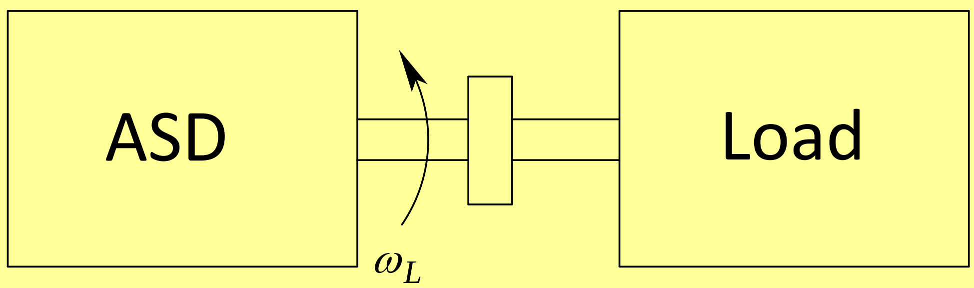
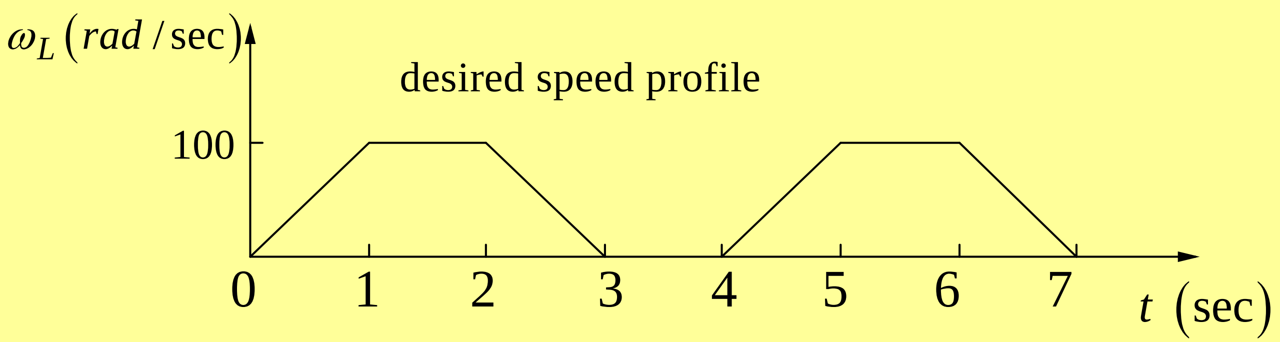
Understanding Mechanical System Requirements

- Torque to meet mechanical System Requirement
- System with Linear Motion
- System with Rotary Motion
- Moment of Inertia
- Acceleration, Speed and Position
- Frictional Torque and Torsional Resonances

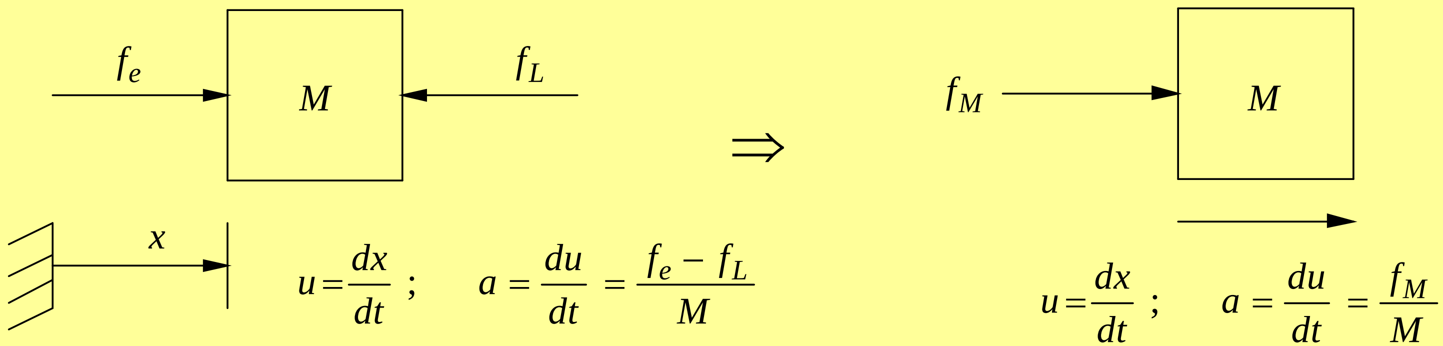
Requirement

- How can the ASD accelerate and decelerate the load to give desired speed profile





