,

$$|X_a(j\Omega)|=\left|\int_{-\infty}^{\infty} x_a(t)e^{-j\Omega t}dt\right|\le \int_{-\infty}^{\infty}\left|x_a(t)e^{-j\Omega t}\right|dt \quad \le \int_{-\infty}^{\infty}|x_a(t)|dt<\infty.$$

3.2 **(a)** $Y_a(j\Omega)=\int_{-\infty}^{\infty}\cos(\Omega_o t)e^{-j\Omega t}dt=\frac{1}{2}\int_{-\infty}^{-\infty}\left(e^{j\Omega_o t}+e^{-j\Omega_o t}\right)e^{-j\Omega t}dt$

$$=\frac{1}{2}\left(\delta(\Omega-\Omega_o)+\delta(\Omega+\Omega_o)\right).$$

(b) $U_a(j\Omega)=\int_{-\infty}^{\infty}e^{-\alpha|t|}e^{-j\Omega t}dt=\int_{-\infty}^{0}e^{\alpha t}e^{-j\Omega t}dt+\int_{0}^{\infty}e^{-\alpha t}e^{-j\Omega t}dt$

$$=\frac{1}{\alpha-j\Omega}\cdot\left[e^{(\alpha-j\Omega)t}\right]_{-\infty}^{0}-\frac{1}{\alpha+j\Omega}\cdot\left[e^{-(\alpha+j\Omega)t}\right]_{-\infty}^{0}=\frac{1}{\alpha-j\Omega}+\frac{1}{\alpha+j\Omega}=\frac{2\alpha}{\alpha^2+\Omega^2}.$$

(c) $V_a(j\Omega)=\int_{-\infty}^{\infty}e^{j\Omega_o t}e^{-j\Omega t}dt=\int_{-\infty}^{\infty}e^{-j(\Omega-\Omega_o)t}dt=\delta(\Omega-\Omega_o).$

(d) $P_a(j\Omega)=\int_{-\infty}^{\infty}\left(\sum_{\ell=-\infty}^{\infty}\delta(t-\ell T)\right)e^{-j\Omega t}dt=\sum_{\ell=-\infty}^{\infty}\left(\int_{-\infty}^{\infty}\delta(t-\ell T)e^{-j\Omega t}dt\right)$ using the linearity property of the CTFT. Next, using the shifting property of the CTFT we get $P_a(j\Omega)=\sum_{\ell=-\infty}^{\infty}e^{-j\Omega\ell T}$ which can be alternately expressed in the form

$P_a(j\Omega)=\frac{2\pi}{T}\sum_{\ell=-\infty}^{\infty}\delta(\Omega-\frac{2\pi\ell}{T})$ making use of the results of Problem 3.2(c).

3.3 **(a)** $V_a(j\Omega)=\int_{-\infty}^{\infty}e^{-j\Omega t}dt=\delta(\Omega).$

(b) $\mu(j\Omega)=\int_{-\infty}^{\infty}\mu(t)e^{-j\Omega t}dt=\int_{-\infty}^{\infty}\left(\int_{-\infty}^{t}\delta(\tau)d\tau\right)e^{-j\Omega t}dt=\frac{1}{j\Omega}+\pi\delta(\Omega).$

(c) The function $x_a(t)$ is also denoted by $\text{rect}(t)$. Thus, $X_a(j\Omega)=\int_{-\infty}^{\infty}x_a(t)e^{-j\Omega t}dt$

$$=\int_{-1/2}^{1/2}e^{-j\Omega t}dt=-\frac{1}{j\Omega}\left(e^{-j\Omega/2}-e^{j\Omega/2}\right)=\frac{2\sin(\Omega/2)}{\Omega}=\frac{\sin(\Omega/2)}{\Omega/2}=\text{sinc}(\Omega/2).$$

(d) $y_a(t) = \begin{cases} 1-2|t|, & |t| < 0.5, \\ 0, & |t| \geq 0.5. \end{cases}$ A more convenient way to determine the CTFT of $y_a(t)$ is to differentiate it twice with respect to t, determine the CTFT of $\frac{d^2 y_a(t)}{dt^2}$ and then make use of the time-differentiation property given in Problem 3.6(e) and time-shifting property given in Problen 3.6(a). Now, $\frac{dy_a(t)}{dt} = \begin{cases} 2, & |t| < 0.5, \\ -2, & |t| \geq 0.5. \end{cases}$ As $\frac{dy_a(t)}{dt}$ has jump discontinuities with a positive jump of value 2 at $t = \pm 0.5,$ a negative jump of value -4 at $t = 0,$ and zero everywhere else, $\frac{d^2 y_a(t)}{dt^2}$ has only impulses of strength 2 at $t = \pm 0.5,$ and an impulse of strength -4 at $t = 0,$ i.e., $\frac{d^2 y_a(t)}{dt^2} = 2\delta(t+0.5) - 4\delta(t) + 2\delta(t-0.5).$ If $Y_a(j\Omega)$ denotes the CTFT of $y_a(t)$, then using of the time-differentiation property we have $\frac{d^2 y_a(t)}{dt^2} \overset{\text{CTFT}}{\leftrightarrow} (j\Omega)^2 Y_a(j\Omega).$ Using the time-shifting property, we arrive at the CTFT of $\frac{d^2 y_a(t)}{dt^2}$ given by $2e^{j\Omega/2} - 4 + 2e^{-j\Omega/2}.$ Therefore $(j\Omega)^2 Y_a(j\Omega)$ = $-\Omega^2 Y_a(j\Omega) = 2e^{j\Omega/2} - 4 + 2e^{-j\Omega/2} = 4\left(\cos(\Omega/2) - 1\right),$ i.e,

$$Y_a(j\Omega) = -\frac{4}{\Omega^2}\left(\cos(\Omega/2) - 1\right) = \frac{8}{\Omega^2}\sin^2\left(\frac{\Omega}{4}\right) = \frac{1}{2}\text{sinc}^2\left(\frac{\Omega}{4}\right).$$

3.4 $h(t) = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-(t-\mu)^2/2\sigma^2}.$ Thus, $H(j\Omega) = \int_{-\infty}^{\infty} \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-(t-\mu)^2/2\sigma^2} e^{-j\Omega t}\, dt.$

Making a change of variable $t - \mu = \tau$ we get $H(j\Omega) = \frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\tau^2/2\sigma^2} e^{-j\Omega(\tau+\mu)} d\tau$

$$= \frac{1}{\sigma\sqrt{2\pi}} e^{-j\Omega\mu} \int_{-\infty}^{\infty} e^{-\tau^2/2\sigma^2} e^{-j\Omega\tau} d\tau = \frac{1}{\sigma\sqrt{2\pi}} \cdot e^{-j\Omega\mu} \cdot \sigma\sqrt{2\pi} \cdot e^{-\sigma^2\Omega^2/2}$$

$= e^{-(\frac{\sigma^2\Omega^2}{2} + j\Omega\mu)}.$ For a zero mean impulse response, we then have the CTFT pair $e^{-t^2/2\sigma^2} \overset{\text{CTFT}}{\leftrightarrow} \sigma\sqrt{2\pi} e^{-\sigma^2\Omega^2/2}.$

3.5 $x_a(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\Omega) e^{j\Omega t} d\Omega = \frac{1}{2\pi}\int_{-1}^{1} e^{j\Omega t} d\Omega = \frac{1}{j2\pi t}\left(e^{j\Omega} - e^{-j\Omega}\right) = \frac{\sin(t)}{\pi t}.$

3.6 **(a)** $\int_{-\infty}^{\infty} x_a(t-t_o)e^{-j\Omega t}dt = \int_{-\infty}^{\infty} x_a(\tau)e^{-j\Omega(\tau+t_o)}d\tau$ obtained using a change of variable $t - t_o = \tau$. Therefore $\int_{-\infty}^{\infty} x_a(t-t_o)e^{-j\Omega t}dt = e^{-j\Omega t_o}\int_{-\infty}^{\infty} x_a(\tau)e^{-j\Omega\tau}d\tau = e^{-j\Omega t_o}X_a(j\Omega).$

(b) $\int_{-\infty}^{\infty} x_a(t)e^{j\Omega_o t}e^{-j\Omega t}dt = \int_{-\infty}^{\infty} x_a(t)e^{-j(\Omega-\Omega_o)t}dt = X\big(j(\Omega-\Omega_o)\big).$

(c) $x_a(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X_a(j\Omega)e^{j\Omega t}d\Omega.$ Therefore $2\pi x_a(-t) = \int_{-\infty}^{\infty} X_a(j\Omega)e^{-j\Omega t}d\Omega.$

Interchanging t and Ω we get $2\pi x_a(-\Omega) = \int_{-\infty}^{\infty} X_a(jt)e^{-j\Omega t}dt.$

(d) For a positive real constant a the CTFT of $x_a(at)$ is given by $\int_{-\infty}^{\infty} x_a(at)e^{-j\Omega t}dt = \frac{1}{a}\int_{-\infty}^{\infty} x_a(\tau)e^{-j(\Omega/a)\tau}d\tau = \frac{1}{a}X_a(j\frac{\Omega}{a}).$ In a similar manner we can show that for a negative constant a the CTFT of $x_a(at)$ is given by $-\frac{1}{a}X_a(j\frac{\Omega}{a}).$

Therefore $x_a(at) \overset{CTFT}{\leftrightarrow} \frac{1}{|a|}X_a\left(j\frac{\Omega}{a}\right).$

(e) Differentiating both sides of $x_a(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X_a(j\Omega)e^{j\Omega t}d\Omega$ get

$\frac{dx_a(t)}{dt} = \frac{1}{2\pi}\int_{-\infty}^{\infty} j\Omega X_a(j\Omega)e^{j\Omega t}d\Omega.$ Therefore $\frac{dx_a(t)}{dt} \overset{\text{CTFT}}{\leftrightarrow} j\Omega X_a(j\Omega).$

3.7 $X_a(j\Omega) = \int_{-\infty}^{\infty} x_a(t)e^{-j\Omega t}dt = |X_a(j\Omega)|e^{j\theta_a(\Omega)}$, where $\theta_a(\Omega) = \arg\{X_a(j\Omega)\}.$ Thus,

$X_a(-j\Omega) = \int_{-\infty}^{\infty} x_a(t)e^{j\Omega t}dt.$ If $x_a(t)$ is a real function of then it follows from the definition of $X_a(j\Omega)$ and the expression for $X_a(-j\Omega)$ that $X_a(j\Omega)$ and $X_a(-j\Omega)$ are complex conjugates. Therefore $|X_a(-j\Omega)| = |X_a(j\Omega)|$ and $\theta_a(-\Omega) = -\theta_a(\Omega).$ Or in other words, for a real , the magnitude spectrum $|X_a(j\Omega)|$ is an even function of Ω and the phase spectrum $\theta_a(\Omega)$ is an odd function of $\Omega.$

3.8

3.9 $\hat{x}(t) = \int_{-\infty}^{\infty} h_{HT}(t-\tau)x(\tau)d\tau.$ where $h_{HT}(t)$ is the impulse response of the Hilbert transformer. Taking the CTFT of both sides we get $\hat{X}(j\Omega) = H_{HT}(j\Omega)X(j\Omega)$ where $\hat{X}(j\Omega)$ and $H_{HT}(j\Omega)$ denote the CTFTs of $\hat{x}(t)$ and $h_{HT}(t)$, respectively. Rewriting $\hat{X}(j\Omega) = H_{HT}(j\Omega)\left(X_p(j\Omega)+X_n(j\Omega)\right) = -j\,X_p(j\Omega) + j\,X_n(j\Omega)$. As the magnitude and phase of $\hat{X}(j\Omega)$ are an even and odd function, $\hat{x}(t)$ is seen to be real signal. Consider the complex signal $y(t) = x(t) + j\,\hat{x}(t)$. Its CTFT is then given by
$Y(j\Omega) = X(j\Omega) + j\,\hat{X}(j\Omega) = 2X_p(j\Omega).$

3.10 The total energy $\varepsilon_x = \int_{-\infty}^{\infty}\left|e^{-\alpha t}\right|^2 dt = \int_{-\infty}^{\infty} e^{-2\alpha t}dt = \frac{1}{-2\alpha}\left[e^{-2\alpha t}\right]_0^{\infty} = \left.\frac{1}{2\alpha}\right|_{\alpha=1/2} = 1.$
The total energy can also be computed using using the Parsevals' theorem

$$\varepsilon_x = \frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{1}{\alpha^2+\Omega^2}d\Omega.$$

Therefore, the 80% bandwidth Ω_c can be found by evaluating $\frac{1}{2\pi}\int_{-\Omega_c}^{\Omega_c}\frac{1}{\alpha^2+\Omega^2}d\Omega$

$$= \frac{1}{2\pi}\left[\frac{1}{\alpha}\cdot\tan^{-1}\left(\frac{\Omega}{\alpha}\right)\right]_{-\Omega_c}^{\Omega_c} = \frac{1}{2\pi\alpha}\left[\tan^{-1}\left(\frac{\Omega_c}{\alpha}\right) - \tan^{-1}\left(-\frac{\Omega_c}{\alpha}\right)\right] = \left.\frac{1}{\pi\alpha}\cdot\tan^{-1}\left(\frac{\Omega_c}{\alpha}\right)\right|_{\alpha=1/2}$$

$= \frac{2}{\pi}\tan^{-1}(2\Omega_c) = 0.8$. Therefore, $\Omega_c = \frac{1}{2}\cdot\tan\left(\frac{0.8\pi}{2}\right) = 1.5388.$

3.11 $y[n] = \mu[n] = y_{ev}[n] + y_{od}[n]$, where $y_{ev}[n] = \frac{1}{2}(y[n]+y[-n]) = \frac{1}{2}+\frac{1}{2}\delta[n]$ and $y_{od}[n] = \frac{1}{2}(y[n]-y[-n]) = \frac{1}{2}(\mu[n]-\mu[-n]) = \mu[n]-\frac{1}{2}-\frac{1}{2}\delta[n]$. Now,
$Y_{ev}(e^{j\omega}) = \frac{1}{2}\left[2\pi\sum_{k=-\infty}^{\infty}\delta(\omega+2\pi k)\right] + \frac{1}{2} = \pi\sum_{k=-\infty}^{\infty}\delta(\omega+2\pi k) + \frac{1}{2}$. Since
$y_{od}[n] = \mu[n]-\frac{1}{2}-\frac{1}{2}\delta[n]$, $y_{od}[n-1] = \mu[n-1]-\frac{1}{2}-\frac{1}{2}\delta[n-1]$. As a result,
$y_{od}[n]-y_{od}[n-1] = \mu[n]-\mu[n-1]-\frac{1}{2}\delta[n-1]+\frac{1}{2}\delta[n] = \frac{1}{2}\left(\delta[n]+\delta[n-1]\right).$
Taking the DTFT of both sides of the above equation, we get
$Y_{od}(e^{j\omega}) - e^{-j\omega}Y_{od}(e^{j\omega}) = \frac{1}{2}\left(1+e^{-j\omega}\right)$ or $Y_{od}(e^{j\omega}) = \frac{1}{2}\left(\frac{1+e^{-j\omega}}{1-e^{-j\omega}}\right) = \frac{1}{1-e^{-j\omega}} - \frac{1}{2}.$

Hence, $Y(e^{j\omega}) = Y_{ev}(e^{j\omega}) + Y_{od}(e^{j\omega}) = \frac{1}{1-e^{-j\omega}} + \pi\sum_{k=-\infty}^{\infty}\delta(\omega+2\pi k).$

3.12 The inverse DTFT of $X(e^{j\omega}) = \sum_{k=-\infty}^{\infty}2\pi\delta(\omega+2\pi k)$ is given by

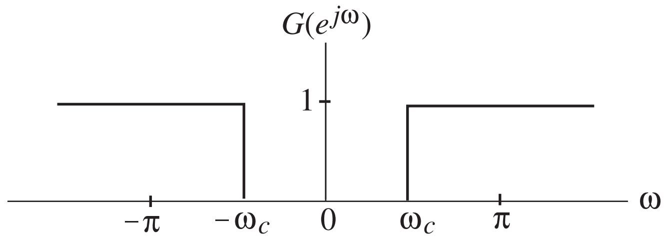
$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} 2\pi\delta(\omega)\, e^{j\omega n} d\omega = \frac{2\pi}{2\pi} = 1.$$

3.13 $Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} \alpha^{|n|} e^{-j\omega n}$ with $|\alpha| < 1$. Rewriting we get

$$Y(e^{j\omega}) = \sum_{n=-\infty}^{-1} \alpha^{-n} e^{-j\omega n} + \sum_{n=0}^{\infty} \alpha^{n} e^{-j\omega n} = \sum_{n=1}^{\infty} (\alpha\, e^{j\omega}) + \sum_{n=0}^{\infty} (\alpha\, e^{-j\omega})$$

$$= \frac{\alpha\, e^{j\omega}}{1 - \alpha\, e^{j\omega}} + \frac{1}{1 - \alpha\, e^{-j\omega}} = \frac{1 - \alpha^2}{1 - 2\alpha\cos\omega + \alpha^2}.$$

3.14 $G(e^{j\omega}) = \sum_{n=-\infty}^{\infty} \left(\delta[n] - \frac{\sin(\omega_c n)}{\pi n} \right) e^{-j\omega n} = 1 - H_{LP}(e^{j\omega}).$



3.15 $x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} d\omega.$ Hence, $x^*[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X^*(e^{j\omega}) e^{-j\omega n} d\omega.$

(a) Since $x[n]$ is real and even, we have $X(e^{j\omega}) = X^*(e^{j\omega})$. Thus

$x[-n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega}) e^{-j\omega n} d\omega.$ Therefore,

$x[n] = \frac{1}{2}(x[n] + x[-n]) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega}) \cos(\omega n) d\omega.$ As $x[n]$ is even, $X(e^{j\omega}) = X(e^{-j\omega}).$

As a result, the term $X(e^{j\omega}) \cos n(\omega n)$ inside the above integral is even, and hence

$$x[n] = \frac{1}{\pi}\int_{0}^{\pi} X(e^{j\omega}) \cos(\omega n) d\omega.$$

(b) Since $x[n]$ is real and odd, we have $x[n] = -x[-n]$ and $X(e^{j\omega}) = -X(e^{-j\omega})$. Thus,

$x[n] = \frac{1}{2}(x[n] - x[-n]) = \frac{j}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega}) \sin(\omega n) d\omega.$ As a result, the term $X(e^{j\omega}) \sin(\omega n)$

inside the above integral is even, and hence $x[n] = \frac{j}{2\pi}\int_{0}^{\pi} X(e^{j\omega}) \sin(\omega n) d\omega.$

3.16 $x[n] = A\alpha^n \sin(\omega_0 n + \phi)\mu[n] = A\alpha^n \left(\dfrac{e^{j\omega_0 n}e^{j\phi} - e^{-j\omega_0 n}e^{-j\phi}}{2j}\right)\mu[n]$

$= \dfrac{A}{2j}e^{j\phi}\left(\alpha e^{j\omega_0}\right)^n \mu[n] - \dfrac{A}{2j}e^{-j\phi}\left(\alpha e^{-j\omega_0}\right)^n \mu[n]$. Therefore, the DTFT of $x[n]$ is given by $X(e^{j\omega}) = \dfrac{A}{j2}e^{j\phi}\dfrac{1}{1-\alpha e^{j\omega}e^{j\omega_0}} - \dfrac{A}{j2}e^{-j\phi}\dfrac{1}{1-\alpha e^{j\omega}e^{-j\omega_0}}$.

3.17 Let $x[n] = \alpha^n \mu[n1]$ with $|\alpha| < 1$. Its DTFT was computed in Example 3.6 and is given by $X(e^{j\omega}) = \dfrac{1}{1-\alpha e^{-j\omega}}$.

