Space Vectors Representation

- Voltage and Current Space Vectors
- Physical Interpretation of Current Space vectors
- Space Vector Components
- Balanced Sinusoidal Steady-State Excitation (Rotor Open-Circuited)
- Relation Between Space Vectors and Phasors
- Voltages in the stator windings

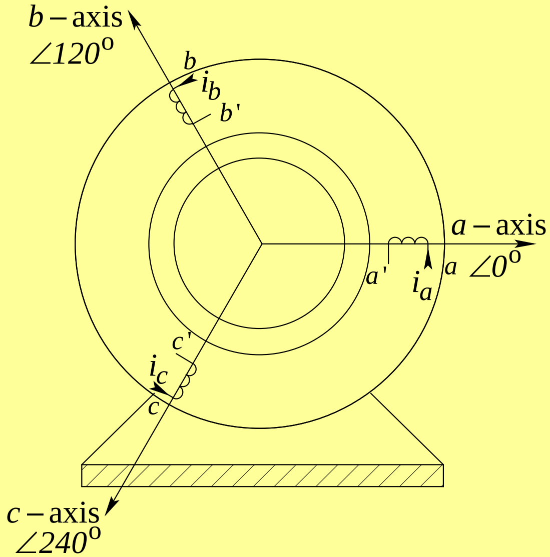
Space Vectors Representation of Combined Phase Currents and Voltages

- Mathematical concept

At time t

$$\vec{i}_s(t) = i_a(t)\angle 0^0 + i_b(t)\angle 120^0 + i_c(t)\angle 240^0$$
$$= \hat{I}_s(t)\angle \theta_{i_s}(t)$$

$$\vec{v}_s(t) = v_a(t)\angle 0^0 + v_b(t)\angle 120^0 + v_c(t)\angle 240^0$$
$$= \hat{V}_s(t)\angle \theta_{v_s}(t)$$



Physical interpretation of $\vec{i}_s(t)$

$$\frac{N_s}{2}\vec{i}_s(t) = \underbrace{\frac{N_s}{2} i_a(t)\angle 0^0}_{\vec{F}_a(t)} + \underbrace{\frac{N_s}{2} i_b(t)\angle 120^0}_{\vec{F}_b(t)} + \underbrace{\frac{N_s}{2} i_c(t)\angle 240^0}_{\vec{F}_c(t)} = \vec{F}_s(t)$$

at time t

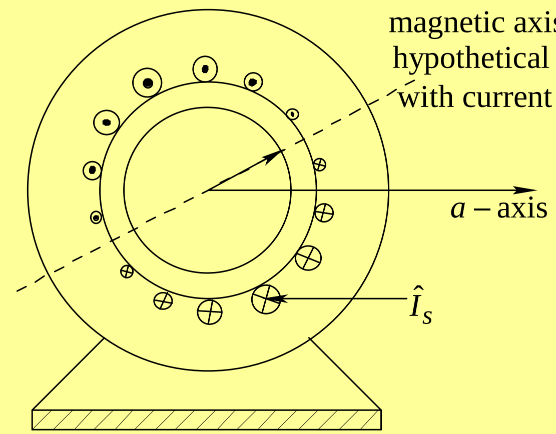
$$\vec{i}_s(t) = \frac{\vec{F}_s(t)}{N_s/2} \Rightarrow \hat{I}_s(t) = \frac{\hat{F}_s(t)}{N_s/2}$$

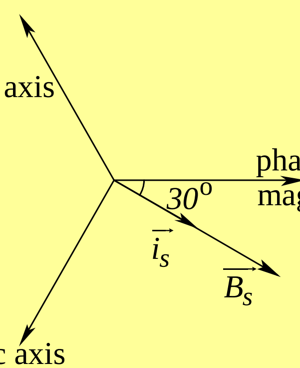
and $\theta_{i_s}(t) = \theta_{F_s}(t)$

$\vec{F}_s(t)$ and $\vec{i}_s(t)$ are collinear

$$\vec{B}_s(t) = \frac{N_s \mu_o}{2 l_g} \vec{i}_s(t)$$

- ☐ Magnetic filed is produced by combined effect of i_a, i_b and i_c but could equivalently be produced by hypothetical winding current $\vec{i}_s(t)$ at θ_{i_s}
- ☐ helps in obtaining expression for torque





Space Vector Components:

Finding Phase Currents from Current Space Vector

$$\mathrm{Re}\left[\vec{i}_s\angle 0^0\right] = i_a + \underbrace{\mathrm{Re}\left[i_b\angle 120^0\right]}_{-\frac{1}{2}i_b} + \underbrace{\mathrm{Re}\left[i_b\angle 240^0\right]}_{-\frac{1}{2}i_c} = \frac{3}{2}i_a$$

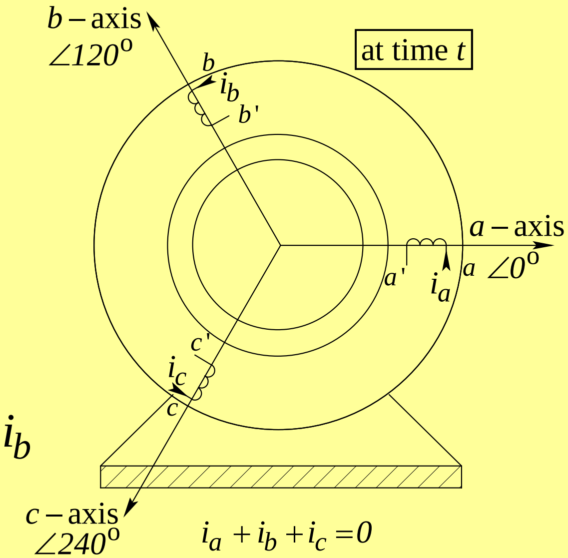
$$\Rightarrow i_a(t) = \frac{2}{3}\mathrm{Re}(\vec{i}_s\angle 0^{\mathrm{o}}) = \frac{2}{3}\hat{I}_s\cos\theta_{i_s}$$

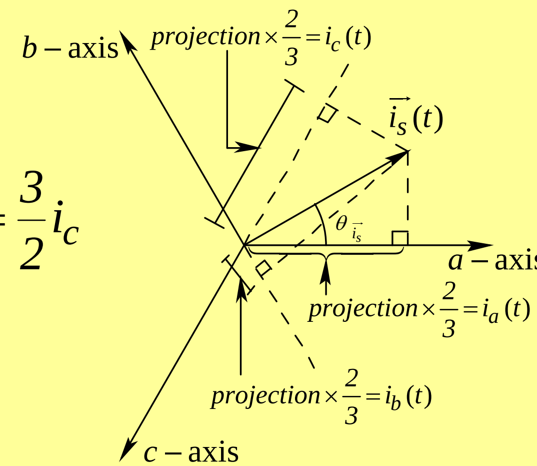
$$\mathrm{Re}\left[\vec{i}_s\angle -120^0\right] = \underbrace{\mathrm{Re}\left[i_a\angle -120^0\right]}_{-\frac{1}{2}i_a} + i_b + \underbrace{\mathrm{Re}\left[i_c\angle 120^0\right]}_{-\frac{1}{2}i_c} = \frac{3}{2}i_b$$

$$\Rightarrow i_b(t) = \frac{2}{3}\mathrm{Re}(\vec{i}_s\angle -120^{\mathrm{o}}) = \frac{2}{3}\hat{I}_s\cos\left(\theta_{i_s} - 120^{\mathrm{o}}\right)$$

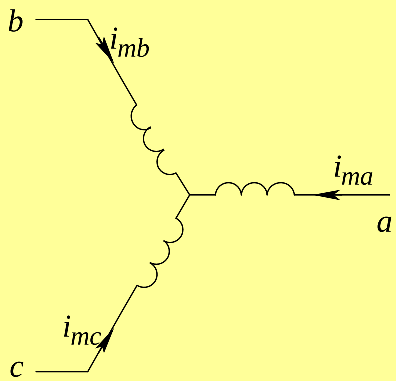
$$\mathrm{Re}\left[\vec{i}_s\angle -240^0\right] = \underbrace{\mathrm{Re}\left[i_a\angle -240^0\right]}_{-\frac{1}{2}i_a} + \underbrace{\mathrm{Re}\left[i_b\angle -240^0\right]}_{-\frac{1}{2}i_b} + i_c = \frac{3}{2}i_c$$