Systems With Linear Motion



$$u = \frac{dx}{dt}\;; \qquad a = \frac{du}{dt} = \frac{f_e - f_L}{M}$$

$$u = \frac{dx}{dt}\;; \qquad a = \frac{du}{dt} = \frac{f_M}{M}$$

- Figure on left includes load force, f_L , that must be overcome
- Figure on right shows only the force, f_M , available to accelerate the mass, *M*

Accelaration

$$a = \frac{f_e - f_L}{M} = \frac{f_M}{M}$$

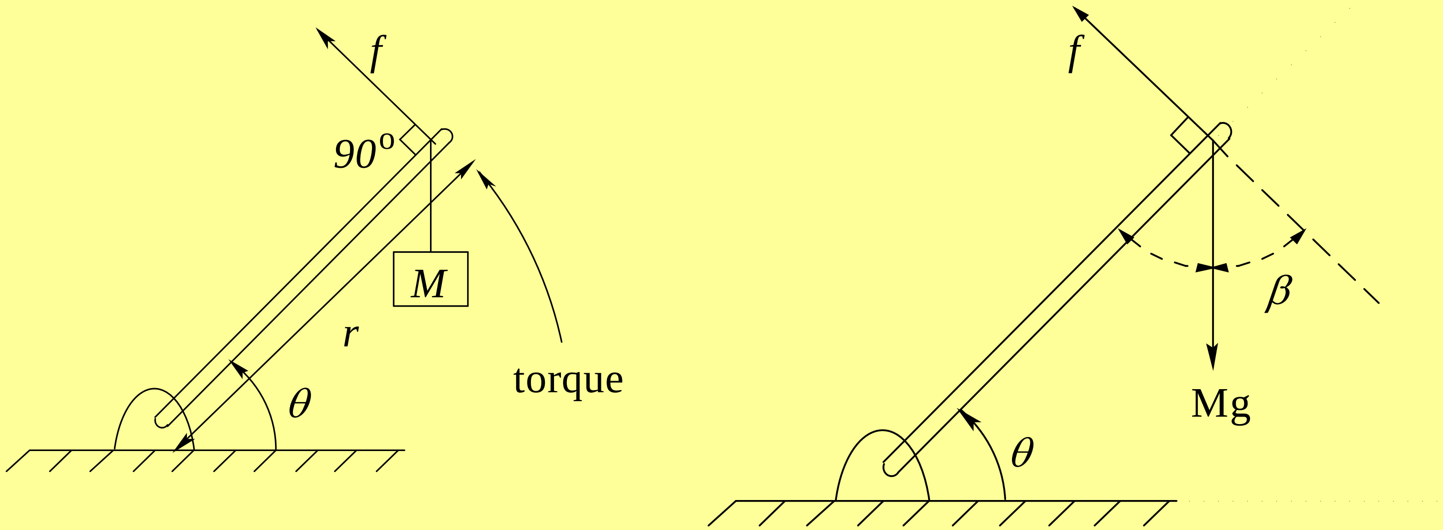
Power Input

$$P_e(t) = f_e \cdot u = f_M \cdot u + f_L \cdot u$$

Kinetic energy

$$W_M = \frac{1}{2} M u^2$$

Rotating Systems

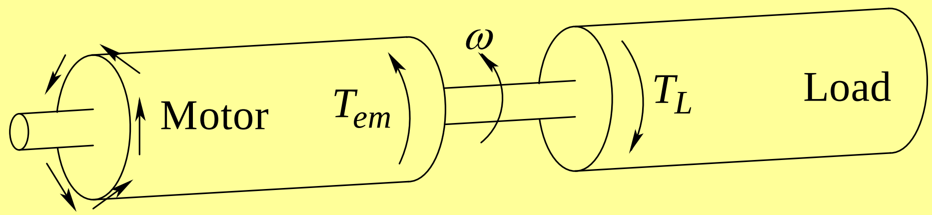


- Torque = force radius

  $[Nm]$ $[N]$ $[m]$

- Example: what torque is needed to hold *M* motionless

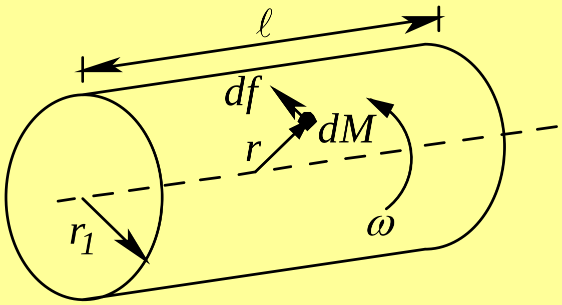
☐ Torque in an electric drive

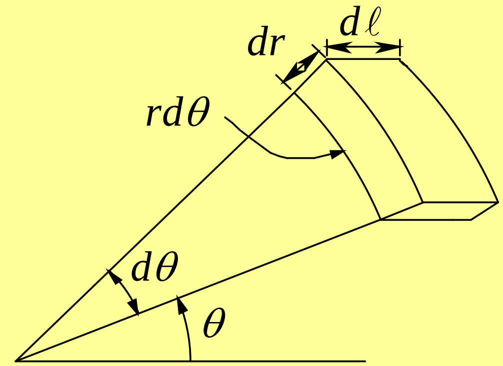


- T_{em} electromagnetic torque produced by motor
- T_{em} is opposed by load torque, T_L