(a) $x_1[n] = \alpha^n \mu[n-1]$ with $|\alpha| < 1$. Its DTFT is given by $X_1(e^{j\omega}) = \sum\limits_{n=1}^{\infty} \alpha^n e^{-j\omega n}$

$$= \sum_{n=1}^{\infty}(\alpha e^{-j\omega})^n = \sum_{n=0}^{\infty}(\alpha e^{-j\omega})^n - 1 = \frac{1}{1-\alpha e^{-j\omega}} - 1 = \frac{\alpha e^{-j\omega}}{1-\alpha e^{-j\omega}}.$$

(b) $x_2[n] = n\alpha^n \mu[n]$ with $|\alpha| < 1$. Note $x_2[n] = n\,x[n]$. Therefore, using the differentiation-in-frequency property in Table 3.4 we get

$$X_2(e^{j\omega}) = j\frac{dX(e^{j\omega})}{d\omega} = j\frac{d}{d\omega}\left(\frac{1}{1-\alpha e^{-j\omega}}\right) = \frac{\alpha e^{-j\omega}}{(1-\alpha e^{-j\omega})^2}.$$

(c) $x_3[n] = \alpha^n \mu[n+1]$ with $|\alpha| < 1$. Its DTFT is given by

$$X_3(e^{j\omega}) = \sum_{n=-1}^{\infty}\alpha^n e^{-j\omega n} = \alpha^{-1}e^{j\omega} + \sum_{n=0}^{\infty}\alpha^n e^{-j\omega n} = \alpha^{-1}e^{j\omega} + \frac{1}{1-\alpha e^{-j\omega}} = \frac{1}{\alpha}\left(\frac{e^{j\omega}-\alpha}{1-\alpha e^{-j\omega}}\right).$$

(d) $x_4[n] = n\alpha^n \mu[n+2]$ with $|\alpha| < 1$. Its DTFT is given by

$X_4(e^{j\omega}) = \sum\limits_{n=-2}^{\infty} n\alpha^n e^{-j\omega n} = \sum\limits_{n=0}^{\infty} n\alpha^n e^{-j\omega n} - 2\alpha^{-2}e^{j2\omega} - \alpha^{-1}e^{j\omega}$. From the results of Part (b) we observe that $\sum\limits_{n=0}^{\infty} n\alpha^n e^{-j\omega n} = \dfrac{\alpha e^{-j\omega}}{(1-\alpha e^{-j\omega})^2}$. Hence,

$$X_4(e^{j\omega}) = \frac{\alpha e^{-j\omega}}{(1-\alpha e^{-j\omega})^2} - 2\alpha^{-2}e^{j2\omega} - \alpha^{-1}e^{j\omega}.$$

(e) $x_5[n] = n\alpha^n \mu[-n-1]$ with $|\alpha| > 1$. Its DTFT is given by

$$X_5(e^{j\omega}) = \sum_{n=-\infty}^{-1}\alpha^n e^{-j\omega n} = \sum_{m=1}^{\infty}\alpha^{-m}e^{j\omega m} = \sum_{m=0}^{\infty}\alpha^{-m}e^{j\omega m} - 1 = \frac{1}{1-\alpha^{-1}e^{j\omega}} - 1 = \frac{e^{j\omega}}{\alpha - e^{j\omega}}.$$

(f) $x_6[n] = \begin{cases}\alpha^{|n|}, & |n| \le M, \\ 0, & \text{otherwise.}\end{cases}$ Its DTFT is given by

$$X_6(e^{j\omega}) = \sum_{n=0}^{M}\alpha^n e^{-j\omega n} + \sum_{n=-M}^{-1}\alpha^{-n}e^{-j\omega n} = \frac{1-\alpha^{M+1}e^{-j\omega(M+1)}}{1-\alpha e^{-j\omega}} + \alpha^M e^{j\omega M}\cdot\frac{1-\alpha^{M+1}e^{-j\omega(M+1)}}{1-\alpha e^{-j\omega}}.$$

3.18 **(a)** $x_a[n] = \mu[n] - \mu[n-5]$. Let denote $\mu(e^{j\omega})$ the DTFT of $\mu[n]$. Using the time-shifting property of the DTFT given in Table 3.4, the DTFT of $x_a[n]$ is thus given by $X_a(e^{j\omega}) = (1-e^{-j5\omega})\,\mu(e^{j\omega})$. From Table 3.3, we have

$\mu(e^{j\omega}) = \dfrac{1}{1-e^{j\omega}} + \sum_{k=-\infty}^{\infty}\pi\delta(\omega+2\pi k)$. Therefore,

$$X_a(e^{j\omega}) = (1-e^{-j5\omega})\left(\frac{1}{1-e^{j\omega}} + \sum_{k=-\infty}^{\infty}\pi\delta(\omega+2\pi k)\right).$$

(b) $x_b[n] = \alpha^n(\mu[n]-\mu[n-8])$. Let $x[n] = \alpha^n\mu[n1]$ with $|\alpha| < 1$. Its DTFT was computed in Example 3.6 and is given by $X(e^{j\omega}) = \dfrac{1}{1-\alpha e^{-j\omega}}$. Now $x_b[n] = x[n] - x[n-8]$. Using the time-shifting property of the DTFT given in Table 3.4, the DTFT of $x_a[n]$ is thus given by $X_b(e^{j\omega}) = (1-e^{-j8\omega})X(e^{j\omega}) = \dfrac{1-e^{-j8\omega}}{1-\alpha e^{-j\omega}}$.

(c) $x_c[n] = (n+1)\alpha^n\mu[n] = n\alpha^n\mu[n] + \alpha^n\mu[n]$ with $|\alpha| > 1$. We can rewrite it as $x_c[n] = x_2[n] + x[n]$. The DTFT of $x_2[n]$ was computed in Problem 3.17(b) and is given by $X_2(e^{j\omega}) = \dfrac{\alpha e^{-j\omega}}{(1-\alpha e^{-j\omega})^2}$ and the DTFT of $x[n]$ was computed in Example 3.6 and is given by $X(e^{j\omega}) = \dfrac{1}{1-\alpha e^{-j\omega}}$. Therefore $X_c(e^{j\omega}) = X_2(e^{j\omega}) + X(e^{j\omega})$

$$= \frac{\alpha e^{-j\omega}}{(1-\alpha e^{-j\omega})^2} + \frac{1}{1-\alpha e^{-j\omega}} = \frac{1}{(1-\alpha e^{-j\omega})^2}.$$

3.19 **(a)** $y_1[n] = \begin{cases}1, & -N \le n \le N, \\ 0, & \text{otherwise.}\end{cases}$ Then $Y_1(e^{j\omega}) = \sum_{n=-N}^{N} e^{-j\omega n} = e^{-j\omega N}\left(\dfrac{1-e^{-j\omega(2N+1)}}{1-e^{-j\omega}}\right)$

$$= \frac{\sin\left(\omega[N+\frac{1}{2}]\right)}{\sin(\omega/2)}.$$

(b) $y_2[n] = \begin{cases} 1, & 0 \le n \le N, \\ 0, & \text{otherwise.} \end{cases}$ Then $Y_2(e^{j\omega}) = \sum_{n=0}^{N} e^{-j\omega n} = \frac{1-e^{-j\omega(N+1)}}{1-e^{-j\omega}}$

$$= e^{-j\omega N/2}\left(\frac{\sin(\omega[N+1]/2)}{\sin(\omega/2)}\right).$$

(c) $y_3[n] = \begin{cases} 1-\frac{|n|}{N}, & -N \le n \le N, \\ 0, & \text{otherwise.} \end{cases}$ Assume N to be odd. Then we can express

$y_3[n] = \frac{1}{N} y_0[n] \circledast y_0[n]$ where $y_0[n] = \begin{cases} 1, & -\frac{N-1}{2} \le n \le \frac{N-1}{2}, \\ 0, & \text{otherwise.} \end{cases}$ Therefore,

$Y_3(e^{j\omega}) = \frac{1}{N} Y_0(e^{j\omega}) Y_0(e^{j\omega}) = \frac{1}{N} Y_0^2(e^{j\omega})$. Now, from the results of Part (a), we have

$Y_0(e^{j\omega}) = \frac{\sin(\omega N/2)}{\sin(\omega/2)}$. Hence, $Y_3(e^{j\omega}) = \frac{1}{N} \cdot \frac{\sin^2(\omega N/2)}{\sin^2(\omega/2)}$.

Note: The above result also holds for N even.

(d) $y_4[n] = \begin{cases} N+1-\frac{|n|}{N}, & -N \le n \le N, \\ 0, & \text{otherwise,} \end{cases} = N y_1[n] + y_3[n]$, where $y_1[n]$ is the sequence considered in Part (a) and $y_3[n]$ is the sequence considered in Part (c). Hence,

$$Y_4(e^{j\omega}) = N \cdot Y_1(e^{j\omega}) + Y_3(e^{j\omega}) = N \cdot \frac{\sin\left(\omega[N+\frac{1}{2}]\right)}{\sin(\omega/2)} + \frac{1}{N} \cdot \frac{\sin^2(\omega N/2)}{\sin^2(\omega/2)}.$$

(e) $y_5[n] = \begin{cases} \cos(\pi n/2N), & -N \le n \le N, \\ 0, & \text{otherwise.} \end{cases}$ Then

$$Y_5(e^{j\omega}) = \frac{1}{2}\sum_{n=-N}^{N} e^{-j(\pi n/2N)} e^{-j\omega n} + \frac{1}{2}\sum_{n=-N}^{N} e^{j(\pi n/2N)} e^{-j\omega n}$$

$$= \frac{1}{2}\sum_{n=-N}^{N} e^{-j\left(\omega-\frac{\pi}{2N}\right)n} + \frac{1}{2}\sum_{n=-N}^{N} e^{-j\left(\omega+\frac{\pi}{2N}\right)n} = \frac{1}{2} \cdot \frac{\sin\left((\omega-\frac{\pi}{2N})(N+\frac{1}{2})\right)}{\sin\left((\omega-\frac{\pi}{2N})/2\right)} + \frac{1}{2} \cdot \frac{\sin\left((\omega+\frac{\pi}{2N})(N+\frac{1}{2})\right)}{\sin\left((\omega+\frac{\pi}{2N})/2\right)}.$$

3.20 Denote $x_m[n] = \frac{(n+m-1)!}{n!(m-1)!}\alpha^n\mu[n]$ with $|\alpha| < 1$. We shall prove by induction that the DTFT of $x_m[n]$ is given by $X_m(e^{j\omega}) = \frac{1}{(1-\alpha e^{-j\omega})^m}$. From Table 3.3, it follows that it holds for $m = 1$. Let $m = 2$. Then

$x_2[n] = \frac{(n+1)!}{n!}\alpha^n\mu[n] = (n+1)x_1[n] = nx_1[n] + x_1[n]$. Therefore,

$X_2(e^{j\omega}) = \frac{\alpha e^{-j\omega}}{(1-\alpha e^{-j\omega})^2} + 1 - \alpha e^{-j\omega} = \frac{1}{(1-\alpha e^{-j\omega})^2}$, and it also holds for $m = 2$.

Now, assume that it holds for m. Consider next $x_{m+1}[n] = \frac{(n+m)!}{n!(m)!}\alpha^n\mu[n]$

$= \left(\frac{n+m}{m}\right)\frac{(n+m-1)!}{n!(m-1)!}\alpha^n\mu[n] = \left(\frac{n+m}{m}\right)x_m[n] = \frac{1}{m}\cdot n\cdot x_m[n] + x_m[n]$. Hence,

$X_{m+1}(e^{j\omega}) = \frac{1}{m} j \frac{d}{d\omega}\left(\frac{1}{(1-\alpha e^{-j\omega})^m}\right) + \frac{1}{(1-\alpha e^{-j\omega})^m} = \frac{\alpha e^{-j\omega}}{(1-\alpha e^{-j\omega})^{m+1}} + \frac{1}{(1-\alpha e^{-j\omega})^m}$

$= \frac{1}{(1-\alpha e^{-j\omega})^{m+1}}$.

3.21 **(a)** $X_a(e^{j\omega}) = \sum_{k=-\infty}^{\infty}\delta(\omega + 2\pi k)$. Hence, $x_a[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi}\delta(\omega)e^{j\omega n}d\omega = 1$.

(b) $X_b(e^{j\omega}) = \frac{e^{j\omega}\left(1 - e^{j\omega N}\right)}{1 - e^{j\omega}} = e^{j\omega}\sum_{n=0}^{N-1} e^{j\omega n}$. Let $m = -n$. $X_b(e^{j\omega}) = e^{j\omega}\sum_{m=0}^{-N+1} e^{-j\omega m}$.

Consider the DTFT $X(e^{j\omega}) = \sum_{m=0}^{-N+1} e^{-j\omega m}$. Its inverse is given by

$x[n] = \begin{cases} 1, & -(N-1) \le n \le 0, \\ 0, & \text{otherwise.} \end{cases}$ Therefore, by the time-shifting property of the DTFT, the inverse DTFT of $X_b(e^{j\omega}) = e^{j\omega}X(e^{j\omega})$ is given by $x_b[n] = x[n+1] = \begin{cases} 1, & -N \le n \le -1, \\ 0, & \text{otherwise.} \end{cases}$

(c) $X_c(e^{j\omega}) = 1 + 2\sum_{\ell=0}^{N}\cos(\omega\ell) = 2 + \sum_{\ell=-N}^{N} e^{-j\omega\ell}$. Hence, $x_c[n] = \begin{cases} 3, & n = 0, \\ 1, & 0 < |n| \le N, \\ 0, & \text{otherwise.} \end{cases}$

(d) $X_d(e^{j\omega}) = \frac{-\alpha e^{-j\omega}}{(1-\alpha e - j^{j\omega})^2}$ with $|\alpha| < 1$. We can rewrite $X_d(e^{j\omega})$ as

$X_d(e^{j\omega}) = \dfrac{dX_o(e^{j\omega})}{d\omega}$ where $X_o(e^{j\omega}) = \dfrac{1}{1-\alpha e^{-j\omega}}$. From Table 3.3, the inverse DTFT of $X_d(e^{j\omega})$ is given by $x_o[n] = \alpha^n \mu[n]$. From Table 3.4, using the differentiation-in-frequency property the inverse DTFT of $X_d(e^{j\omega})$ is thus given by $x_d[n] = n\, x_o[n] = n\,\alpha^n \mu[n]$.

3.22 **(a)** $H_a(e^{j\omega}) = \sin(4\omega) = \dfrac{e^{j4\omega} - e^{-j4\omega}}{2j} = \dfrac{1}{2j}e^{j4\omega} - \dfrac{1}{2j}e^{-j4\omega}$. Therefore,
$h_a[n] = \{-j0.5,\quad 0,\quad 0,\quad 0,\quad 0,\quad 0,\quad 0,\quad 0,\quad j0.5\},\ -4 \le n \le 4.$

(b) $H_b(e^{j\omega}) = \cos(4\omega) = \dfrac{e^{j4\omega} + e^{-j4\omega}}{2} = 0.5e^{j4\omega} + 0.5e^{-j4\omega}$. Therefore,
$h_b[n] = \{0.5,\quad 0,\quad 0,\quad 0,\quad 0,\quad 0,\quad 0,\quad 0,\quad 0.5\},\ -4 \le n \le 4.$

(c) $H_c(e^{j\omega}) = \sin(5\omega) = \dfrac{e^{j5\omega} - e^{-j5\omega}}{2j} = \dfrac{1}{2j}e^{j5\omega} - \dfrac{1}{2j}e^{-j5\omega}$. Therefore,
$h_c[n] = \{-j0.5,\quad 0,\quad 0,\quad 0,\quad 0,\quad 0,\quad 0,\quad 0,\quad 0,\quad 0,\quad j0.5\},\ -5 \le n \le 5.$

(d) $H_d(e^{j\omega}) = \cos(5\omega) = \dfrac{e^{j5\omega} + e^{-j5\omega}}{2} = 0.5e^{j5\omega} + 0.5e^{-j5\omega}$. Therefore,
$h_d[n] = \{0.5,\quad 0,\quad 0,\quad 0,\quad 0,\quad 0,\quad 0,\quad 0,\quad 0,\quad 0,\quad 0.5\},\ -5 \le n \le 5.$

3.23 **(a)** $H_1(e^{j\omega}) = 1 + 2\cos(\omega) + 3\cos(2\omega) = 1 + 2\left(\dfrac{e^{j\omega} + e^{-j\omega}}{2}\right) + 3\left(\dfrac{e^{j2\omega} + e^{-j2\omega}}{2}\right)$

$= 1 + e^{j\omega} + e^{-j\omega} + 1.5e^{j2\omega} + 1.5e^{-j2\omega}$. Therefore,
$\{h_1[n]\} = \{1.5,\quad 1,\quad 1,\quad 1,\quad 1.5\},\ -2 \le n \le 2.$

(b) $H_2(e^{j\omega}) = \big(3 + 2\cos(\omega) + 4\cos(2\omega)\big)\cos\left(\dfrac{\omega}{2}\right)e^{-j\omega/2}$

$$= \left[3 + 2\left(\frac{e^{j\omega} + e^{-j\omega}}{2}\right) + 4\left(\frac{e^{j2\omega} + e^{-j2\omega}}{2}\right)\right]\left(\frac{e^{j\omega/2} + e^{-j\omega/2}}{2}\right)e^{-j\omega/2}$$

$$= \frac{1}{2}\left(3 + e^{j\omega} + e^{-j\omega} + 2e^{j2\omega} + 2e^{-j2\omega}\right)\left(1 + e^{-j\omega}\right)$$

$= 2 + 1.5e^{j\omega} + 2e^{-j\omega} + e^{j2\omega} + 1.5e^{-j2\omega} + e^{-j3\omega}$. Hence,
$\{h_2[n]\} = \{1,\quad 1.5,\quad 2,\quad 2,\quad 1.5,\quad 1\},\ -2 \le n \le 3.$

(c) $H_3(e^{j\omega}) = j\big[3 + 4\cos(\omega) + 2\cos(2\omega)\big]\sin(\omega)$

$$= j\left[3+4\left(\frac{e^{j\omega}+e^{-j\omega}}{2}\right)+2\left(\frac{e^{j2\omega}+e^{-j2\omega}}{2}\right)\right]\left(\frac{e^{j\omega}-e^{-j\omega}}{2j}\right)$$

$$= \frac{1}{2}\left(3+2e^{j\omega}+2e^{-j\omega}+e^{j2\omega}+e^{-j2\omega}\right)\left(e^{j\omega}-e^{-j\omega}\right)$$

$= 3+2e^{j\omega}+2e^{-j\omega}+0.5e^{j2\omega}+0.5e^{-j2\omega}$. Hence,

$\{h_c[n]\} = \{0.5,\ \ 2,\ \ 3,\ \ 2,\ \ 0.5\}, -2 \le n \le 2.$

(d) $H_4(e^{j\omega}) = j[4+2\cos(\omega)+3\cos(2\omega)]\sin(\omega/2)\,e^{j\omega/2}$

$$= j\left[4+2\left(\frac{e^{j\omega}+e^{-j\omega}}{2}\right)+3\left(\frac{e^{j2\omega}+e^{-j2\omega}}{2}\right)\right]\left(\frac{e^{j\omega/2}-e^{-j\omega/2}}{2j}\right)e^{-j\omega/2}$$

$$= \frac{1}{2}\left(4+e^{j\omega}+e^{-j\omega}+1.5e^{j2\omega}+1.5e^{-j2\omega}\right)\left(1-e^{-j\omega}\right)$$

$= 1.5-0.25e^{j\omega}-1.5e^{-j\omega}+1.5e^{j2\omega}-0.5e^{-j2\omega}-0.75e^{-j3\omega}$. Hence,