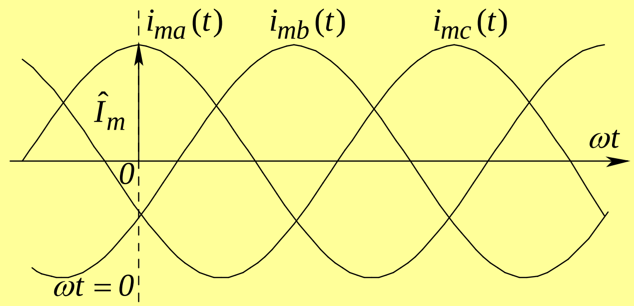
$$\Rightarrow i_c(t) = \frac{2}{3}\mathrm{Re}(\vec{i}_s\angle -240^{\mathrm{o}}) = \frac{2}{3}\hat{I}_s\cos\left(\theta_{i_s} - 240^{\mathrm{o}}\right)$$

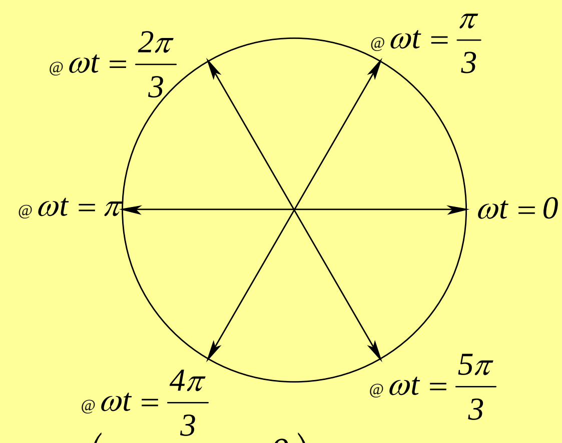




Balanced Sinusoidal Steady-State Excitation (Rotor Open-Circuited)







$$i_{ma} = \hat{I}_m \cos \omega t; \qquad i_{mb} = \hat{I}_m \cos\left(\omega t - 120^0\right); \quad i_{mc} = \hat{I}_m \cos\left(\omega t - 240^0\right)$$

$$\vec{i_{ms}}(t) = \hat{I}_m\left[\cos \omega t \angle 0^0 + \cos(\omega t - 120^0)\angle 120^0 + \cos(\omega t - 240^0)\angle 240^0\right]$$

$$\Rightarrow \vec{i_{ms}}(t) = \hat{I}_{ms}\angle \omega t \qquad \text{where} \quad \hat{I}_{ms} = \frac{3}{2}\hat{I}_m$$

- ☐ Rotating MMF

  & Flux density

- ☐ Constant amplitude

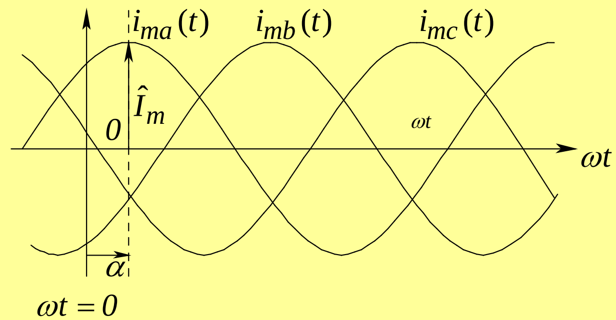
$$\vec{F_{ms}}(t) = \frac{N_s}{2}\vec{i_s}(t) = \hat{F}_{ms}\angle \omega t \qquad \text{where} \quad \hat{F}_{ms} = \frac{3}{2}\frac{N_s}{2}\hat{I}_m = \frac{N_s}{2}\hat{I}_{ms}$$

$$\vec{B_{ms}}(t) = \left(\frac{\mu_o}{\ell_g}\right)\frac{N_s}{2}\vec{i_{ms}}(t)$$

Relation Between Space Vectors and Phasors

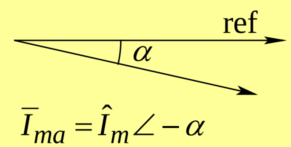
❒ Time domain

$$i_a(t) = \hat{I}_m \cos(\omega t - \alpha)$$



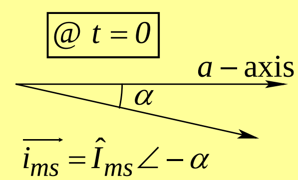
❒ Phasor

$$\bar{I}_a = \hat{I}_m \angle -\alpha$$



❒ Space Vector

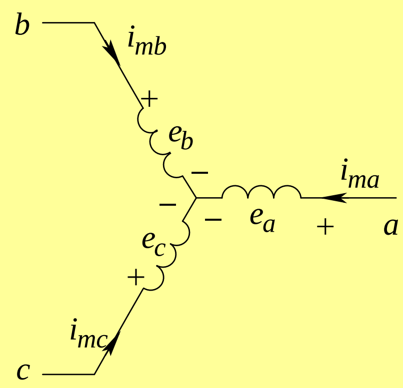
$$\vec{i}_{ms}\Big|_{t=0} = \hat{I}_{ms} \angle -\alpha \quad ; \quad \hat{I}_{ms} = \frac{3}{2} \hat{I}_m$$