- The difference, $T_{em} - T_L = T_J$, will accelerate the system
- $$\frac{d\omega}{dt} = \frac{T_{em} - T_L}{J} = \frac{T_J}{J}$$

where *J* is the moment of inertia

Calculation of Moment of Inertia *J* of a Uniform Cylinder





$$dM = \rho \underbrace{rd\theta}_{arc} \; \underbrace{dr}_{height} \; \underbrace{d\ell}_{length}$$

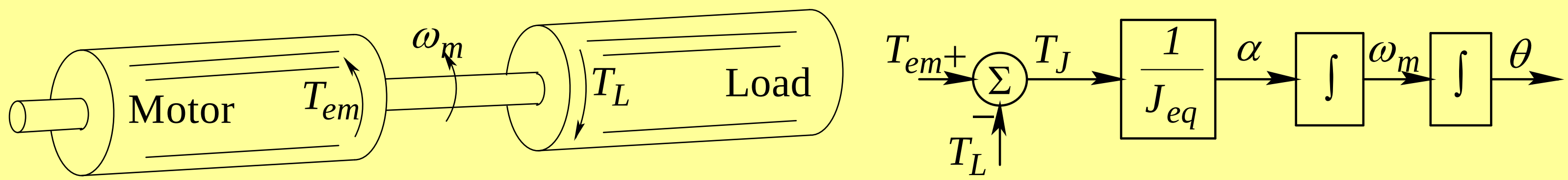
$$d\,f = dM \frac{d}{dt} v$$

$$\Rightarrow dT = r^2\, dM \frac{d}{dt}\omega \; = \rho\,(\,r^3 dr\, d\theta\, d\ell\,)\frac{d}{dt}\omega$$

$$T = \rho\,(\int_0^{r_1} r^3 dr \int_0^{2\pi} d\theta \int_0^{\ell} d\ell\,)\frac{d}{dt}\omega \; = \underbrace{(\frac{\pi}{2}\rho\,\ell\, r_1^4\,)}_{J}\frac{d}{dt}\omega$$

$$J_{solid} = \frac{\pi}{2}\rho\,\ell\, r_1^4 = \frac{1}{2} M\, r_1^2$$

Accelaration, Speed and Position, Power and Energy



$$\text{acceleration}, \quad \alpha = \frac{d\omega_m}{dt} = \frac{1}{(J_m + J_L)}(T_{em} - T_L) = \frac{T_J}{J_{eq}}$$