$\{h_4[n]\} = \{-1.5,\ \ -0.5,\ \ -3,\ \ 3,\ \ -0.5,\ \ -3\}, -3 \le n \le 2.$

3.24 Let $H(e^{j\omega})$ and $G(e^{j\omega})$ denote the DTFTs of the sequences $h[n]$ and $g[n]$, respectively.

(a) Linearity Theorem: $\mathcal{F}\{\alpha\, h[n]+\beta\, g[n]\} = \sum_{n=-\infty}^{\infty}(\alpha\, h[n]+\beta\, g[n])\, e^{-j\omega n}$

$$= \alpha \sum_{n=-\infty}^{\infty} h[n]\, e^{-j\omega n} + \beta \sum_{n=-\infty}^{\infty} g[n]\, e^{-j\omega n} = \alpha\, H(e^{j\omega})+\beta\, G(e^{j\omega}).$$

(b) Time-reversal Theorem: $\sum_{n=-\infty}^{\infty} h[-n]e^{-j\omega n} = \sum_{m=-\infty}^{\infty} h[m]e^{j\omega m} = H(e^{-j\omega}).$

(c) Time-shifting Theorem: $\sum_{n=-\infty}^{\infty} h[n-n_o]e^{-j\omega n} = \sum_{m=-\infty}^{\infty} h[m]e^{-j\omega(m+n_o)}$

$$= e^{-j\omega n_o}\sum_{m=-\infty}^{\infty} h[m]e^{-j\omega m} = e^{-j\omega n_o} H(e^{j\omega}).$$

(d) Frequency-shifting Theorem: $\sum_{n=-\infty}^{\infty}\left(e^{j\omega_o n}h[n]\right)e^{-j\omega n} = \sum_{n=-\infty}^{\infty} h[n]e^{-j(\omega-\omega_o)n}$

$$= H(e^{j(\omega-\omega_o)}).$$

3.25 Let $H_1(e^{j\omega}) = \mathcal{F}\{h_1[n]\}$ and $H_2(e^{j\omega}) = \mathcal{F}\{h_2[n]\}$. From Example 3.8 we have

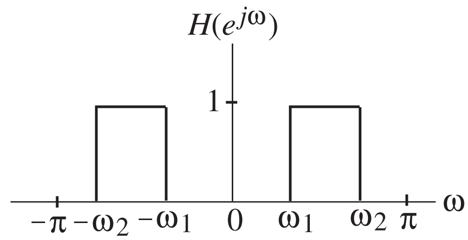
$$H_2(e^{j\omega}) = \sum_{n=-\infty}^{\infty}\left(\frac{\sin(\omega_2 n)}{\pi n}\right)$$

$$= \begin{cases} 1, & 0 \le |\omega| \le \omega_2, \\ 0, & \omega_2 < |\omega| \le \pi. \end{cases}$$ From the result of Problem 3.14 we get

$$H_1(e^{j\omega}) = \sum_{n=-\infty}^{\infty} \left(\delta[n] - \frac{\sin(\omega_1 n)}{\pi n} \right) e^{-j\omega n} = \begin{cases} 0, & 0 \le |\omega| \le \omega_1, \\ 1, & \omega_1 < |\omega| \le \pi. \end{cases}$$

As the impulse response of the cascade is given by $h[n] = h_1[n] \circledast h_2[n]$, using the convolution theorem we obtain the DTFT of the cascade: $H(e^{j\omega}) = H_1(e^{j\omega}) H_2(e^{j\omega})$

$$= \begin{cases} 0, & 0 \le |\omega| < \omega_1, \\ 1, & \omega_1 \le |\omega| \le \omega_2, \\ 0, & \omega_2 < |\omega| \le \pi. \end{cases}$$



3.26 $Y(e^{j\omega}) = X(e^{j4\omega}) = X\left((e^{j\omega})^4\right)$. Now, $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$. Hence,

$$Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} y[n] e^{-j\omega n} = X\left((e^{j\omega})^4\right) = \sum_{n=-\infty}^{\infty} x[n] (e^{-j\omega n})^4 = \sum_{m=-\infty}^{\infty} x[m/4] e^{-j\omega m}.$$

Therefore $y[n] = \begin{cases} x[n], & n = 0, \pm 4, \pm 8, \pm 16, \ldots \\ 0, & \text{otherwise.} \end{cases}$

3.27 $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$. Therefore, $X(e^{j\omega/2}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j(\omega/2)n}$ and

$X(-e^{j\omega/2}) = \sum_{n=-\infty}^{\infty} x[n] (-1)^n e^{-j(\omega/2)n}$. Thus, we can write

$$Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} y[n] e^{-j\omega n} = \frac{1}{2}\left\{ X(e^{j\omega/2}) + X(-e^{j\omega/2}) \right\} = \frac{1}{2} \sum_{n=-\infty}^{\infty} \left(x[n] + x[n](-1)^n \right) e^{-j\omega n}.$$

Hence, $y[n] = \frac{1}{2}\left(x[n] + x[n](-1)^n \right) = \begin{cases} x[n], & \text{for } n \text{ even,} \\ 0, & \text{for } n \text{ odd.} \end{cases}$

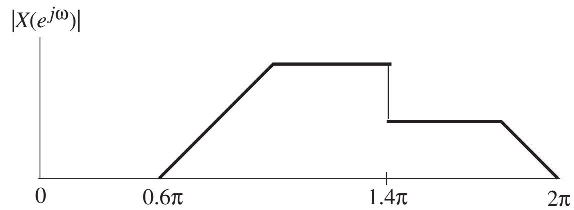
3.28 $\mathcal{F}\{x^*[-n]\} = \sum_{n=-\infty}^{\infty} x^*[-n] e^{-j\omega n} = \sum_{n=-\infty}^{\infty} x^*[n] e^{j\omega n} = \left(\sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n} \right)^* = X^*(e^{j\omega})$.

3.29 $X_{ca}(e^{j\omega}) = \frac{1}{2}\left[X(e^{j\omega}) - X^*(e^{-j\omega})\right] =$

$$= \frac{1}{2}\left(\sum_{n=-\infty}^{\infty} x_{re}[n]e^{-j\omega n} + j\sum_{n=-\infty}^{\infty} x_{im}[n]e^{-j\omega n} - \sum_{n=-\infty}^{\infty} x_{re}[n]e^{-j\omega n} - j\sum_{n=-\infty}^{\infty} x_{im}[n]e^{-j\omega n} \right)$$

$$\omega, = j\sum_{n=-\infty}^{\infty} x_{im}[n]e^{-j\omega n} = \mathcal{F}\{j\,x_{im}[n]\}.$$

3.30



3.31 From Table 3.2 we observe that an even real-valued sequence has a real-valued DTFT and an odd real-valued sequence has an imaginary-valued DTFT.

(a) Since $x_1[n]$ is an odd sequence, it has an imaginary-valued DTFT.

(b) Since $x_2[n]$ is an even sequence, it has a real-valued DTFT.

(c) $x_3[-n] = \frac{\sin(-\omega_c n)}{-\pi n} = \frac{-\sin(\omega_c n)}{-\pi n} = \frac{\sin(\omega_c n)}{\pi n} = x_3[n]$. Since, $x_3[n]$ is an even sequence, it has a real-valued DTFT.

(d) Since $x_4[n]$ is an odd sequence, it has an imaginary-valued DTFT.

(e) Since $x_5[n]$ is an odd sequence, it has an imaginary-valued DTFT.

3.32 From Table 3.2 we observe that an even real-valued sequence has a real-valued DTFT and an odd real-valued sequence has an imaginary-valued DTFT.

(a) Since $Y_1(e^{j\omega})$ is a real-valued function of ω, its inverse is an even sequence.

(b) Since $Y_2(e^{j\omega})$ is an imaginary-valued function of ω, its inverse is an odd sequence.

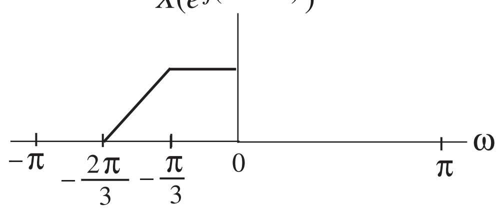
(c) Since $Y_3(e^{j\omega})$ is an imaginary-valued function of ω, its inverse is an odd sequence.

3.33 **(a)** Since $H_1(e^{j\omega})$ is a real-valued function of ω, its inverse is an even sequence.

(b) Since $H_2(e^{j\omega})$ is a real-valued function of ω, its inverse is an even sequence.

3.34 Let $u[n] = x^*[-n]$, and let $X(e^{j\omega})$ and $U(e^{j\omega})$ denote the DTFTs of $x[n]$ and $u[n]$, respectively. From the convolution property of the DTFT given in Table 3.4, the DTFT of $y[n] = x[n] \circledast u[n]$ is given by $Y(e^{j\omega}) = X(e^{j\omega})U(e^{j\omega})$. From Table 3.1, $U(e^{j\omega})$ $= X^*(e^{j\omega})$. Therefore, $Y(e^{j\omega}) = X(e^{j\omega})X^*(e^{j\omega}) = \left|X(e^{j\omega})\right|^2$ which is a real-valued function of ω.

3.35 From the frequency-shifting property of the DTFT given in Table 3.4, $\mathcal{F}\{x[n]e^{-j\pi n/3}\} = X(e^{j(\omega+\pi/3)})$. A sketch of this DTFT is shown below



3.36 $\{-\alpha^n \mu[-n-1]\} = X(e^{j\omega}) = \sum_{n=-\infty}^{-1} -\alpha^n e^{-j\omega n} = -\sum_{n=1}^{\infty} \alpha^{-n} e^{j\omega n} = -\alpha^{-1} e^{j\omega} \sum_{n=0}^{\infty} \left(\frac{e^{j\omega}}{\alpha}\right)^n$.

For $|\alpha| > 1$, $X(e^{j\omega}) = -\alpha^{-1} e^{j\omega} \frac{1}{1-(e^{j\omega}/\alpha)} = \frac{1}{1-\alpha\, e^{j\omega}}$. $\Rightarrow \left|X(e^{j\omega})\right|^2 = \frac{1}{1+\alpha^2 - 2\alpha\cos(\omega)}$.

From Parseval's relation, $\frac{1}{2\pi}\int_{-\pi}^{\pi} \left|X(e^{j\omega})\right|^2 d\omega = \sum_{n=-\infty}^{\infty} |x[n]|^2$.

(a) $\left|X(e^{j\omega})\right|^2 = \frac{1}{5+4\cos(\omega)}$. Hence $\alpha = -2$. Therefore, $x[n] = (-2)^n \mu[-n-1]$.

Now, $4\int_0^{\pi} \left|X(e^{j\omega})\right|^2 d\omega = 2\int_{-\pi}^{\pi} \left|X(e^{j\omega})\right|^2 d\omega = 4\pi \sum_{n=-\infty}^{\infty} |x[n]|^2 = 4\pi \sum_{n=-\infty}^{-1} \left|(-2)^n\right|^2$

$= 4\pi \sum_{n=1}^{\infty} \left(\frac{1}{4}\right)^n = \pi \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n = \frac{4\pi}{3}$.

(b) $\left|X(e^{j\omega})\right|^2 = \frac{1}{3.25-3\cos(\omega)}$. Hence $\alpha = 1.5$. Therefore, $x[n] = -(1.5)^n \mu[-n-1]$.

Now, $\int_0^{\pi} \left|X(e^{j\omega})\right|^2 d\omega = \frac{1}{2}\int_{-\pi}^{\pi} \left|X(e^{j\omega})\right|^2 d\omega = \pi \sum_{n=-\infty}^{\infty} |x[n]|^2 = \pi \sum_{n=-\infty}^{-1} \left|(1.5)^n\right|^2$

$= \pi \sum_{n=1}^{\infty} \left(\frac{4}{9}\right)^n = \frac{4\pi}{9} \sum_{n=0}^{\infty} \left(\frac{4}{9}\right)^n = \frac{4\pi}{9} \cdot \frac{9}{5} = \frac{4\pi}{5}$.

(c) Using the differentiation-in-frequency property of the DTFT as given in Table 3.4, the inverse DTFT of $X(e^{j\omega}) = j\frac{d}{d\omega}\left(\frac{1}{1-\alpha e^{-j\omega}}\right) = \frac{\alpha e^{-j\omega}}{(1-\alpha e^{-j\omega})^2}$ is $x[n] = -n\alpha^n \mu[-n-1]$.

Hence, the inverse DTFT of $\frac{1}{(1-\alpha e^{-j\omega})^2}$ is $-(n+1)\alpha^n \mu[-n-1]$.

$\left|Y(e^{j\omega})\right|^2 = \frac{1}{(5-4e^{-j\omega})^2}$. Hence $\alpha = 2$. Therefore, $y[n] = -(n+1)2^n \mu[-n-1]$.

Now, $4\int_0^{\pi}\left|X(e^{j\omega})\right|^2 d\omega = 2\int_{-\pi}^{\pi}\left|X(e^{j\omega})\right|^2 d\omega = 4\pi\sum_{n=-\infty}^{\infty}\left|x[n]\right|^2 = 4\pi\sum_{n=-\infty}^{-1}(n+1)^2 \cdot 2^{2n}$

$= \pi\sum_{n=0}^{\infty}\left(\frac{1}{4}\right)^n \cdot n^2 = \pi\frac{9/4}{9/16} = 4\pi.$

3.37 $\int_{-\pi}^{\pi}\left|\frac{dX(e^{j\omega})}{d\omega}\right|^2 d\omega = 2\pi\sum_{-\infty}^{\infty}\left|n\,x[n]\right|^2 = 2\pi\sum_{-3}^{6}\left|n\,x[n]\right|^2 = 152\pi.$ (Using Parseval's relation with differentiation-in-frequency property)

3.38 **(a)** $X(e^{j0}) = \sum_{n=-\infty}^{\infty}x[n] = \sum_{n=-2}^{6}x[n] = 10.$

(b) $X(e^{j\pi}) = \sum_{n=-\infty}^{\infty}x[n]e^{j\pi n} = \sum_{n=-2}^{6}x[n]e^{j\pi n} = -6.$

(c) $\int_{-\pi}^{\pi}X(e^{j\omega})d\omega = 2\pi x[0] = -2\pi.$

(d) $\int_{-\pi}^{\pi}\left|X(e^{j\omega})\right|^2 d\omega = 2\pi\sum_{n=-\infty}^{\infty}\left|x[n]\right|^2 = 2\pi\sum_{n=-2}^{6}\left|x[n]\right|^2 = 120\pi.$

(e) $\int_{-\pi}^{\pi}\left|\frac{dX(e^{j\omega})}{d\omega}\right|^2 d\omega = 2\pi\sum_{-\infty}^{\infty}\left|n\,x[n]\right|^2 = 2\pi\sum_{-2}^{6}\left|n\,x[n]\right|^2 = 124\pi.$

3.39 **(a)** $X(e^{j0}) = \sum_{n=-\infty}^{\infty}x[n] = \sum_{n=-6}^{2}x[n] = 12.$

(b) $X(e^{j\pi}) = \sum_{n=-\infty}^{\infty}x[n]e^{j\pi n} = \sum_{n=-6}^{2}x[n]e^{j\pi n} = -12.$

(c) $\int_{-\pi}^{\pi}X(e^{j\omega})d\omega = 2\pi x[0] = -4\pi.$

(d) $\int_{-\pi}^{\pi}\left|X(e^{j\omega})\right|^2 d\omega = 2\pi\sum_{n=-\infty}^{\infty}\left|x[n]\right|^2 = 2\pi\sum_{n=-6}^{2}\left|x[n]\right|^2 = 160\pi.$

(e) $\int_{-\pi}^{\pi}\left|\frac{dX(e^{j\omega})}{d\omega}\right|^2 d\omega = 2\pi\sum_{-\infty}^{\infty}|n\,x[n]|^2 = 2\pi\sum_{-6}^{2}|n\,x[n]|^2 = 136\pi.$

3.40 From the differentiation-in-frequency property of the DTFT given in Table 3.4 we have $\sum_{n=-\infty}^{\infty} n\,x[n]e^{-j\omega n} = j\frac{dX(e^{j\omega})}{d\omega}$ where $X(e^{j\omega}) = \mathcal{F}\{x[n]\}$. Therefore, $\sum_{n=-\infty}^{\infty} n\,x[n] = j\left.\frac{dX(e^{j\omega})}{d\omega}\right|_{\omega=0}$. From the definition of the DTFT $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega n}$ it follows that $\sum_{n=-\infty}^{\infty} x[n] = X(e^{j0})$. Therefore, $C_g = \frac{j\left.\frac{dX(e^{j\omega})}{d\omega}\right|_{\omega=0}}{X(e^{j0})}$.

From Table 3.3, $X(e^{j\omega}) = \mathcal{F}\{\alpha^n\mu[n]\} = \frac{1}{1-\alpha e^{-j\omega}}$, $|\alpha| < 1$. As a result,

$j\left.\frac{dX(e^{j\omega})}{d\omega}\right|_{\omega=0} = \left.\frac{\alpha e^{-j\omega}}{(1-\alpha e^{-j\omega})^2}\right|_{\omega=0} = \frac{\alpha}{(1-\alpha)^2}$ and $X(e^{j0}) = \frac{1}{1-\alpha}$. Hence, $C_g = \frac{\alpha}{1-\alpha}$.

3.41 Let $G_1(e^{j\omega}) = \mathcal{F}\{g_1[n]\}$.

(b) Note $g_2[n] = g_1[n] + g_1[n-4]$. Hence, $\mathcal{F}\{g_2[n]\} = G_2(e^{j\omega})$ $= G_1(e^{j\omega}) + e^{-j4\omega}G_1(e^{j\omega}) = \left(1+e^{-j4\omega}\right)G_1(e^{j\omega})$.

(c) Note $g_3[n] = g_1[-(n-3)] + g_1[n-4]$. Now, $\mathcal{F}\{g_1[-n]\} = G_1(e^{-j\omega})$. Hence, $\mathcal{F}\{g_3[n]\} = G_3(e^{j\omega}) = e^{-j3\omega}G_1(e^{-j\omega}) + e^{-j4\omega}G_1(e^{j\omega})$.

(d) Note $g_4[n] = g_1[n] + g_1[-(n-7)]$. Hence, $\mathcal{F}\{g_4[n]\} = G_4(e^{j\omega})$ $= G_1(e^{j\omega}) + e^{-j7\omega}G_1(e^{-j\omega})$.

3.42 $Y(e^{j\omega}) = X_1(e^{j\omega})X_2(e^{j\omega})$, i.e., $\sum_{n=-\infty}^{\infty} y[n]e^{-j\omega n} = \left(\sum_{n=-\infty}^{\infty} x_1[n]e^{-j\omega n}\right)\left(\sum_{n=-\infty}^{\infty} x_2[n]e^{-j\omega n}\right)$.

(a) Setting $\omega = 0$ in the above we get $\sum_{n=-\infty}^{\infty} y[n] = \left(\sum_{n=-\infty}^{\infty} x_1[n]\right)\left(\sum_{n=-\infty}^{\infty} x_2[n]\right)$.

(b) Setting $\omega = \pi$ we get $\sum_{n=-\infty}^{\infty}(-1)^n y[n] = \left(\sum_{n=-\infty}^{\infty}(-1)^n x_1[n]\right)\left(\sum_{n=-\infty}^{\infty}(-1)^n x_2[n]\right)$.