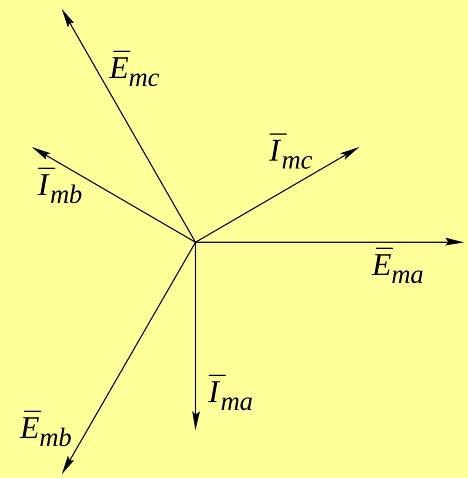


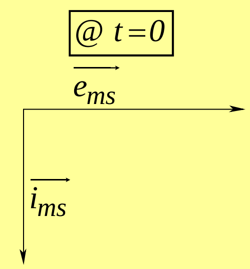
❒ Space Vector $\Leftrightarrow$ phasor

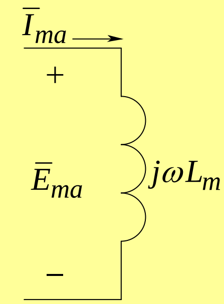
$$\vec{i}_{ms}\Big|_{t=0} \Leftrightarrow \frac{3}{2} \bar{I}_{ma}$$

Voltages in the stator windings









$$e_{ma}(t) = L_m \frac{d}{dt} i_{ma}(t) \quad \text{etc.}$$

Where the three phase magnetizing inductance (2 pole), $L_m = \frac{3}{2}\frac{\pi\mu_o r l}{l_g}\left(\frac{N_s}{2}\right)^2$

$$\Rightarrow \; \vec{e}_{ms}(t) = j\omega L_m \vec{i}_{ms}(t) \; = \; j\omega\left(\frac{3}{2}\pi r l \frac{N_s}{2}\right)\vec{B}_{ms}(t)$$

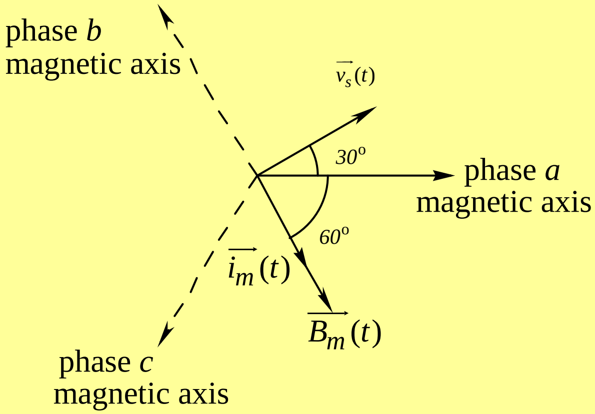
Example

$$v_a(t) = 120\sqrt{2}\cos\omega t$$

$$v_b(t) = 120\sqrt{2}\cos(\omega t - 120^\circ)$$

$$v_c(t) = 120\sqrt{2}\cos(\omega t - 240^\circ)$$

$$\vec{v}_s = \frac{3}{2} \times 120\sqrt{2}\angle 30^0 = 254.56\angle 30^0 \ V$$



$$\overrightarrow{i_{ms}} = \frac{\vec{v}_s}{j\omega L_m} = \frac{254.56\angle(30^0 - 90^0)}{2\pi \times 60 \times 0.777} = 0.869\angle -60^0 \ A$$

$$\overrightarrow{B_{ms}} = \frac{\mu_o N_s \vec{i}_{ms}}{2\ell_g} = \frac{4\pi \times 10^{-7} \times 50 \times 0.869\angle -60^0}{10^{-3}} = 0.055\angle -60^0 \ Wb/m^2$$

Summary

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