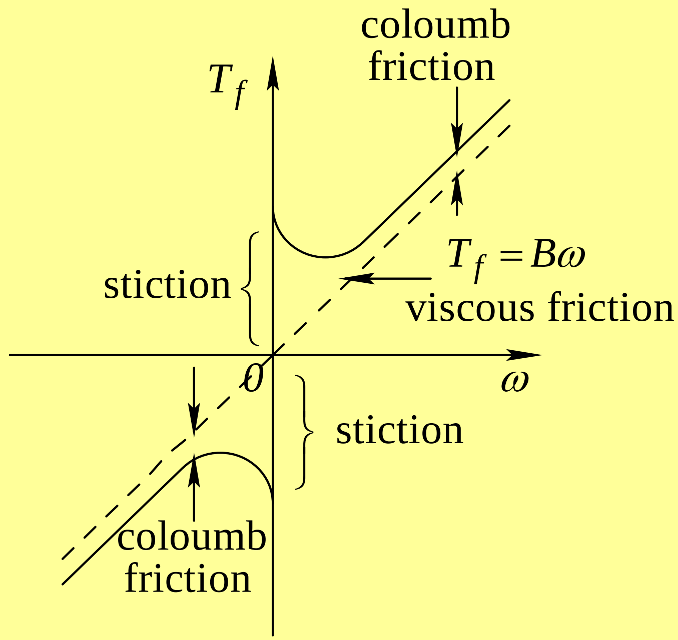
$$\Rightarrow \text{speed}, \quad \omega_m(t) = \omega_m(0) + \int_0^t \alpha(\tau)\, d\tau$$

$$\Rightarrow \text{position}, \quad \theta(t) = \theta(0) + \int_0^t \omega(\tau)\, d\tau$$

$$\text{Power} \quad P_{em} = T_{em} \cdot \omega_m \;\; ; \quad P_L = T_L \cdot \omega_m$$

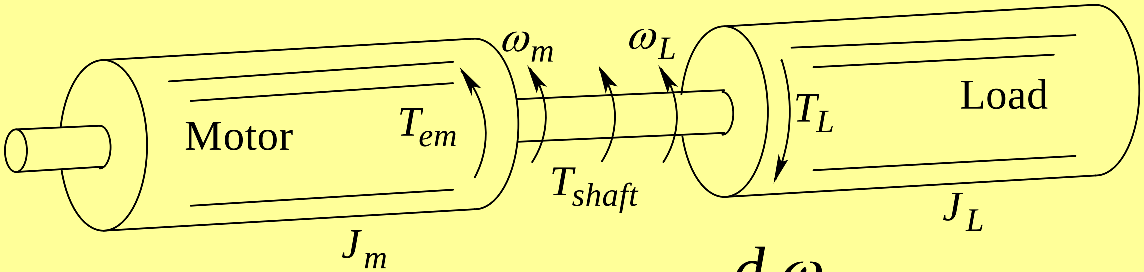
$$\text{Kinetic Energy} \quad W = \frac{1}{2} J \omega^2$$

Frictional Torque



- Stiction: static component
- Coulomb friction: dynamic component
- Viscous friction: speed dependent
- In general, friction is non-linear

Torsional Resonances



At motor end $T_{shaft} = T_{em} - J_m \dfrac{d\,\omega_m}{dt}$

At load end $T_{shaft} = T_L + J_L \dfrac{d\,\omega_L}{dt}$

$$(\theta_m - \theta_L) = \frac{T_{shaft}}{K}$$

θ_m and θ_L : angular displacement at the two ends of the shaft

- If $K \to \infty$, $\theta_m = \theta_L$

  (J_M and J_L can be treated as one inertial mass)

- Finite K may lead to resonances

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