3.43 $x[n] = \alpha^n\mu[n]$, $|\alpha| < 1$. From Table 3.3, $\mathcal{F}\{x[n]\} = X(e^{j\omega}) = \frac{1}{1-\alpha e^{j\omega}}$. The total energy of $x[n]$ is $\mathcal{E}_x = \frac{1}{2\pi}\int_{-\pi}^{\pi}\left|\frac{1}{1-\alpha e^{-j\omega}}\right|^2 d\omega = \sum_{n=0}^{\infty}(\alpha^2)^n = \left.\frac{1}{1-\alpha^2}\right|_{\alpha=1/2} = \frac{4}{3}$. To determine the

80% bandwidth of the signal, we set $\mathcal{E}_{x,80} = \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c}\left|\frac{1}{1-\alpha e^{-j\omega}}\right|^2 d\omega = 0.8\,\mathcal{E}_x = 0.8\cdot\frac{4}{3}$

and solve for ω_c, i.e., set $\mathcal{E}_{x,80} = \left[\frac{1}{2\pi}\int_{-\omega_c}^{\omega_c}\frac{1}{(1-\alpha\cos\omega)^2+\alpha^2\sin^2\omega}d\omega\right]_{\alpha=1/2} = \frac{3.2}{3}$.

A numerical solution of the above equation yields $\omega_c = 0.5081\pi$.

3.44 Recall $x[n] = x_{ev}[n] + x_{od}[n]$, where $x_{ev}[n] = \frac{1}{2}(x[n]+x[-n])$ and As $x[n]$ is causal, $x[n] = 0$ for $n < 0$ and $x[-n] = 0$ for $n > 0$. Hence, there is no overlap between the nonzero portions of $x[n]$ and $x[-n]$ except at $n = 0$, and we have $x[n] = 2x_{ev}[n]\mu[n] - x_{ev}[0]\delta[n]$ and $x[n] = 2x_{od}[n]\mu[n] + x_{od}[0]\delta[n]$. Moreover, since $x[n]$ is real, it follows from Table 3.2 that $\mathcal{F}\{x_{ev}[n]\} = X_{re}(e^{j\omega})$ and $\mathcal{F}\{x_{ev}[n]\} = jX_{im}(e^{j\omega})$. Taking the DTFT of $x[n] = 2x_{ev}[n]\mu[n] - x_{ev}[0]\delta[n]$ we arrive at

$$X(e^{j\omega}) = \frac{1}{\pi}\int_{-\pi}^{\pi}X_{re}(e^{j\nu})\,\mu(e^{j(\omega-\nu)})d\nu - x[0].$$

From Table 3.3 we have

$$\mu(e^{j\omega}) = \mathcal{F}\{\mu[n]\} = \frac{1}{1-e^{-j\omega}} + \sum_{k=-\infty}^{\infty}\pi\delta(\omega+2\pi k) = \frac{1}{2}\left(1-j\cot(\frac{\omega}{2})\right) + \sum_{k=-\infty}^{\infty}\pi\delta(\omega+2\pi k).$$

Substituting the above in the equation preceding it we get

$$X(e^{j\omega}) = \frac{1}{\pi}\int_{-\pi}^{\pi}X_{re}(e^{j\nu})\left[\frac{1}{2}\left(1-j\cot\left(\frac{\omega-\nu}{2}\right)\right)\right]d\nu$$

$$+\sum_{k=-\infty}^{\infty}\int_{-\pi}^{\pi}X_{re}(e^{j\nu})\,\delta\big((\omega-\nu)+2\pi k\big)d\nu - x[0]$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi}X_{re}(e^{j\nu})\,d\nu - \frac{j}{2\pi}\int_{-\pi}^{\pi}X_{re}(e^{j\nu})\cot\left(\frac{\omega-\nu}{2}\right)d\nu + X_{re}(e^{j\omega}) - x[0].$$

Comparing the last equation with $X(e^{j\omega}) = X_{re}(e^{j\omega}) + jX_{im}(e^{j\omega})$, we arrive at

$$X_{im}(e^{j\omega}) = -\frac{1}{2\pi}\int_{-\pi}^{\pi}X_{re}(e^{j\nu})\cot\left(\frac{\omega-\nu}{2}\right)d\nu.$$

Likewise, taking the DTFT of $x[n] = 2x_{od}[n]\mu[n] + x_{od}[0]\delta[n]$ we get

$$X(e^{j\omega}) = \frac{j}{\pi}\int_{-\pi}^{\pi}X_{im}(e^{j\nu})\,\mu(e^{j(\omega-\nu)})d\nu + x[0].$$

Substituting the expression for $\mu(e^{j\omega})$ given earlier in the above equation we get

$$X(e^{j\omega}) = \frac{j}{\pi}\int_{-\pi}^{\pi} X_{im}(e^{j\nu})\left[\frac{1}{2}\left(1 - j\cot\left(\frac{\omega-\nu}{2}\right)\right)\right]d\nu$$

$$+ j\sum_{k=-\infty}^{\infty}\int_{-\pi}^{\pi} X_{im}(e^{j\nu})\delta\big((\omega-\nu)+2\pi k\big)d\nu + x[0]$$

$$= \frac{j}{2\pi}\int_{-\pi}^{\pi} X_{im}(e^{j\nu})\,d\nu + \frac{1}{2\pi}\int_{-\pi}^{\pi} X_{im}(e^{j\nu})\cot\left(\frac{\omega-\nu}{2}\right)d\nu + jX_{im}(e^{j\omega}) + x[0].$$

Comparing the last equation with $X(e^{j\omega}) = X_{re}(e^{j\omega}) + j\,X_{im}(e^{j\omega})$, we arrive at

$$X_{re}(e^{j\omega}) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X_{im}(e^{j\nu})\cot\left(\frac{\omega-\nu}{2}\right)d\nu + x[0].$$

3.45 If $u[n] = z^n$ is the input to the LTI discrete-time system, then its output is given by

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]u[n-k] = \sum_{k=-\infty}^{\infty} h[k]z^{n-k} = z^n\sum_{k=-\infty}^{\infty} h[k]z^{-k} = z^n H(z).$$

where $H(z) = \sum_{k=-\infty}^{\infty} h[k]z^{-k}$. Hence is an eigenfunction of the system.

If $v[n] = z^n\mu[n]$ is the input to the LTI discrete-time system, then its output is given by

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]v[n-k] = z^n\sum_{k=-\infty}^{\infty} h[k]\mu[n-k]z^{n-k} = z^n\sum_{k=-\infty}^{n} h[k]z^{-k}.$$

Since in this case the summation depends upon is not an eigenfunction of the system.

3.46 $\mathcal{F}\{h_1[n]\} = H_1(e^{j\omega}) = \mathcal{F}\{\delta[n] + \frac{1}{2}\delta[n-1]\} = 1 + 0.5e^{-j\omega}$,

$\mathcal{F}\{h_2[n]\} = H_2(e^{j\omega}) = \mathcal{F}\{\frac{1}{2}\delta[n] - \frac{1}{4}\delta[n-1]\} = 0.5 - 0.25e^{-j\omega}$,

$\mathcal{F}\{h_3[n]\} = H_3(e^{j\omega}) = \mathcal{F}\{2\delta[n]\} = 2$,

$\mathcal{F}\{h_4[n]\} = H_4(e^{j\omega}) = \mathcal{F}\{-2\left(\frac{1}{2}\right)^n\mu[n]\} = \frac{-2}{1-0.5e^{-j\omega}}$.

The overall frequency response of the structure of Figure 2.35 is given by

$$H(e^{j\omega}) = H_1(e^{j\omega}) + H_2(e^{j\omega})H_3(e^{j\omega}) + H_2(e^{j\omega})H_4(e^{j\omega})$$

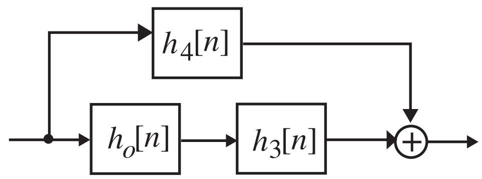
$$= 1 + 0.5e^{-j\omega} + 2(0.5 - 0.25e^{-j\omega}) - \frac{2(0.5-0.25e^{-j\omega})}{1-0.5e^{-j\omega}} = 1.$$

3.46 Denote $H_i(e^{j\omega}) = \mathcal{F}\{h_1[n]\}, 1 \le i \le 5$.

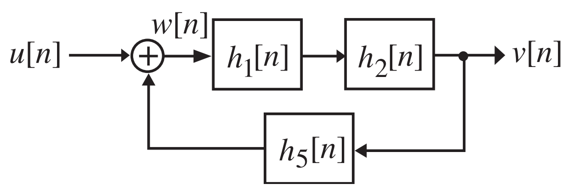
(a) The overall frequency of Figure P2.2(a) is then given by

$H_i(e^{j\omega}) = H_1(e^{j\omega})H_2(e^{j\omega}) + H_3(e^{j\omega})H_4(e^{j\omega}) + H_1(e^{j\omega})H_2(e^{j\omega})H_3(e^{j\omega})H_5(e^{j\omega})$.

(b) The structure of Figure P2.2(b) can be redrawn as shown below



where the block with an impulse response represents the part of Figure P2.2(b) with a feedback loop as shown below



Let $U(e^{j\omega}) = \mathcal{F}\{u[n]\}$, $V(e^{j\omega}) = \mathcal{F}\{v[n]\}$, and $W(e^{j\omega}) = \mathcal{F}\{w[n]\}$. Then we have $W(e^{j\omega}) = U(e^{j\omega}) + H_3(e^{j\omega})V(e^{j\omega})$ and $V(e^{j\omega}) = H_1(e^{j\omega})H_2(e^{j\omega})W(e^{j\omega})$.

Eliminating $W(e^{j\omega})$ from these two equations we get

$$\left(1 - H_1(e^{j\omega})H_2(e^{j\omega})H_5(e^{j\omega})\right)V(e^{j\omega}) = H_1(e^{j\omega})H_2(e^{j\omega})U(e^{j\omega})$$

which leads to the frequency response of the feedback structure given by

$$H_o(e^{j\omega}) = \frac{V(e^{j\omega})}{U(e^{j\omega})} = \frac{H_1(e^{j\omega})H_2(e^{j\omega})}{1 - H_1(e^{j\omega})H_2(e^{j\omega})H_5(e^{j\omega})}$$

The overall frequency of Figure P2.2(a) is thus given by

$$H(e^{j\omega}) = H_4(e^{j\omega}) + H_o(e^{j\omega})H_3(e^{j\omega}) = H_4(e^{j\omega}) + \frac{H_1(e^{j\omega})H_2(e^{j\omega})}{1 - H_1(e^{j\omega})H_2(e^{j\omega})H_5(e^{j\omega})}.$$

3.48 $\mathcal{F}\{h_1[n]\} = H_1(e^{j\omega}) = \mathcal{F}\{2\delta[n-2] - 3\delta[n+1]\} = 2e^{-j2\omega} - 3e^{j\omega}$,

$\mathcal{F}\{h_2[n]\} = H_2(e^{j\omega}) = \mathcal{F}\{\delta[n-1] + 2\delta[n+2]\} = e^{-j\omega} + 2e^{j2\omega}$,

$\mathcal{F}\{h_3[n]\} = H_3(e^{j\omega}) = \mathcal{F}\{5\delta[n-5] + 7\delta[n-3] + 2\delta[n-1] - \delta[n] + 3\delta[n+1]\}$

$= 5e^{-j5\omega} + 7e^{-j3\omega} + 2e^{-j\omega} - 1 + 3e^{j\omega}$.

The overall frequency of Figure P2.3 is given by

$$H(e^{j\omega}) = H_3(e^{j\omega}) + H_1(e^{j\omega})H_2(e^{j\omega}) = 5e^{-j5\omega} + 7e^{-j3\omega} + 2e^{-j\omega} - 1 + 3e^{j\omega}$$
$$+ \left(2e^{-j2\omega} - 3e^{j\omega}\right)\left(e^{-j\omega} + 2e^{j2\omega}\right) = 5e^{-j5\omega} + 9e^{-j3\omega} + 2e^{-j\omega} + 3e^{j\omega} - 6e^{j3\omega}.$$

3.49 Now, $h_{ev}[n]$ is the inverse DTFT of $H_{re}(e^{j\omega})$. Rewriting we get

$$H_{re}(e^{j\omega}) = 1 + 2\left(\frac{e^{j\omega}+e^{-j\omega}}{2}\right) + 3\left(\frac{e^{j2\omega}+e^{-j2\omega}}{2}\right) + 4\left(\frac{e^{j3\omega}+e^{-j3\omega}}{2}\right)$$

$= 1 + e^{j\omega} + e^{-j\omega} + 1.5e^{j2\omega} + 1.5e^{-j2\omega} + 2e^{j3\omega} + 2e^{-j3\omega}$. Its inverse DTFT is $h_{ev}[n] = \delta[n] + \delta[n+1] + \delta[n-1] + 1.5\delta[n+2] + 1.5\delta[n-2] + 2\delta[n+3] + 2\delta[n-3]$.

Since $h[n]$ is real and causal, and its DTFT $H(e^{j\omega})$ exists, it is also absolutely summable. Hence, we can reconstruct $h[n]$ from $h_{ev}[n]$ as

$$h[n] = 2h_{ev}[n]\mu[n] - h_{ev}[0]\delta[n] = 2\big(\delta[n] + \delta[n-1] + 1.5\delta[n-2] + 2\delta[n-3]\big) - \delta[n]$$
$$= \delta[n] + 2\delta[n-1] + 3\delta[n-2] + 4\delta[n-3].$$

3.50 **(a)** $y_a[n] = \sin\left(\frac{\pi n}{3}\right)\cos(\omega_o n) = \frac{1}{2}\sin\left((\omega_o + \frac{\pi}{3})n\right) - \frac{1}{2}\sin\left((\omega_o - \frac{\pi}{3})n\right)$. Hence, the angular frequencies present in the output are $(\omega_o \pm \frac{\pi}{3})n$.

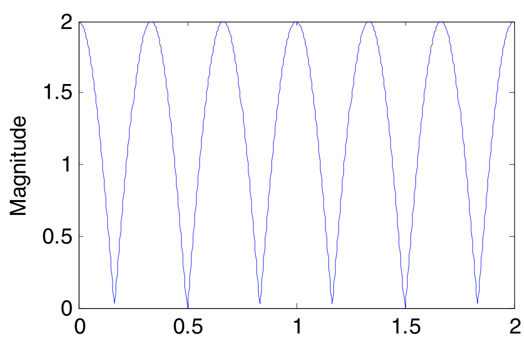
(b) $y_b[n] = \cos^3(\omega_o n) = \cos^2(\omega_o n)\cos(\omega_o n) = \left[\frac{1}{2} + \frac{1}{2}\cos(2\omega_o n)\right]\cos(\omega_o n)$

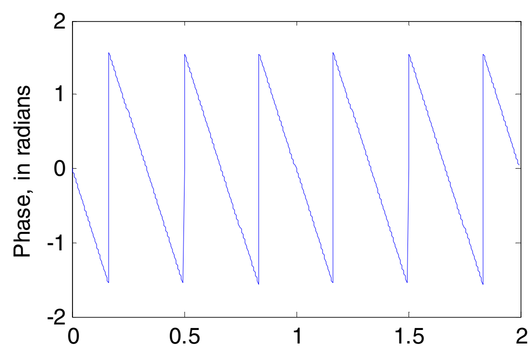
$= \frac{1}{2}\cos(\omega_o n) + \frac{1}{2}\cos(2\omega_o n)\cos(\omega_o n) = \frac{1}{2}\cos(\omega_o n) + \frac{1}{4}\left[\cos(3\omega_o n) + \cos(\omega_o n)\right]$

$= \frac{3}{4}\cos(\omega_o n) + \frac{1}{4}\cos(3\omega_o n)$. Hence, the angular frequencies present in the output are $3\omega_o$ and ω_o.

(c) $y_c[n] = \cos(3\omega_o n)$. Hence, the angular frequency present in the output is $3\omega_o$.

3.51 $\mathcal{F}\{\delta[n] - \alpha\delta[n-R]\} = H(e^{j\omega}) = 1 - \alpha\, e^{-j\omega R}$. Let $\alpha = |\alpha|e^{j\phi}$. Then the maximum value of $|H(e^{j\omega})|$ is $1+|\alpha|$ and the minimum value is $1-|\alpha|$. There are R peaks of $|H(e^{j\omega})|$ located at $\omega = 2\pi k/R,\ 0 \le k \le R-1$, and R dips located at $\omega = (2k+1)\pi/R,\ \ 0 \le k \le R-1$ in the frequency range $0 \le \omega < 2\pi$.





3.52 $G(e^{j\omega}) = \sum_{n=0}^{M-1} \alpha^n e^{-j\omega n} = \frac{1-\alpha^M e^{-j\omega M}}{1-\alpha e^{-j\omega}}$. Note $G(e^{j\omega}) = H(e^{j\omega})$ for $\alpha = 1$. In order to have $G(e^{j0}) = 1,$ the impulse response should be multiplied by a factor $K,$ where $K = \left|\frac{1-\alpha}{1-\alpha^M}\right|.$

3.53 $H(e^{j\omega}) = [a_1\cos(2\omega) + (a_2 + a_3)\cos(\omega) + a_3] + j[-a_1\sin(2\omega) + (a_2 - a_3)\sin(\omega)].$
The frequency response will have zero-phase for $a_2 = a_3$ and $a_1 = 0.$

3.54 $H(e^{j\omega}) = a_1 + a_2 e^{-j\omega} + a_3 e^{-j2\omega} + a_4 e^{-j3\omega} + a_5 e^{-j4\omega}$

$$= \left(a_1 e^{j2\omega} + a_5 e^{-j2\omega}\right)e^{-j2\omega} + \left(a_2 e^{j\omega} + a_4 e^{-j\omega}\right)e^{-j2\omega} + a_3 e^{-j2\omega}.$$

If $a_1 = a_5$ and $a_2 = a_4,$ then we can rewrite the above equation as

$H(e^{j\omega}) = [2a_1\cos(2\omega) + 2a_2\cos(\omega) + a_3]e^{-j2\omega}$ which is seen to have a linear phase.

3.55 $H(e^{j\omega}) = a_1 e^{jk\omega} + a_2 e^{j(k-1)\omega} + a_2 e^{-j(k-2)\omega} + a_1 e^{-j(k-3)\omega}$

$$= e^{jk\omega}(a_1 + a_2 e^{-j\omega} + a_2 e^{-j2\omega} + a_1 e^{-j3\omega})$$

$= e^{j(k-\frac{3}{2})\omega}\left[a_1(e^{j3\omega/2} + e^{-j3\omega/2}) + a_2(e^{j\omega/2} + e^{-j\omega/2})\right]$ Hence, $H(e^{j\omega})$ will be a real function of ω if $k = 3/2,$ in which case we have

$H(e^{j\omega}) = a_1(e^{j3\omega/2} + e^{-j3\omega/2}) + a_2(e^{j\omega/2} + e^{-j\omega/2}).$

3.56 $\mathcal{F}\{a\delta[n] + b\delta[n-1] + \delta[n-2]\} = H_1(e^{j\omega}) = a + b e^{-j\omega} + e^{-j2\omega},$

$\mathcal{F}\{c^n\mu[n]\} = H_2(e^{j\omega}) = \dfrac{1}{1 - c e^{-j\omega}},$ and $\mathcal{F}\{d^n\mu[n]\} = H_3(e^{j\omega}) = \dfrac{1}{1 - d e^{-j\omega}}.$

The overall frequency response is then $H(e^{j\omega}) = H_1(e^{j\omega})H_2(e^{j\omega})H_3(e^{j\omega})$

$= \dfrac{a + b e^{-j\omega} + e^{-j2\omega}}{(1 - c e^{-j\omega})(1 - d\, e^{-j\omega})}.$ Therefore,

$$\left|H(e^{j\omega})\right|^2 = \frac{a + b e^{-j\omega} + e^{-j2\omega}}{(1 - c e^{-j\omega})(1 - d\, e^{-j\omega})}\,\frac{a + b e^{j\omega} + e^{j2\omega}}{(1 - c e^{j\omega})(1 - d\, e^{j\omega})}$$

$$= \frac{(a^2 + b^2 + 1) + 2b(a+1)\cos(\omega) + 2a\cos(2\omega)}{\left(1 - 2c\cos(\omega) + c^2\right)\left(1 - 2d\cos(\omega) + d^2\right)}$$

$= \dfrac{(a^2 + b^2 + 1) + 2b(a+1)\cos(\omega) + 2a\cos(2\omega)}{(1 + c^2 + d^2 + c^2d^2 + 2cd) - 2(c+d)(1+cd)\cos(\omega) + 2cd\cos(2\omega)}.$ Hence,

$\left|H(e^{j\omega})\right|^2 = 1,$ if $a^2 + b^2 + 1 = 1 + c^2 + d^2 + c^2d^2 + 2cd,$

$b(a+1)\cos(\omega) = -(c+d)(1+cd),$ and $a = cd.$ Substituting $a = cd$ in the equation on the left we get $b = -(c+d).$

3.57 $Y(e^{j\omega}) = \left|X(e^{j\omega})\right|^{\alpha} e^{j\arg X(e^{j\omega})}$. Therefore, $H(e^{j\omega}) = \dfrac{Y(e^{j\omega})}{X(e^{j\omega})} = \left|X(e^{j\omega})\right|^{\alpha-1}$.

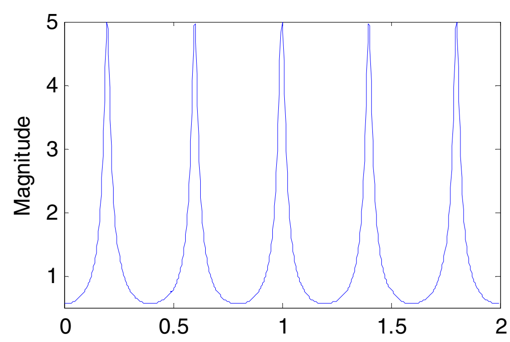
Since $H(e^{j\omega})$ is real function of ω, it has zero-phase.

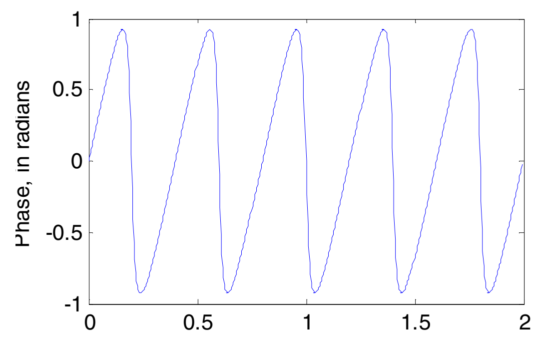
3.58 $y[n] = x[n] - \alpha\, y[n-R]$. Taking the DTFT of both sides we get

$Y(e^{j\omega}) = X(e^{j\omega}) - \alpha\, e^{-j\omega R}\, Y(e^{j\omega})$. Hence, $H(e^{j\omega}) = \dfrac{Y(e^{j\omega})}{X(e^{j\omega})} = \dfrac{1}{1+\alpha\, e^{-j\omega R}}$.

The maximum value of $H(e^{j\omega})$ is $\dfrac{1}{1-|\alpha|}$ and the minimum value is $\dfrac{1}{1+|\alpha|}$. There are peaks and dips in the range $0 \le \omega < 2\pi$. The locations of the peaks and the dips are given by $1-\alpha\, e^{-j\omega R} = 1 \pm |\alpha|$ or $e^{-j\omega R} = \pm\dfrac{|\alpha|}{\alpha}$. The locations of the peaks are given by $\omega = \omega_k = \dfrac{2\pi k}{R}$ and the locations of the dips are given by $\omega = \omega_k = \dfrac{(2\pi+1)k}{R}, 0 \le k \le R-1.$

Plots of the magnitude and the phase responses of $H(e^{j\omega})$ for $\alpha = 0.8$ and $R = 5$ are shown below:





In this case the maximum value is $\dfrac{1}{1-0.8} = 5$ and the minimum value is $\dfrac{1}{1+0.8} = 0.5556$.

3.59 $A(e^{j\omega}) = \dfrac{b_0 + b_1 e^{-j\omega}}{1+a_1 e^{-j\omega}}$. Thus, we set $\left|A(e^{j\omega})\right|^2 = A(e^{j\omega})A^*(e^{j\omega}) = A(e^{j\omega})A(e^{-j\omega})$

$$= \frac{(b_0 + b_1 e^{-j\omega})(b_0 + b_1 e^{j\omega})}{(1+a_1 e^{-j\omega})(1+a_1 e^{j\omega})} = \frac{b_0^2 + b_1^2 + 2b_0 b_1 \cos(\omega)}{1+a_1^2 + 2a_1\cos(\omega)} = 1.$$

Solution #1: $b_0 = \pm 1$ and $b_1 = \text{sgn}(b_0)a_1$. In which case $A(e^{j\omega}) = \pm\dfrac{1+a_1 e^{-j\omega}}{1+a_1 e^{-j\omega}} = 1$, a trivial solution.

Solution #2: $b_1 = \pm 1$ and $b_0 = \text{sgn}(b_1)a_1$. In which case $A(e^{j\omega}) = \pm\dfrac{a_1 + e^{-j\omega}}{1+a_1 e^{-j\omega}}$.

3.60 $A(e^{j\omega}) = \dfrac{b_0 + b_1 e^{-j\omega}}{1 + a_1 e^{-j\omega}} = \dfrac{B_0 e^{j\phi_0} + B_1 e^{j\phi_1} e^{-j\omega}}{1 + A_1 e^{j\theta} e^{-j\omega}}$. Thus, we set

$$\left|A(e^{j\omega})\right|^2 = A(e^{j\omega})A^*(e^{j\omega}) = \frac{B_0 e^{j\phi_0} + B_1 e^{j\phi_1} e^{-j\omega}}{1 + A_1 e^{j\theta} e^{-j\omega}} \cdot \frac{B_0 e^{-j\phi_0} + B_1 e^{-j\phi_1} e^{j\omega}}{1 + A_1 e^{-j\theta} e^{j\omega}}$$

$$= \frac{B_0^2 + B_1^2 + 2B_0 B_1 \cos(\omega - \phi_1 + \phi_0)}{1 + A_1^2 + 2A_1 \cos(\omega - \theta)} = 1.$$

<u>Solution #1</u>: $B_0 = \pm 1$, $B_1 = \text{sgn}(B_0)A_1$, $\phi_1 - \phi_0 = \theta$. In which case

$$A(e^{j\omega}) = \frac{\pm e^{j\phi_0} \pm A_1 e^{j(\phi_0 + \theta)} e^{-j\omega}}{1 + A_1 e^{j\theta} e^{-j\omega}} = \pm e^{j\phi_0} \left(\frac{1 + A_1 e^{j\theta} e^{-j\omega}}{1 + A_1 e^{j\theta} e^{-j\omega}} \right) = \pm e^{j\phi_0}$$ implying

$\phi_0 = \pm\pi$. A trivial solution.

<u>Solution #2</u>: $B_1 = \pm 1$, $B_0 = \text{sgn}(B_1)A_1$, $\phi_1 - \phi_0 = \theta$. In which case

$$A(e^{j\omega}) = \frac{\pm A_1 e^{j(\phi_1 - \theta)} \pm e^{-j\omega}}{1 + A_1 e^{j\theta} e^{-j\omega}} = \pm e^{j\phi_1} \left(\frac{A_1 e^{-j\theta} + e^{-j\omega}}{1 + A_1 e^{j\theta} e^{-j\omega}} \right) = \pm e^{j\phi_1}$$ implying $\phi_1 = \pm\pi$.

Hence, a non-trivia solution is $A(e^{j\omega}) = e^{j\pi} \left(\dfrac{a_1^* + e^{-j\omega}}{1 + a_1 e^{-j\omega}} \right)$.

3.61 **(a)** $H_a(e^{j\omega}) = \text{cosec}(\omega) = \cot\left(\dfrac{\omega}{2}\right) - \cot(\omega) = \dfrac{\cos(\omega/2)}{\sin(\omega/2)} - \dfrac{\cos(\omega)}{\sin(\omega)}$

$$= \frac{j(e^{j\omega/2} + e^{-j\omega/2})}{(e^{j\omega/2} - e^{-j\omega/2})} - \frac{j(e^{j\omega} + e^{-j\omega})}{(e^{j\omega} - e^{-j\omega})}$$

$$= j\frac{(e^{j\omega/2} + e^{-j\omega/2})(e^{j\omega} - e^{-j\omega}) - (e^{j\omega/2} - e^{-j\omega/2})(e^{j\omega} + e^{-j\omega})}{(e^{j\omega/2} - e^{-j\omega/2})(e^{j\omega} - e^{-j\omega})}$$

$$= j2\left(\frac{e^{-j\omega} - e^{-j2\omega}}{1 - e^{-j\omega} - e^{-j4\omega} + e^{-j3\omega}} \right).$$ Therefore, the input-output relation is given by

$y[n] - y[n-1] - y[n-2] + y[n-3] = j2\,x[n-1] - j2\,x[n-2]$.

(b) $H_b(e^{j\omega}) = \sec(\omega) = \dfrac{1}{\cos(\omega)} = \dfrac{2}{e^{j\omega} + e^{-j\omega}} = \dfrac{2e^{-j\omega}}{1 + e^{-j2\omega}}$. Therefore, the input-output relation is given by $y[n] + y[n-2] = 2\,x[n-1]$.

(c) $H_c(e^{j\omega}) = \cot(\omega) = \dfrac{\cos(\omega)}{\sin(\omega)} = j\left(\dfrac{e^{j\omega} + e^{-j\omega}}{e^{j\omega} - e^{-j\omega}} \right) = \dfrac{j(e^{j\omega} + e^{-j\omega})}{e^{j\omega} - e^{-j\omega}} = \dfrac{je^{-j\omega}(e^{j\omega} + e^{-j\omega})}{1 - e^{-j2\omega}}$

$= \dfrac{j(1 + e^{-j2\omega})}{1 - e^{-j2\omega}}$. Therefore, the input-output relation is given by

$y[n] - y[n-2] = j\,x[n] + j\,x[n-2]$.

(d) $H_d(e^{j\omega}) = \tan\left(\frac{\omega}{2}\right) = \frac{\sin(\omega/2)}{\cos(\omega/2)} = \frac{e^{j\omega/2} - e^{-j\omega/2}}{j(e^{j\omega/2} + e^{-j\omega/2})} = \frac{1 - e^{-j\omega}}{j(1 + e^{-j\omega})} == \frac{-j + j\,e^{-j\omega}}{1 + e^{-j\omega}}.$

Therefore, the input-output relation is given by $y[n] + y[n-1] = -j\,x[n] + j\,x[n-1].$

3.62 From Eq. (2.20), the input-output relation of a factor-of-L up-sampler is given by $y[n] = \begin{cases} x[n/L], & n = 0, \pm L, \pm 2L, \pm 3L, \ldots \\ 0, & \text{otherwise.} \end{cases}$ The DTFT of $y[n]$ is thus given by

$$Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty} y[n]e^{-j\omega n} = \sum_{\substack{n=-\infty \\ n=mL}}^{\infty} x[n/L]e^{-j\omega n} = \sum_{m=-\infty}^{\infty} x[m]e^{-j\omega mL} = X(e^{jL\omega}),$$ where

$X(e^{j\omega}) = \mathcal{F}\{x[n]\}.$

3.63 $G(e^{j\omega}) = \frac{1}{1 - \alpha\,e^{-jL\omega}}, \; |\alpha| < 1.$ Thus, we can write $G(e^{j\omega}) = X(e^{jL\omega}),$ where

$X(e^{j\omega}) = \frac{1}{1 - \alpha\,e^{-j\omega}}.$ From Table 3.3, the inverse DTFT of $X(e^{j\omega})$ is $x[n] = \alpha^n \mu[n].$

Hence, from the results of Problem 3.62, it follows that

$$g[n] = \begin{cases} x[n/L], & n = 0, \pm L, \pm 2L, \pm 3L, \ldots \\ 0, & \text{otherwise.} \end{cases}$$

3.64 From Table 3.3, $H(e^{j\omega}) = \frac{1}{1 - 0.5\,e^{-j\omega}}.$ Thus, $\left|H(e^{j\omega})\right| = \frac{1}{\sqrt{1.25 - \cos(\omega)}}$ and

$\arg\{H(e^{j\omega})\} = \theta(\omega) = \tan^{-1}\left(\frac{-0.5\sin(\omega)}{1 - 0.5\cos(\omega)}\right).$ $H(e^{\pm j\pi/5}) = 1.3504 \mp j0.6664.$ Therefore

$\left|H(e^{\pm j\pi/5})\right| = 1.5059$ and $\theta(\pm\pi/5) = \mp 0.4585$ radians.

Now, for an input $x[n] = \sin(\omega_o n)\mu[n],$ the steady-state output is given by

$y[n] = \left|H(e^{j\omega_o})\right| \sin\left(\omega_o n + \theta(\omega_o)\right).$ For $\omega_o = \pi/5,$ the steady-state output is therefore

given by $y[n] = \left|H(e^{j\pi/5})\right| \sin\left(\frac{\pi}{5}n + \theta(\frac{\pi}{5})\right) = 1.5059 \sin\sin\left(\frac{\pi}{5}n - 0.4585\right).$

3.65 $H(e^{j\omega}) = h[0] + h[1]e^{-j\omega} + h[0]e^{-j2\omega} = h[0](1 + e^{-j2\omega}) + h[1]e^{-j\omega}$

$= e^{-j\omega}\left(2h[0]\cos(\omega) + h[1]\right).$ We require $\left|H(e^{j0.3})\right| = 2h[0]\cos(0.3) + h[1] = 0$

and $\left|H(e^{j0.6})\right| = 2h[0]\cos(0.6) + h[1] = 1.$ Solving these two equations we get

$h[0] = 3.8461$ and $h[1] = -6.3487.$

3.66 $H(e^{j\omega}) = h[0](1+e^{-j2\omega}) + h[1]e^{-j\omega} = e^{-j\omega}\left(2h[0]\cos(\omega) + h[1]\right)$. We require $\left|H(e^{j0.3})\right| = 2h[0]\cos(0.3) + h[1] = 1$ and $\left|H(e^{j0.6})\right| = 2h[0]\cos(0.6) + h[1] = 0$. Solving these two equations we get $h[0] = -3.8461$ and $h[1] = 7.3487$.

3.67

3.68 $H(e^{j\omega}) = h[0](1+e^{-j4\omega}) + h[1](e^{-j\omega} + e^{-j3\omega}) + h[2]e^{-j2\omega}$
$= e^{-j2\omega}\left(2h[0]\cos(2\omega) + 2h[1]\cos(\omega) + h[2]\right)$. We require
$\left|H(e^{j0.2})\right| = 2\cos(0.4)h[0] + 2\cos(0.2)h[1] + h[2] = 0,$

$\left|H(e^{j0.5})\right| = 2\cos(1.0)h[0] + 2\cos(0.5)h[1] + h[2] = 1,$

$\left|H(e^{j0.8})\right| = 2\cos(1.6)h[0] + 2\cos(0.8)h[1] + h[2] = 0$. Solving these three equations we get $h[0] = -13.4866,\ h[1] = 45.228,\ h[2] = -63.8089,$ i.e.,
$\{h[n]\} = \{-13.4866, \quad 45.228, \quad -63.8089\},\ 0 \le n \le 2.$

3.69 $H(e^{j\omega}) = h[0] + h[1]e^{-j\omega} + h[2]e^{-j2\omega} + h[3]e^{-j3\omega}$. Therefore,
$H(e^{j0}) = h[0] + h[1] + h[2] + h[3] = 2,$
$H(e^{j\pi/2}) = h[0] + h[1]e^{-j\pi/2} + h[2]e^{-j\pi} + h[3]e^{-j3\pi/2}$
$= h[0] - j\,h[1] - h[2] + j\,h[3] = 7 - j3,$
$H(e^{j\pi}) = h[0] - h[1] + h[2] - h[3] = 0$. Since the impulse response is real, the value of $H(e^{j\omega})$ at $\omega = 3\pi/2$ is the conjugate of its value at $\omega = \pi/2$, i.e.,
$H(e^{j3\pi/2}) = H^*(e^{j\pi/2}) = h[0] + j\,h[1] - h[2] - j\,h[3] = 7 + j3$. Writing the four equations in matrix form we get

$$\begin{bmatrix} h[0] \\ h[1] \\ h[2] \\ h[3] \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & -j & -1 & j \\ 1 & -1 & 1 & -1 \\ 1 & j & -1 & -j \end{bmatrix}^{-1} \begin{bmatrix} 2 \\ 7-j3 \\ 0 \\ 7+j3 \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & j & -1 & -j \\ 1 & -1 & 1 & -1 \\ 1 & -j & -1 & j \end{bmatrix}\begin{bmatrix} 2 \\ 7-j3 \\ 0 \\ 7+j3 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ -3 \\ -1 \end{bmatrix}, \text{ and}$$

hence $\{h[n] = \{4, \quad 2, \quad -3, \quad -1\},\ 0 \le n \le 3.$

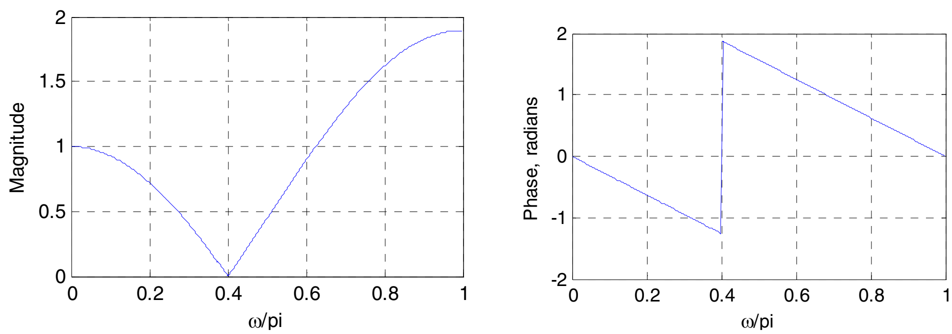
3.70 **(a)** $H(e^{j\omega}) = h[0] + h[1]e^{-j\omega} + h[2]e^{-j2\omega} + h[3]e^{-j3\omega}$
$= h[0] + h[1]e^{-j\omega} - h[1]e^{-j2\omega} - h[0]e^{-j3\omega}$. Therefore,
$H(e^{j\pi/2}) = h[0] + h[1]e^{-j\pi/2} - h[1]e^{-j\pi} - h[0]e^{-j3\pi/2}$
$= h[0] - j\,h[1] + h[1] - j\,h[0] = -2 + j2,$
$H(e^{j\pi}) = h[0] - h[1] - h[1] + h[0] = 8$. Solving these two equations we get $h[0] = 1$ and $h[1] = -3$. Hence, $\{h[n]\} = \{1, \quad -3, \quad 3, \quad -1\},\ 0 \le n \prec 3.$

3.71 **(a)** $H(e^{j\omega}) = h[0] + h[1]e^{-j\omega} + h[2]e^{-j2\omega} = h[0] + h[1]e^{-j\omega} + h[0]e^{-j2\omega}$. The two conditions to be satisfied by the filter are:

$H(e^{j0.4\pi}) = h[0] + h[1]e^{-j0.4\pi} + h[0]e^{-j0.8\pi} = 0$ and

$H(e^{j0}) = h[0] + h[1] + h[0] = 1$. Solving these two equations we get $h[0] = 0.7236$ and $h[1] = -0.4472$.

(b) $H(e^{j\omega}) = 0.7236 - 0.4472\,e^{-j\omega} + 0.7236\,e^{-j2\omega}$.



3.72 **(a)** $H(e^{j\omega}) = h[0] + h[1]e^{-j\omega} + h[2]e^{-j2\omega} + h[3]e^{-j3\omega}$

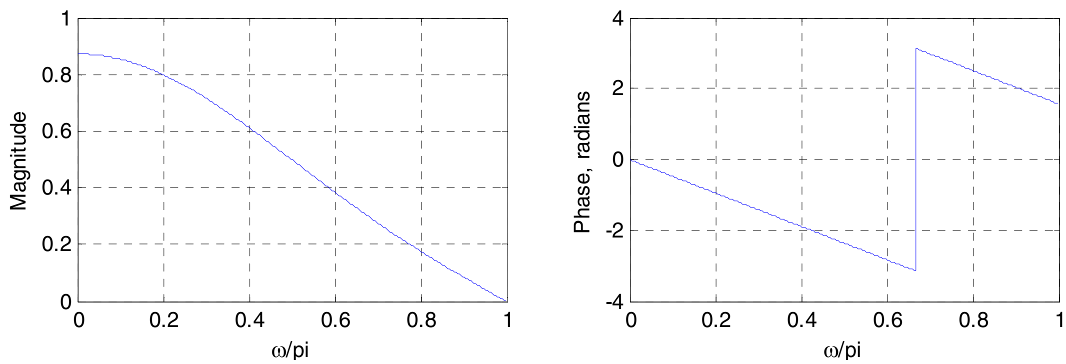
$= h[0] + h[1]e^{-j\omega} + h[1]e^{-j2\omega} + h[0]e^{-j3\omega} = e^{-j3\omega/2}\left(2h[0]\cos(3\omega/2) + 2h[1]\cos(\omega/2)\right)$.

The two conditions to be satisfied by the filter are:

$\left|H(e^{j0.2\pi})\right| = 2h[0]\cos(0.3\pi) + 2h[1]\cos(0.1\pi) = 0.8,$

$\left|H(e^{j0.5\pi})\right| = 2h[0]\cos(0.75\pi) + 2h[1]\cos(0.25\pi) = 0.5$. Solving these two equations we get $h[0] = 0.0414$ and $h[1] = 0.395$.

(b) $H(e^{j\omega}) = 0.0414 + 0.395\,e^{-j\omega} + 0.395\;e^{-j2\omega} + 0.0414\,e^{-j3\omega}$.



3.73 **(a)** $H(e^{j\omega}) = h[0] + h[1]e^{-j\omega} + h[2]e^{-j2\omega} + h[3]e^{-j3\omega}$

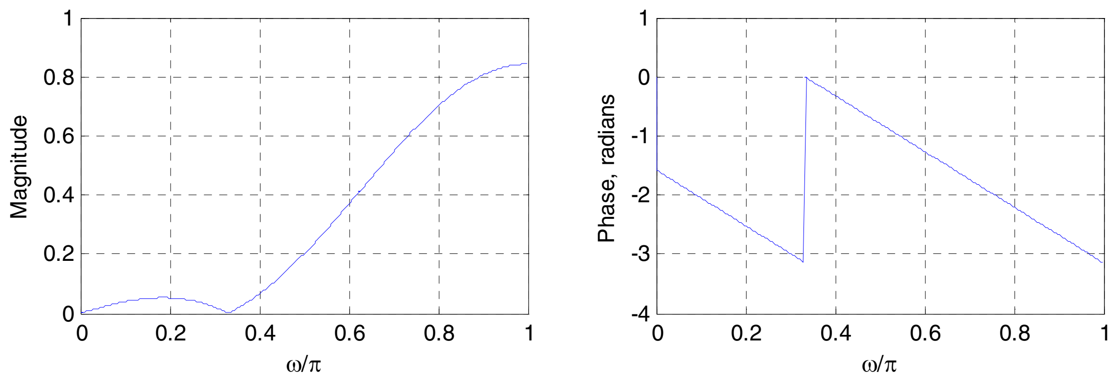
$= h[0] + h[1]e^{-j\omega} - h[1]e^{-j2\omega} - h[0]e^{-j3\omega} = j\,e^{-j3\omega/2}\left(2h[0]\sin s(3\omega/2) + 2h[1]\sin(\omega/2)\right).$

The two conditions to be satisfied by the filter are:

$\left|H(e^{j0.5\pi})\right| = 2h[0]\sin(0.75\pi) + 2h[1]\sin s(0.25\pi) = 0.2,$

$\left|H(e^{j0.8\pi})\right| = 2h[0]\sin(1.2\pi) + 2h[1]\cos(0.4\pi) = 0.7.$ Solving these two equations we get $h[0] = -0.14$ and $h[1] = 0.2815.$

(b) $H(e^{j\omega}) = -0.14 + 0.2815\,e^{-j\omega} - 0.2815\,e^{-j2\omega} + 0.14\,e^{-j3\omega}.$



3.74 (a) $H(e^{j\omega}) = h[0] + h[1]e^{-j\omega} + h[2]e^{-j2\omega} + h[3]e^{-j3\omega} + h[4]e^{-j4\omega}$

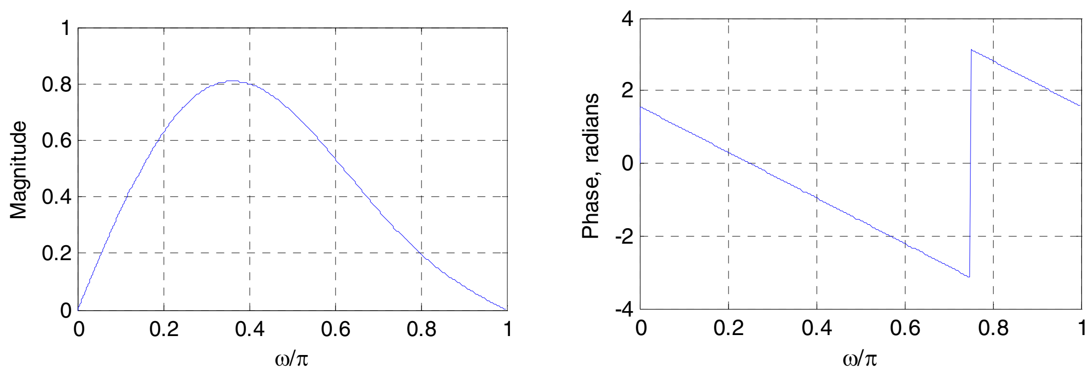
$= h[0] + h[1]e^{-j\omega} - h[1]e^{-j3\omega} - h[0]e^{-j4\omega} = j\,e^{-j2\omega}\left(2h[0]\sin s(2\omega) + 2h[1]\sin(\omega)\right).$

The two conditions to be satisfied by the filter are:

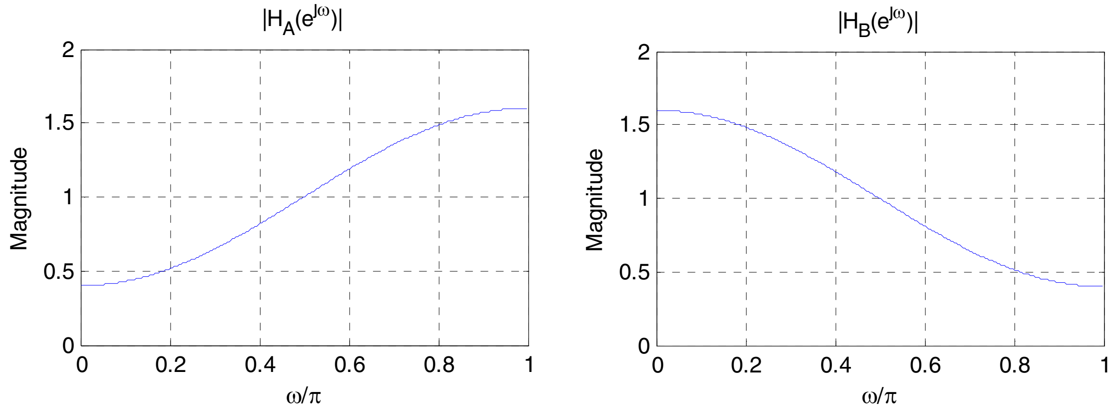
$\left|H(e^{j0.5\pi})\right| = 2h[0]\sin(0.8\pi) + 2h[1]\sin s(0.4\pi) = 0.8,$

$\left|H(e^{j0.8\pi})\right| = 2h[0]\sin(1.6\pi) + 2h[1]\cos(0.8\pi) = 0.2.$ Solving these two equations we get $h[0] = 0.112$ and $h[1] = 0.3514.$

(b) $H(e^{j\omega}) = 0.112 + 0.3514e^{-j\omega} - 0.3514\,e^{-j3\omega} - 0.112\,e^{-j4\omega}.$



3.75 **(a)** $H_A(e^{j\omega}) = 0.3 - e^{-j\omega} + 0.3e^{-j2\omega}$, $H_B(e^{j\omega}) = 0.3 + e^{-j\omega} + 0.3e^{-j2\omega}$.



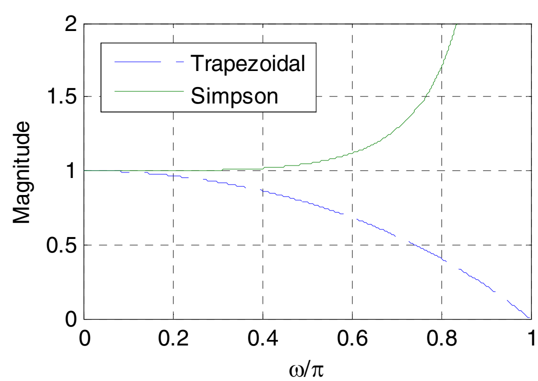
It can be seen from the above plots that $H_A(e^{j\omega})$ is a highpass filter, whereas $H_B(e^{j\omega})$ is a lowpass filter.

(b) $H_C(e^{j\omega}) = H_B(e^{j\omega}) = H_A(e^{j(\omega+\pi)})$.

3.76 $y[n] = y[n-1] + 0.5\big(x[n] + x[n-1]\big)$. Taking the DTFT of both sides we get $Y(e^{j\omega}) = e^{-j\omega}Y(e^{j\omega}) + 0.5\big(X(e^{j\omega}) + e^{-j\omega}X(e^{j\omega})\big)$. Hence, the frequency response is given by $H_{trap}(e^{j\omega}) = \dfrac{Y(e^{j\omega})}{X(e^{j\omega})} = \dfrac{1}{2}\left(\dfrac{1+e^{-j\omega}}{1-e^{-j\omega}}\right)$.

3.77 $y[n] = y[n-2] + \frac{1}{3}\big(x[n] + 4x[n-1] + x[n-2]\big)$. Hence,

$$H_{simpson}(e^{j\omega}) = \frac{1}{3}\left(\frac{1+4e^{-j\omega}+e^{-j2\omega}}{1-e^{-j2\omega}}\right).$$



Note: To compare the performances of the Trapezoidal numerical integration formula with that of the Simpson's formula, we first observe that if the input is $x_a(t) = e^{j\omega t}$, then the result of integration is $y_a(t) = \frac{1}{j\omega} e^{j\omega t}$. Thus, the desired ideal frequency response is $H(e^{j\omega}) = \frac{1}{j\omega}$. Hence, we take the ratio of the frequency responses of the approximation to the ideal, and plot the two curves as indicated on the previous page. From this plot, it is evident that the Simpson's formula amplifies high frequencies, whereas, the trapezoidal formula attenuates them. In the very low frequency range, both formulae yield results close to the ideal. However, Simpson's formula is reasonably accurate for frequencies close to the midband range.

3.78 $G(e^{j\omega}) = g_0 + g_1 e^{-j\omega} + g_2 e^{-j2\omega} + g_3 e^{-j3\omega}$

$$= e^{-j3\omega/2}\left(g_0 e^{j3\omega/2} + g_1 e^{j\omega/2} + g_2 e^{-j\omega/2} + g_0 e^{-j3\omega/2}\right)$$

$$= e^{-j3\omega/2}[g_0 \cos(3\omega/2) + jg_0 \sin(3\omega/2) + g_3 \cos(-3\omega/2) + jg_3 \sin(-3\omega/2)$$
$$+ g_1 \cos(\omega/2) + jg_1 \sin(\omega/2) + g_2 \cos(-\omega/2) + jg_2 \sin(-\omega/2)]]$$

$$= e^{-j3\omega/2}[(g_0 + g_3)\cos(3\omega/2) + (g_1 + g_2)\cos(\omega/2)$$
$$+ j(g_0 - g_3)\sin(3\omega/2) + j(g_1 - g_2)\cos(\omega/2)].$$

Thus, if $g_0 = g_3$ and $g_1 = g_2$, then

$G(e^{j\omega}) = e^{-j3\omega/2}[(g_0 + g_3)\cos(3\omega/2) + (g_1 + g_2)\cos(\omega/2)]$ which has a linear phase $\theta(\omega) = -\frac{3\omega}{2}$, and hence, a constant group delay.

Alternately, if $g_0 = -g_3$ and $g_1 = -g_2$, then

$G(e^{j\omega}) = j\, e^{-j3\omega/2}[(g_0 + g_3)\sin(3\omega/2) + (g_1 + g_2)\sin(\omega/2)]$ which has a linear phase $\theta(\omega) = -\frac{3\omega}{2} + \frac{\pi}{2}$, and hence, a constant group delay.

3.79 **(a)** $H_a(e^{j\omega}) = a + b\,e^{-j\omega} = a + b\cos\omega - j\,b\sin\omega$. Thus, $\theta_{H_a}(\omega) = \tan^{-1}\left(\frac{-b\sin\omega}{a + b\cos\omega}\right)$.

Hence, $\tau_{H_a}(\omega) = -\frac{d\theta_{H_a}(\omega)}{d\omega} = -\frac{1}{1 + \left(\frac{-b\sin\omega}{a + b\cos\omega}\right)^2} \cdot \frac{d}{d\omega}\left(\frac{-b\sin\omega}{a + b\cos\omega}\right)$

$$= -\frac{(a + b\cos\omega)^2}{(a + b\cos\omega)^2 + (b\sin\omega)^2} \cdot \frac{-(a + b\cos\omega)b\cos\omega - (-b\sin\omega)(-b\sin\omega)}{(a + b\cos\omega)^2}$$

$$= -\frac{-ab\cos\omega - b^2\cos^2\omega - b^2\sin^2\omega}{a^2 + b^2\cos^2\omega + 2ab\cos\omega + b^2\sin^2\omega} = \frac{b^2 + ab\cos\omega}{a^2 + b^2 + 2ab\cos\omega}.$$

(b) Let $G_b(e^{j\omega}) = 1 + c\,e^{-j\omega} = 1 + c\cos\omega - j\,c\sin\omega$. From the results of Part (a) we have $\tau_{G_b}(\omega) = \dfrac{c^2 + c\cos\omega}{1 + c^2 + 2c\cos\omega}$. Since $H_b(e^{j\omega}) = \dfrac{1}{G_b(e^{j\omega})} = \dfrac{1}{1 + c\,e^{-j\omega}}$, $\theta_{H_b}(\omega) = -\theta_{G_b}(\omega)$, we have $\tau_{H_b}(\omega) = -\tau_{G_b}(\omega) = -\dfrac{c^2 + c\cos\omega}{1 + c^2 + 2c\cos\omega}$.

(c) $H_c(e^{j\omega}) = \dfrac{a + b\,e^{-j\omega}}{1 + c\,e^{-j\omega}} = H_a(e^{j\omega})H_b(e^{j\omega})$, where $H_a(e^{j\omega})$ is the frequency response of Part (a) and $H_b(e^{j\omega})$ is the frequency response of Part (b). Thus, $\theta_{H_c}(\omega) = \theta_{H_a}(\omega) + \theta_{H_b}(\omega)$, and therefore, $\tau_{H_c}(\omega) = \tau_{H_a}(\omega) + \tau_{H_b}(\omega)$
$= \dfrac{b^2 + ab\cos\omega}{a^2 + b^2 + 2ab\cos\omega} - \dfrac{c^2 + c\cos\omega}{1 + c^2 + 2c\cos\omega}$.

(d) $H_d(e^{j\omega}) = \dfrac{1}{1 + c\,e^{-j\omega}} \cdot \dfrac{1}{1 + d\,e^{-j\omega}} = H_b(e^{j\omega})H_e(e^{j\omega})$, where $H_b(e^{j\omega})$ is the frequency response of Part (b) and $H_e(e^{j\omega})$ is similar in form to $H_b(e^{j\omega})$. Thus, $\theta_{H_d}(\omega) = \theta_{H_d}(\omega) + \theta_{H_e}(\omega)$, and therefore, $\tau_{H_d}(\omega) = \tau_{H_b}(\omega) + \tau_{H_e}(\omega)$
$= \dfrac{c^2 + c\cos\omega}{1 + c^2 + 2c\cos\omega} + \dfrac{d^2 + d\cos\omega}{1 + d^2 + 2d\cos\omega}$.

3.80 The group delay of a causal LTI discrete-time system with a frequency response $H(e^{j\omega}) = \left|H(e^{j\omega})\right| e^{j\,\theta(\omega)}$ is given by $\tau_g(\omega) = -\dfrac{d\theta(\omega)}{d\omega}$. Now,

$$\frac{dH(e^{j\omega})}{d\omega} = e^{j\theta(\omega)}\frac{d\left|H(e^{j\omega})\right|}{d\omega} + j\left|H(e^{j\omega})\right| e^{j\theta(\omega)}\frac{d\theta(\omega)}{d\omega}.$$ Hence,

$$-j\left|H(e^{j\omega})\right| e^{j\theta(\omega)}\frac{d\theta(\omega)}{d\omega} = e^{j\theta(\omega)}\frac{d\left|H(e^{j\omega})\right|}{d\omega} - \frac{dH(e^{j\omega})}{d\omega},$$ or, equivalently,

$$-\frac{d\theta(\omega)}{d\omega} = \frac{e^{j\theta(\omega)}}{j\left|H(e^{j\omega})\right| e^{j\theta(\omega)}}\cdot\frac{d\left|H(e^{j\omega})\right|}{d\omega} - \frac{1}{j\left|H(e^{j\omega})\right| e^{j\theta(\omega)}}\cdot\frac{dH(e^{j\omega})}{d\omega}$$

$$= \frac{1}{j\left|H(e^{j\omega})\right|}\cdot\frac{d\left|H(e^{j\omega})\right|}{d\omega} + j\frac{1}{H(e^{j\omega})}\cdot\frac{dH(e^{j\omega})}{d\omega}.$$ The first term on the right-hand side of the above equation is purely imaginary. Hence,

$$\tau_g(\omega) = -\frac{d\theta(\omega)}{d\omega} = \text{Re}\left\{\frac{j\frac{dH(e^{j\omega})}{d\omega}}{H(e^{j\omega})}\right\}.$$

3.81 Since $G(e^{j\omega})$ is the Fourier transform of $ng[n]$, $G(e^{j\omega}) = j\frac{dH(e^{j\omega})}{d\omega}$. Rewriting Eq. (3.127) we get $\tau_g(\omega) = \frac{1}{2}\left\{\frac{j\frac{dH(e^{j\omega})}{d\omega}}{H(e^{j\omega})} + \left(\frac{j\frac{dH(e^{j\omega})}{d\omega}}{H(e^{j\omega})}\right)^*\right\} = \frac{1}{2}\left\{\frac{j\frac{dH(e^{j\omega})}{d\omega}}{H(e^{j\omega})} + \frac{\left(j\frac{dH(e^{j\omega})}{d\omega}\right)^*}{H^*(e^{j\omega})}\right\}$

$$= \frac{1}{2}\left\{\frac{j\frac{dH(e^{j\omega})}{d\omega}H^*(e^{j\omega}) + \left(j\frac{dH(e^{j\omega})}{d\omega}\right)^* H(e^{j\omega})}{|H(e^{j\omega})|^2}\right\}$$

$$= \frac{1}{2|H(e^{j\omega})|^2}\Big[\left(G_{re}(e^{j\omega}) + jG_{im}(e^{j\omega})\right)\left(H_{re}(e^{j\omega}) + jH_{im}(e^{j\omega})\right)^*$$

$$+\left(G_{re}(e^{j\omega}) + jG_{im}(e^{j\omega})\right)\left(H_{re}(e^{j\omega}) + jH_{im}(e^{j\omega})\right)^*\Big]$$

$$= \frac{1}{2|H(e^{j\omega})|^2}\left[2G_{re}(e^{j\omega})H_{re}(e^{j\omega}) + 2G_{im}(e^{j\omega})H_{im}(e^{j\omega})\right]$$

$$= \frac{G_{re}(e^{j\omega})H_{re}(e^{j\omega}) + G_{im}(e^{j\omega})H_{im}(e^{j\omega})}{|H(e^{j\omega})|^2}.$$

3.82 **(a)** $H_a(e^{j\omega}) = 1 + 0.4e^{-j\omega} = 1 + 0.4\cos\omega - j0.4\sin\omega$ and thus,
$\text{Re}\{H_a(e^{j\omega})\} = 1 + 0.4\cos\omega$ and $\text{Im}\{H_a(e^{j\omega})\} = -0.4\sin\omega$..

$G_a(e^{j\omega}) = j\frac{dH_a(e^{j\omega})}{d\omega} = j(-j0.4e^{-j\omega}) = 0.4\cos\omega - j0.4\sin\omega$. Thus,
$\text{Re}\{G_a(e^{j\omega})\} = 0.4\cos\omega$ and $\text{Im}\{G_a(e^{j\omega})\} = -0.4\sin\omega$.. Therefore, using Eq. (3.128) we get $\tau_{H_a}(\omega) = \frac{(1+0.4\cos\omega)(0.4\cos\omega) + (-0.4\sin\omega)^2}{(1+0.4\cos\omega)^2 + (-0.4\sin\omega)^2}$

$$= \frac{0.16\cos^2\omega + 0.16\sin^2\omega + 0.4\cos\omega}{1 + 0.16\cos^2\omega + 0.16\sin^2\omega + 0.8\cos\omega} = \frac{0.16 + 0.4\cos\omega}{1.16 + 0.8\cos\omega}.$$

(b) Let $G_b(e^{j\omega}) = \dfrac{1}{H_b(e^{j\omega})} = 1 + 0.6\,e^{-j\omega}$. Then $\tau_{G_b}(\omega) = -\tau_{H_b}(\omega)$. Then using the same procedure as in Part (a) we get $\tau_{G_b}(\omega) = \dfrac{0.36 + 0.6\cos\omega}{1.36 + 1.2\cos\omega}$. Therefore, $\tau_{H_b}(\omega) = -\dfrac{0.36 + 0.6\cos\omega}{1.36 + 1.2\cos\omega}$.

(c) Let $H_c(e^{j\omega}) = (1 - 0.5\,e^{-j\omega})\left(\dfrac{1}{1 + 0.3\,e^{-j\omega}}\right) = G_a(e^{j\omega})G_b(e^{j\omega})$, where $G_a(e^{j\omega}) = 1 - 0.5\,e^{-j\omega}$ and $G_b(e^{j\omega}) = \dfrac{1}{1 + 0.3\,e^{-j\omega}}$. Therefore, $\tau_{H_c}(\omega) = \tau_{G_a}(\omega) + \tau_{G_b}(\omega)$. Then using the same procedure as in Part (a) we get $\tau_{G_a}(\omega) = \dfrac{0.25 - 0.5\cos\omega}{1.25 - \cos\omega}$ and using the same procedure as in Part (b) we get $\tau_{G_b}(\omega) = -\dfrac{0.09 + 0.3\cos\omega}{1.09 + 0.6\cos\omega}$. Hence, $\tau_{H_c}(\omega) = \dfrac{0.25 - 0.5\cos\omega}{1.25 - \cos\omega} - \dfrac{0.09 + 0.3\cos\omega}{1.09 + 0.6\cos\omega}$.

(d) Let $H_d(e^{j\omega}) = \left(\dfrac{1}{1 - 0.3\,e^{-j\omega}}\right)\left(\dfrac{1}{1 + 0.5\,e^{-j\omega}}\right) = G_a(e^{j\omega})G_b(e^{j\omega})$, where $G_a(e^{j\omega}) = \dfrac{1}{1 - 0.3\,e^{-j\omega}}$ and $G_b(e^{j\omega}) = \dfrac{1}{1 + 0.5\,e^{-j\omega}}$. Therefore, $\tau_{H_d}(\omega) = \tau_{G_a}(\omega) + \tau_{G_b}(\omega)$. Then using the same procedure as in Part (b) we get $\tau_{G_a}(\omega) = \dfrac{-0.09 + 0.3\cos\omega}{1.09 - 0.6\cos\omega}$ and $\tau_{G_a}(\omega) = \dfrac{-0.25 - 0.5\cos\omega}{1.25 + \cos\omega}$. Hence,

$$\tau_{G_a}(\omega) = -\left(\frac{0.09 - 0.3\cos\omega}{1.09 - 0.6\cos\omega} + \frac{0.25 + 0.5\cos\omega}{1.25 + \cos\omega}\right).$$

3.83 From Table 3.3, $H(e^{j\omega}) = \dfrac{1}{1 + 0.5\,e^{-j\omega}} = \dfrac{1}{1 + 0.5\cos\omega - j0.5\sin\omega}$. Thus,

$\left|H(e^{j\omega})\right| = \dfrac{1}{\sqrt{(1 + 0.5\cos\omega)^2 + (0.5\sin\omega)^2}} = \dfrac{1}{\sqrt{1.25 + \cos\omega}}$ and

$\arg\{H(e^{j\omega})\} = \theta(\omega) = -\tan^{-1}\left(\dfrac{-0.5\sin\omega}{1 + 0.5\cos\omega}\right) = \tan^{-1}\left(\dfrac{0.5\sin\omega}{1 + 0.5\cos\omega}\right)$. Now

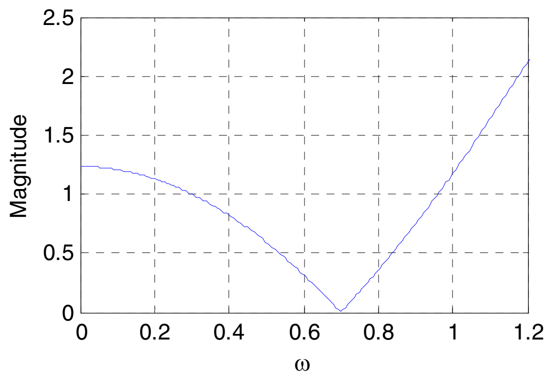
$H(e^{j\pi/5}) = \dfrac{1}{1 + 0.5\cos(\pi/5) - j0.5\sin(\pi/5)} = 0.6821 + \text{j}\ 0.1427$. Therefore,

$\left|H(e^{j\pi/5})\right| = 0.6969$ and $\arg\{H(e^{j\pi/5})\} = \theta(\pi/5) = \tan^{-1}\left(\frac{0.1427}{0.6821}\right) = 0.2063$ radians.

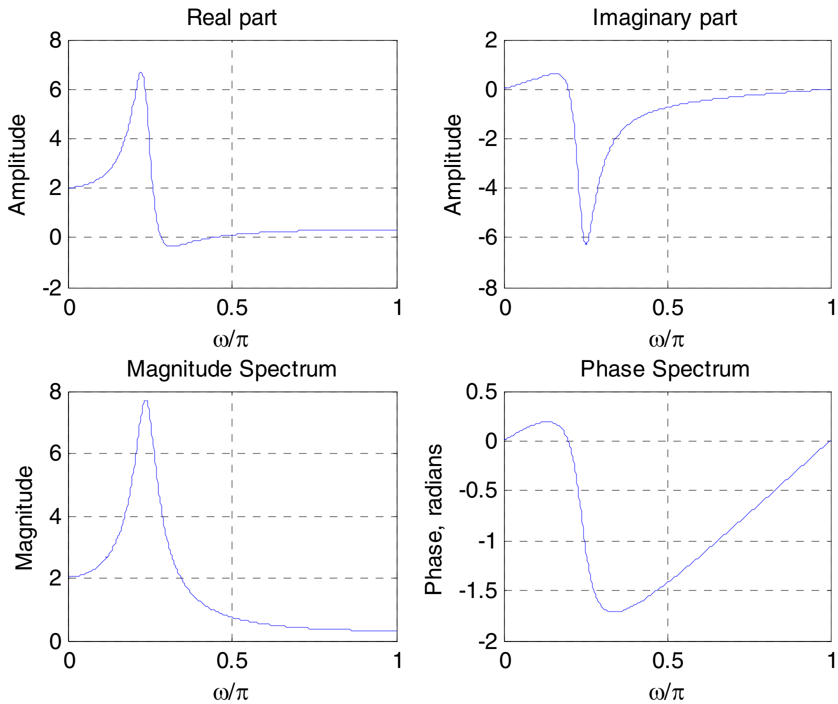
Since for a frequency response with real coefficient impulse response, $\left|H(e^{j\omega})\right|$ is an even function of ω and $\theta(\omega)$ is an odd function of ω, we have $\left|H(e^{-j\pi/5})\right| = 0.6969$ and $\theta(-\pi/5) = -0.2063.$

Now, for an input $x[n] = \sin(\omega_o n)\mu[n]$, the steady-state output is given by $y[n] = \left|H(e^{j\omega_o})\right|\sin\left(\omega_o n + \theta(\omega_o)\right).$ Thus, for $\omega_o = \pi/5$, the steady-state output is given by $y[n] = \left|H(e^{j\pi/5})\right|\sin\left(\frac{\pi}{5}n + \theta(\pi/5)\right) = 0.6969\sin\left(\frac{\pi}{5}n + 0.2063\right).$

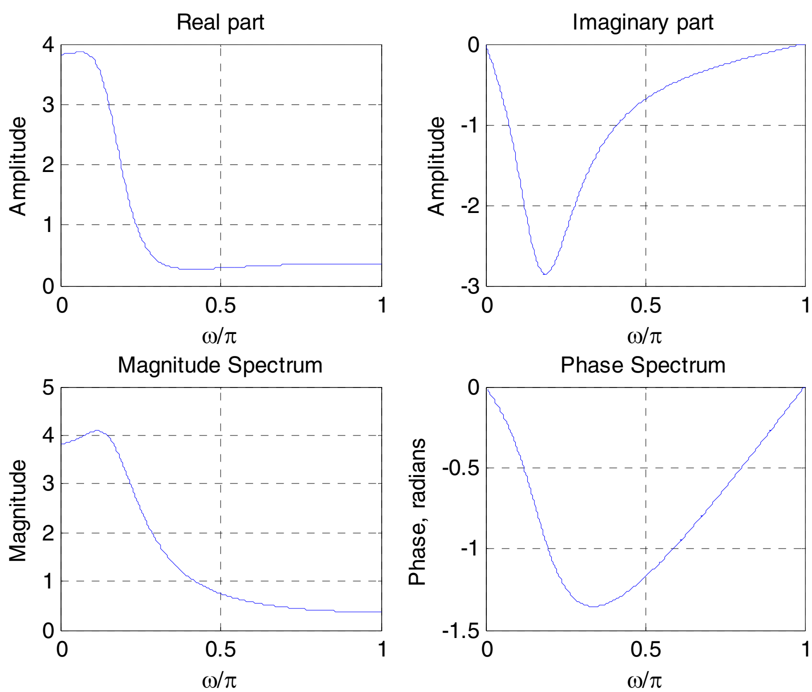
3.84 $H(e^{j\omega}) = h[0]\left(1 + e^{-j2\omega}\right) + h[1]e^{-j\omega} = e^{-j\omega}\left(2h[0]\cos\omega + h[1]\right).$ We require $2h[0]\cos(0.3) + h[1] = 1$ and $2h[0]\cos(0.7) + h[1] = 0.$ Solving these two equations we get $h[0] = 2.6248$ and $h[1] = -4.015.$



M3.1 $r = 0.9$, $\theta = 0.75$



$r = 0.7$, $\theta = 0.5$

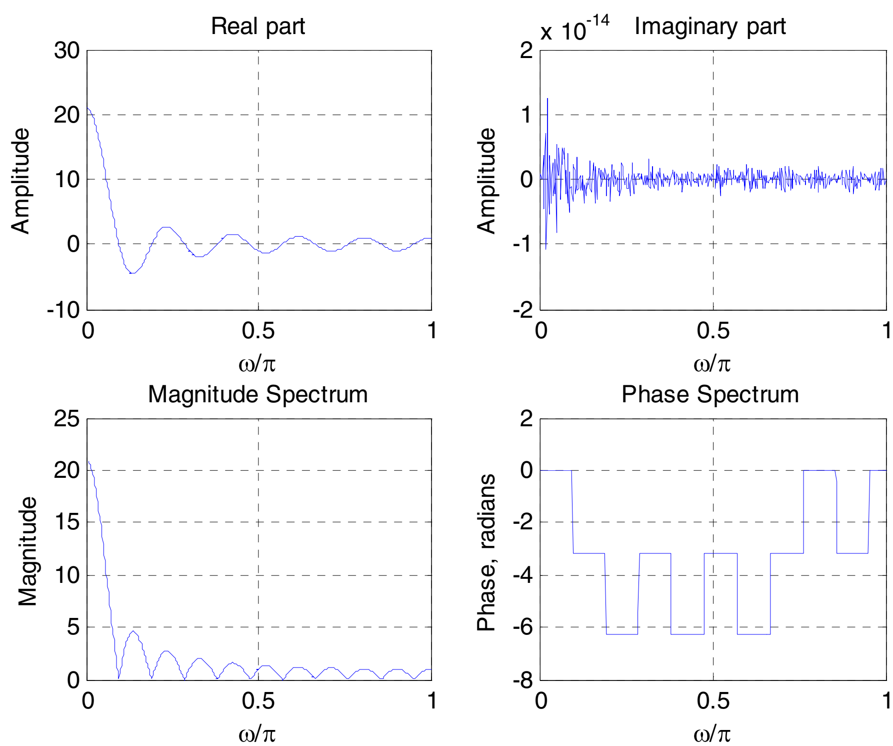


M3.2 It should be noted that Program 3_1.m uses the function `freqz` to determine the samples of a DTFT that is rational function in $e^{-j\omega}$, i.e., a ratio of polynomials in $e^{-j\omega}$. Their inverse DTFTs are two-sided sequences. However, all sequences of

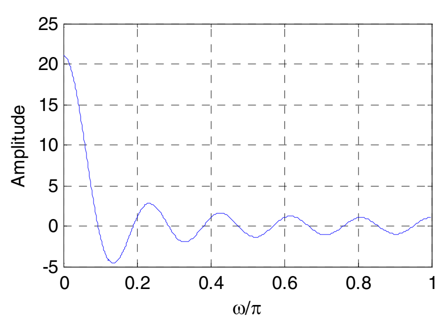
Problem 3.19 except that in Part (b) are two-sided finite-length sequences of length $2N+1$, and their DTFTs have both positive and negative powers of $e^{j\omega}$. As a result, the frequency sample computed using `freqz` should be multiplied by the vector $e^{j\omega_n N}$ evaluated at the frequency points ω_n used in `freqz`. In Parts (a), (c) and (d), the phase spectra are the plots of the unwrapped phase obtained using the function `unwrap`.

Moreover, the DTFTs of the sequences in Parts (a), (c) and (d) are real functions of ω and thus have zero phase. More accurate plots of the DTFTs are obtained using the function `zerophase`.

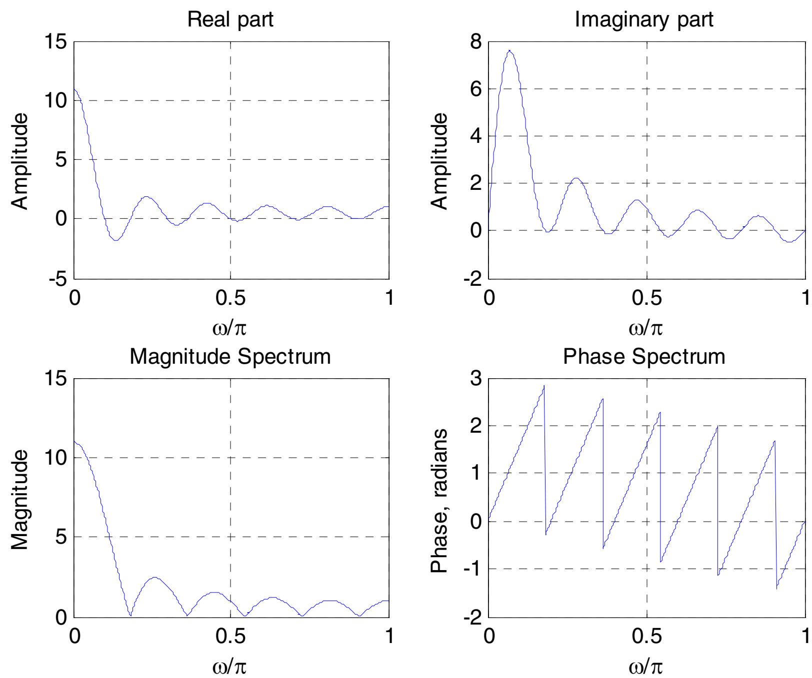
(a) $y_1[n] = \begin{cases} 1, & -10 \le n \le 10, \\ 0, & \text{otherwise}, \end{cases}$ $Y_1(e^{j\omega}) = \dfrac{\sin(21\omega/2)}{\sin(\omega/2)}$. The plots obtained using Program 3_1.m are shown below:



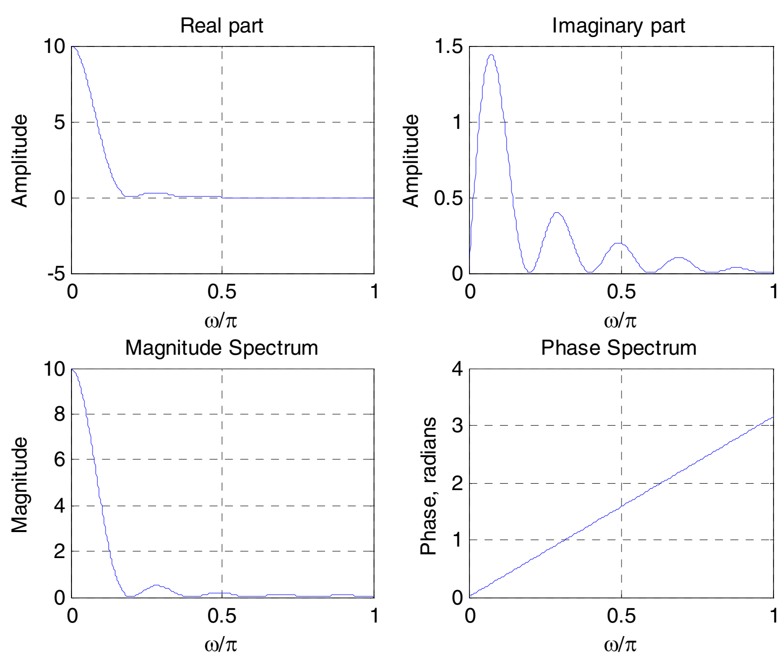
The plot obtained using the function `zerophase` is shown below:



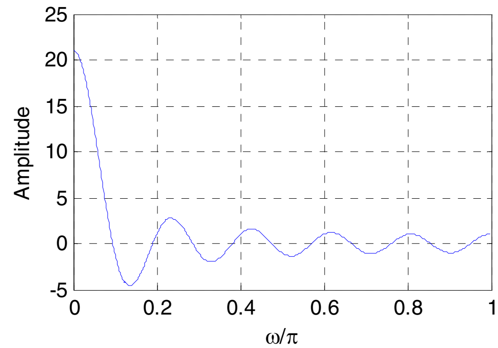
(b) $y_2[n] = \begin{cases} 1, & 0 \le n \le N, \\ 0, & \text{otherwise.} \end{cases}$ Then $Y_2(e^{j\omega}) = e^{-j\omega N/2}\left(\frac{\sin(\omega[N+1]/2)}{\sin(\omega/2)}\right)$. The plots obtained using Program 3_1.m are shown below:



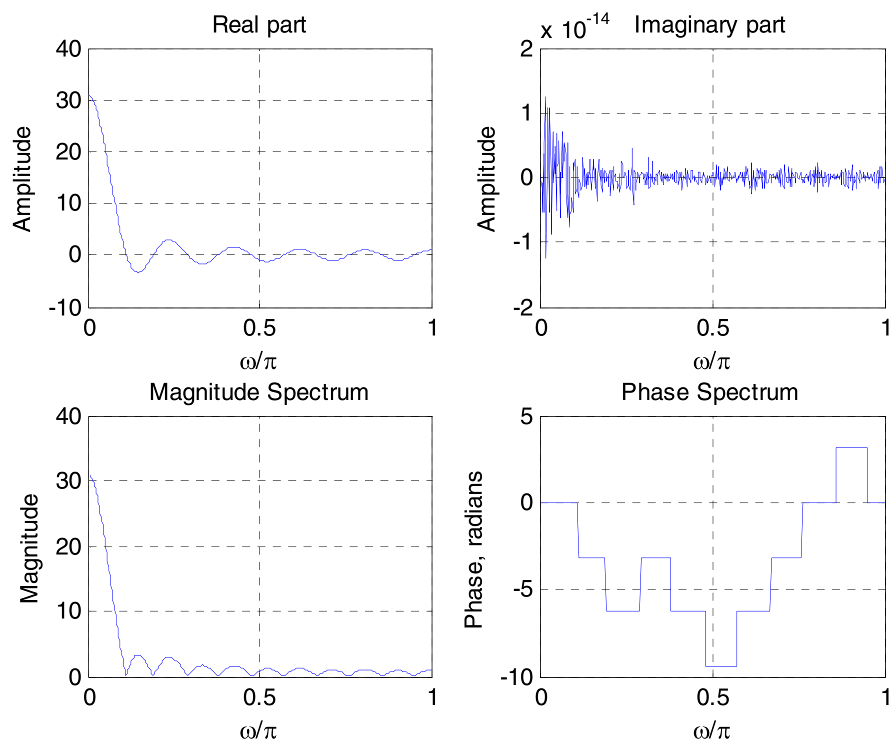
(c) $y_3[n] = \begin{cases} 1 - \frac{|n|}{N}, & -N \le n \le N, \\ 0, & \text{otherwise.} \end{cases}$ $Y_3(e^{j\omega}) = \frac{1}{N} \cdot \frac{\sin^2(\omega N/2)}{\sin^2(\omega/2)}$. The plots obtained using Program 3_1.m are shown below:



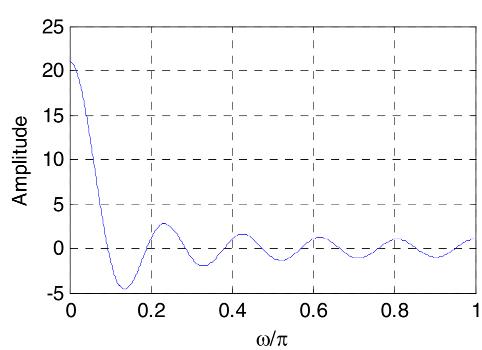
The plot obtained using the function `zerophase` is shown below:



(d) $y_4[n] = \begin{cases} N + 1 - \frac{|n|}{N}, & -N \le n \le N, \\ 0, & \text{otherwise.} \end{cases}$ $Y_4(e^{j\omega}) = N \cdot \frac{\sin\left(\omega[N + \frac{1}{2}]\right)}{\sin(\omega/2)} + \frac{1}{N} \cdot \frac{\sin^2(\omega N/2)}{\sin^2(\omega/2)}.$

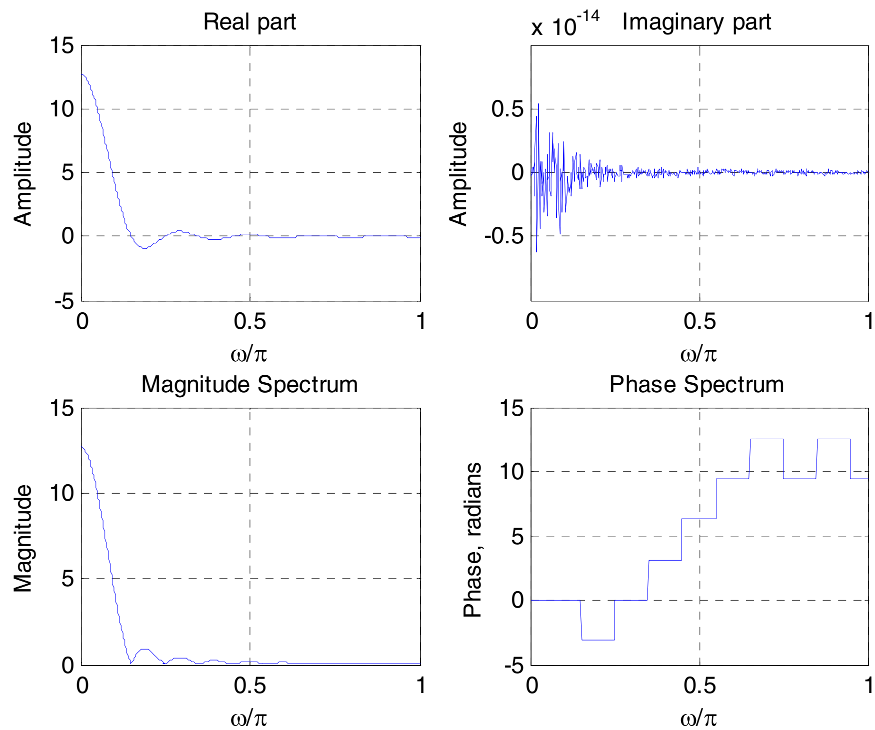


The plot obtained using the function `zerophase` is shown below:

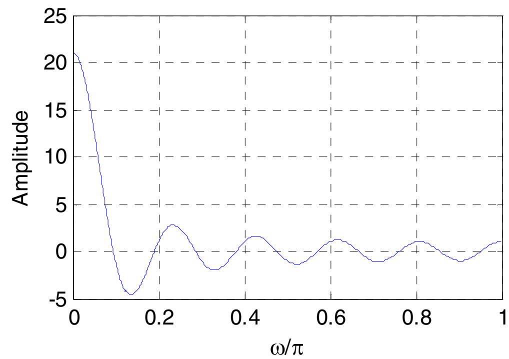


(e) $y_5[n] = \begin{cases} \cos(\pi n / 2N), & -N \le n \le N, \\ 0, & \text{otherwise.} \end{cases}$

$Y_5(e^{j\omega}) = \frac{1}{2} \cdot \frac{\sin\left((\omega - \frac{\pi}{2N})(N+\frac{1}{2})\right)}{\sin\left((\omega - \frac{\pi}{2N})/2\right)} + \frac{1}{2} \cdot \frac{\sin\left((\omega + \frac{\pi}{2N})(N+\frac{1}{2})\right)}{\sin\left((\omega + \frac{\pi}{2N})/2\right)}$. The plots obtained using Program 3_1.m are shown below:

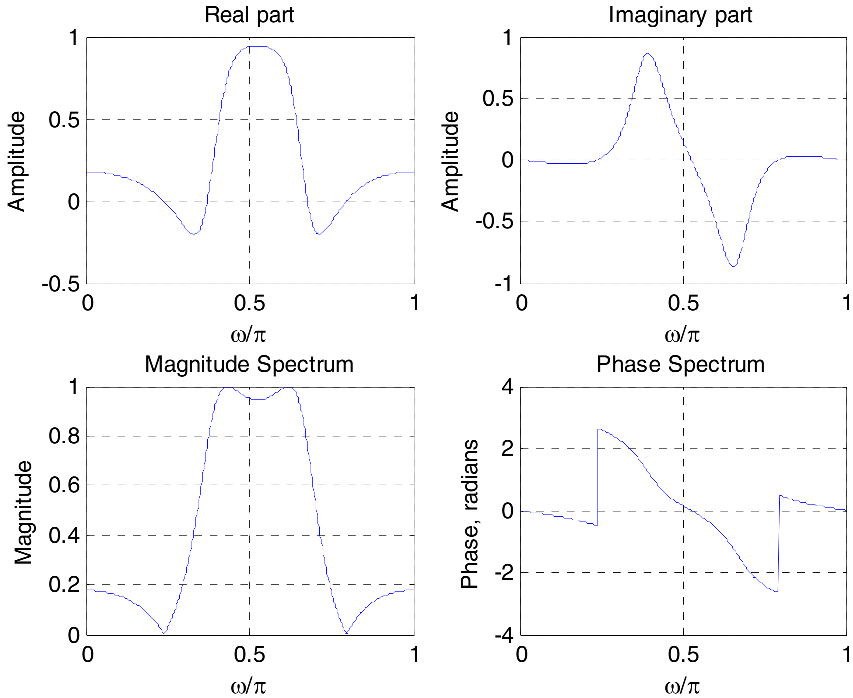


The plot obtained using the function `zerophase` is shown below:



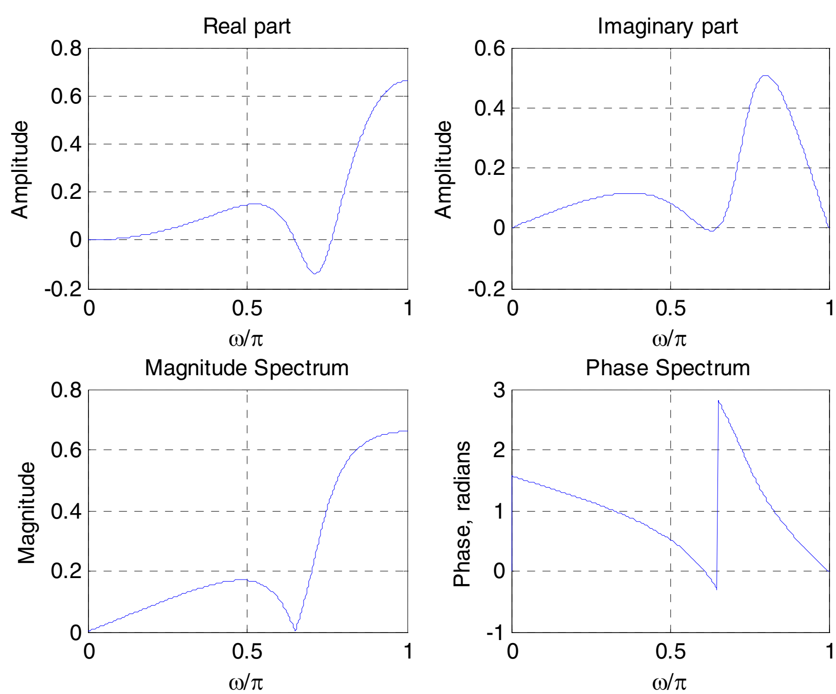
M3.3 **(a)** $X(e^{j\omega}) = \frac{0.2418(1 + 0.139 e^{-j\omega} - 0.3519\, e^{-j2\omega} + 0.139 e^{-j3\omega} + e^{-j4\omega})}{1 + 0.2386 e^{-j\omega} + 0.8258\, e^{-j2\omega} + 0.1393 e^{-j3\omega} + 0.4153 e^{-j4\omega}}$

The plots obtained using Program 3_1.m are shown below:



(b) $X(e^{j\omega}) = \dfrac{0.1397(1 - 0.0911\,e^{-j\omega} + 0.0911\,e^{-j2\omega} - e^{-j3\omega})}{1 + 1.1454\,e^{-j\omega} + 0.7275\,e^{-j2\omega} + 0.1205\,e^{-j3\omega}}.$

The plots obtained using Program 3_1.m are shown below:



M3.4 `% Property 1`

```
N = 8; % Number of samples in sequence
gamma = 0.5; k = 0:N-1;
x = exp(-j*gamma*k); y = exp(-j*gamma*fliplr(k));
% r = x[-n] then y = r[n-(N-1)]
% so if X1(exp(jw)) is DTFT of x[-n], then
% X1(exp(jw)) = R(exp(jw)) = exp(jw(N-1))Y(exp(jw))
[Y,w] = freqz(y,1,512);
X1 = exp(j*w*(N-1)).*Y;
m = 0:511; w = -pi*m/512;
X = freqz(x,1,w);
% Verify X = X1

% Property 2
k = 0:N-1; y = exp(j*gamma*fliplr(k));
[Y,w] = freqz(y,1,512);
X1 = exp(j*w*(N-1)).*Y;
[X,w] = freqz(x,1,512);
% Verify X1 = conj(X)

% Property 3
y = real(x);
[Y3,w] = freqz(y,1,512);
m = 0:511; w0 = -pi*m/512;
X1 = freqz(x,1,w0);
[X,w] = freqz(x,1,512);
% Verify Y3 = 0.5*(X+conj(X1))

% Property 4
y = j*imag(x); [Y4,w] = freqz(y,1,512);
% Verify Y4 = 0.5*(X-conj(X1))

% Property 5
k = 0:N-1; y = exp(-j*gamma*fliplr(k));
xcs = 0.5*[zeros(1,N-1) x] + 0.5*[conj(y) zeros(1,N-1)];
xacs = 0.5*[zeros(1,N-1) x] - 0.5*[conj(y) zeros(1,N-1)];
[Y5,w] = freqz(xcs,1,512);
[Y6,w] = freqz(xacs,1,512);
Y5 = Y5.*exp(j*w*(N-1));
Y6 = Y6.*exp(j*w*(N-1));
% Verify Y5 = real(X) and Y6 = j*imag(X)
```

M3.5

```
N = 8; % Number of samples in sequence
gamma = 0.5; k = 0:N-1;
x = exp(gamma*k); y = exp(gamma*fliplr(k));
xev = 0.5*([zeros(1,N-1) x] + [y zeros(1,N-1)]);
xod = 0.5*([zeros(1,N-1) x] - [y zeros(1,N-1)]);
[X,w] = freqz(x,1,512);
[Xev,w] = freqz(xev,1,512);
[Xod,w] = freqz(xod,1,512);
Xev = exp(j*w*(N-1)).*Xev;
```

```
Xod = exp(j*w*(N-1)).*Xod;
% Verify real(X) = Xev, and j*imag(X) = Xod
```

M3.5

```
N = input('The length of the seqeunce = ');
k = 0:N-1; gamma = -0.5;
g = exp(gamma*k);
% g is an exponential seqeunce
h = sin(2*pi*k/(N/2));
% h is a sinusoidal sequence with period  = N/2
[G,w] = freqz(g,1,512); [H,w] = freqz(h,1,512);

% Property 1
alpha = 0.5; beta = 0.25;
y = alpha*g+beta*h; [Y,w] =
freqz(y,1,512);
% Plot Y and alpha*G+beta*H to verify that they are equal

% Property 2
n0 = 12; % Sequence shifted by 12 samples
y2 = [zeros(1,n0) g];
[Y2,w] = freqz(y2,1,512);
G0 = exp(-j*w*n0).*G;
% Plot G0 and Y2 to verify they are equal

% Property 3
w0 = pi/2; % the value of omega0 = pi/2
r = 256; % the value of omega0 in terms of number of
samples
k = 0:N-1; y3 = g.*exp(j*w0*k);
[Y3,w] = freqz(y3,1,512);
k = 0:511;
w = -w0+pi*k/512; % creating G(exp(w-w0))
G1 = freqz(g,1,w);
% Compare G1 and Y3

% Property 4
k = 0:N-1; y4 = k.*g;
[Y4,w] = freqz(y4,1,512);
% To compute derivative we need sample at pi
y0 = ((-1).^k).*g;
G2 = [G(2:512)' sum(y0)]';
delG = (G2-G)*512/pi;
% Compare Y4, delG

% Property 5
y5 = conv(g,h);
[Y5,w] = freqz(y5,1,512);
% Compare Y5 and G.*H

% Property 6
```

```
y6 = g.*h;
[Y6,w] = freqz(y6,1,512,'whole');
[G0,w] = freqz(g,1,512,'whole');
[H0,w] = freqz(h,1,512,'whole');
% Evaluate the sample value at w = pi/2
% and verify with Y6 at pi/2
H1 = [fliplr(H0(1:129)') fliplr(H0(130:512)')]';
val = 1/(512)*sum(G0.*H1);
% Compare val with Y6(129) i.e., sample at pi/2
% Can extend this to other points similarly

% Parsevals theorem
val1 m = sum(g.*conj(h)); val2 = sum(G0.*conj(H0))/512;
% Compare val1 with val2
```

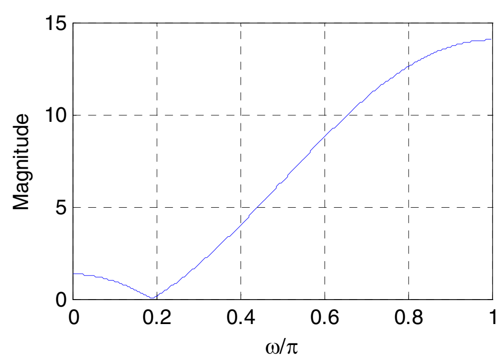
M3.7 The DTFT of $nh[n]$ is $j\frac{dH(e^{j\omega})}{d\omega}$. Hence, the group delay $\tau_g(\omega)$ can be computed at a set of N discrete frequency points $\omega_k = 2\pi k/N$, $0 \le k \le N-1$, as follows:

$$\tau_g(\omega_k) = \mathrm{Re}\left(\frac{DFT\{n\,h[n]\}}{DFT\{h[n]\}}\right),$$

where all DFTs are N–points in length with N greater than or equal to the length of $\{h[n]\}$.

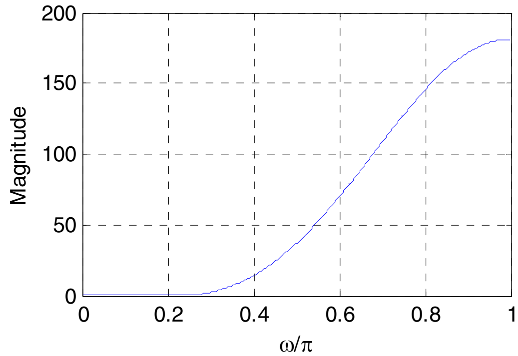
M3.8

```
h = [3.8461 -6.3487 3.8461];
[H,w] = freqz(h,1,512);
plot(w/pi,abs(H)); grid
xlabel('\omega/\pi'); ylabel('Magnitude');
```



M3.9

```
h = [-13.4866 45.228 -63.8089 45.228 -13.4866];
[H,w] = freqz(h,1,512);
plot(w/pi,abs(H)); grid
xlabel('\omega/\pi'); ylabel(Magnitude);
```


200
150
100
50
0
Magnitude
0
0.2
0.4
0.6
0.8
1
ω/π